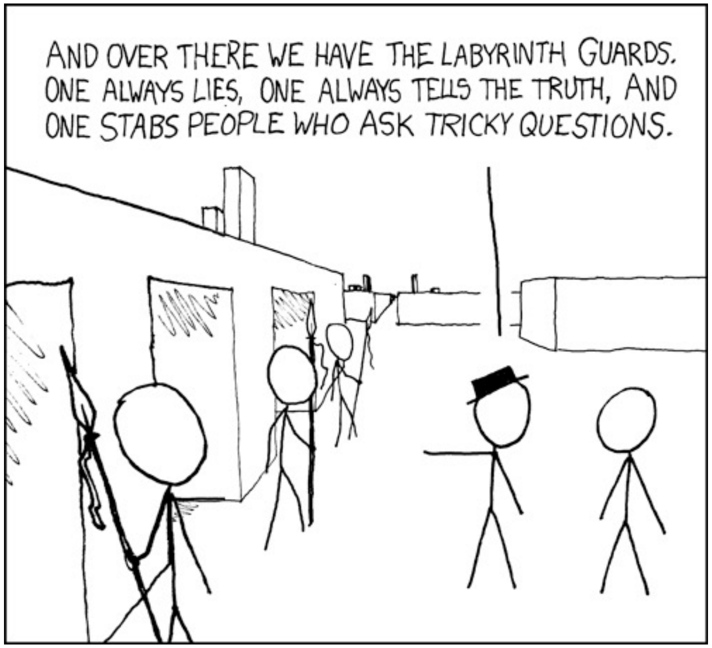
CSE 311: Foundations of Computing

Lecture 3: Digital Circuits & Equivalence



Review: Propositional Logic

Propositions

- atomic propositions are T/F-valued variables
- combined using logical connectives (not, and, or, etc.)
- can be described by a truth table

  shows the truth value of the proposition in

  each combination of truth values of the atomic propositions

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Applications

- understanding complex English sentences
- modeling the input/output behavior of circuits
- (more to come)

Last class: Logical Equivalence $A \equiv B$

$A \equiv B$ is an assertion that ***two propositions*** A and B always have the same truth values.

$$p \wedge r \equiv r \wedge p$$

p	r	$p \wedge r$	$r \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Last class: Logical Equivalence $A \equiv B$

$A = B$ means A and B are identical "strings":

- $p \wedge r = p \wedge r$

- $p \wedge r \neq r \wedge p$

$A \equiv B$ means A and B have identical truth values:

- $p \wedge r \equiv p \wedge r$

- $p \wedge r \equiv r \wedge p$

- $p \wedge r \not\equiv r \vee p$

Last class: Logical Equivalence $A \equiv B$

$A \equiv B$ is an assertion that ***two propositions*** A and B always have the same truth values.

tautology

$A \equiv B$ and $\underbrace{(A \leftrightarrow B)} \equiv T$ have the same meaning.

$p \wedge r \equiv r \wedge p$

p	r	$p \wedge r$	$r \wedge p$	$(p \wedge r) \leftrightarrow (r \wedge p)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

$p \wedge r \not\equiv r \vee p$

When p=T and r=F, $p \wedge r$ is false, but $r \vee p$ is true

De Morgan's Laws

$$\neg(p \wedge r) \equiv \neg p \vee \neg r$$
$$\neg(p \vee r) \equiv \neg p \wedge \neg r$$

Negate the statement:

"My code compiles or there is a bug."

To negate the statement,

ask "when is the original statement false".

It's false when not(my code compiles) AND not(there is a bug).

Translating back into English, we get:

My code doesn't compile and there is not a bug.

De Morgan's Laws

Example: $\neg(p \wedge r) \equiv \neg p \vee \neg r$

p	r	$\neg p$	$\neg r$	$\neg p \vee \neg r$	$p \wedge r$	$\neg(p \wedge r)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

De Morgan's Laws

$$\neg(p \wedge r) \equiv \neg p \vee \neg r$$

$$\neg(p \vee r) \equiv \neg p \wedge \neg r$$

```
if (!(front != null && value > front.data)) {
    front = new ListNode(value, front);
} else {
    ListNode current = front;
    while (current.next != null && current.next.data < value))
        current = current.next;
    current.next = new ListNode(value, current.next);
}
```

De Morgan's Laws

$$\neg(p \wedge r) \equiv \neg p \vee \neg r$$
$$\neg(p \vee r) \equiv \neg p \wedge \neg r$$

```
!(front != null && value > front.data)
```

$$\equiv$$

```
front == null || value <= front.data
```

You've been using these for a while!

Law of Implication

$$p \rightarrow r \equiv \neg p \vee r$$

p	r	$p \rightarrow r$	$\neg p$	$\neg p \vee r$
T	T			
T	F			
F	T			
F	F			

Law of Implication

$$p \to r \equiv \neg p \vee r$$

p	r	$p \to r$	$\neg p$	$\neg p \vee r$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Some Familiar Properties of Arithmetic

- $x + y = y + x$ **(Commutativity)**
- $x \cdot (y + z) = x \cdot y + x \cdot z$ **(Distributivity)**
- $(x + y) + z = x + (y + z)$ **(Associativity)**

Important Equivalences

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

Some Familiar Properties of Arithmetic

- $x \cdot 1 = x$ (Identity)
- $x + 0 = x$

- $x \cdot 0 = 0$ (Domination)

Important Equivalences

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

Some Familiar Properties of Arithmetic

- Usual properties hold under relabeling:
  - 0, 1 becomes F, T
  - "+" becomes "$\vee$"
  - " $\cdot$ " becomes "$\wedge$"
- But there are some new facts:
  - Distributivity works for both "$\wedge$" and "$\vee$"
  - Domination works with T
- There are some other facts specific to logic…

Important Equivalences

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

Important Equivalences

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

Using Equivalences

- Note that p, q, and r can be **any** propositions (not just atomic propositions)

- Ex: $(r \rightarrow s) \wedge (\neg t) \equiv (\neg t) \wedge (r \rightarrow s)$

  – apply commutativity: $p \wedge q \equiv q \wedge p$
  with $p = r \rightarrow s$
  and $q = \neg t$

One more easy equivalence

Double Negation

$$p \equiv \neg\neg p$$

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

Understanding logic and circuits

When do two logic formulas mean the same thing?

When do two circuits compute the same function?

What logical properties can we infer from other ones?

Basic rules of reasoning and logic

- Working with logical formulas
  - Simplification
  - Testing for equivalence

- Applications
  - Query optimization
  - Search optimization and caching
  - Artificial Intelligence
  - Program verification

Computing Equivalence

Given two propositions, can we write an algorithm to determine if they are equivalent?

What is the runtime of our algorithm?

Computing Equivalence

Given two propositions, can we write an algorithm to determine if they are equivalent?

Yes! Generate the truth tables for both propositions and check if they are the same for every entry.

What is the runtime of our algorithm?

Every atomic proposition has two possibilities (T, F). If there are n atomic propositions, there are 2^n rows in the truth table.

Another approach: Logical Proofs

To show A is equivalent to B

- Apply a series of logical equivalences to sub-expressions to convert A to B

To show A is a tautology

- Apply a series of logical equivalences to sub-expressions to convert A to **T**

Another approach: Logical Proofs

To show A is equivalent to B

– Apply a series of logical equivalences to sub-expressions to convert A to B

Example:

Let A be "$p \vee (p \wedge p)$", and B be "p".

Our general proof looks like:

$$p \vee (p \wedge p) \equiv (\qquad\qquad\qquad\qquad)$$
$$\equiv p$$

Another approach: Logical Proofs

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Law of Implication

$$p \rightarrow q \equiv \neg p \vee q$$

Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Double Negation

$$p \equiv \neg\neg p$$

Example:

Let A be "$p \vee (p \wedge p)$", and B be "p".

Our general proof looks like:

$$p \vee (p \wedge p) \equiv (\qquad\qquad\qquad\qquad)$$
$$\equiv p$$

Logical Proofs

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
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Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Double Negation

$$p \equiv \neg\neg p$$

Example:

Let A be "$p \vee (p \wedge p)$", and B be "p".

Our general proof looks like:

$p \vee (p \wedge p)$	$\equiv (\quad p \vee p \quad)$	Idempotent
	$\equiv p$	Idempotent

Logical Proofs

To show A is a tautology

– Apply a series of logical equivalences to sub-expressions to convert A to **T**

Example:

Let A be "$\neg p \vee (p \vee p)$".

Our general proof looks like:

$$\begin{aligned} \neg p \vee (p \vee p) &\equiv (\qquad\qquad\qquad) \\ &\equiv (\qquad\qquad\qquad) \\ &\equiv \mathbf{T} \end{aligned}$$

Logical Proofs

- Identity
  - $p \land \mathrm{T} \equiv p$
  - $p \lor \mathrm{F} \equiv p$
- Domination
  - $p \lor \mathrm{T} \equiv \mathrm{T}$
  - $p \land \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \lor p \equiv p$
  - $p \land p \equiv p$
- Commutative
  - $p \lor q \equiv q \lor p$
  - $p \land q \equiv q \land p$

- Associative
  - $(p \lor q) \lor r \equiv p \lor (q \lor r)$
  - $(p \land q) \land r \equiv p \land (q \land r)$
- Distributive
  - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
  - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
- Absorption
  - $p \lor (p \land q) \equiv p$
  - $p \land (p \lor q) \equiv p$
- Negation
  - $p \lor \neg p \equiv \mathrm{T}$
  - $p \land \neg p \equiv \mathrm{F}$

De Morgan's Laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$

Law of Implication

$$p \to q \equiv \neg p \lor q$$

Contrapositive

$$p \to q \equiv \neg q \to \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

Double Negation

$$p \equiv \neg\neg p$$

Example:

Let A be "$\neg p \lor (p \lor p)$".

Our general proof looks like:

$$\neg p \lor (p \lor p) \equiv (\qquad\qquad\qquad\qquad)$$
$$\equiv (\qquad\qquad\qquad\qquad)$$
$$\equiv \mathbf{T}$$

Logical Proofs

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Law of Implication

$$p \rightarrow q \equiv \neg p \vee q$$

Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Double Negation

$$p \equiv \neg\neg p$$

Example:

Let A be "$\neg p \vee (p \vee p)$".

Our general proof looks like:

$$\neg p \vee (p \vee p) \equiv (\quad \neg p \vee p \quad) \quad \text{Idempotent}$$
$$\equiv (\quad p \vee \neg p \quad) \quad \text{Commutative}$$
$$\equiv \mathbf{T} \quad \text{Negation}$$

Prove these propositions are equivalent: Option 1

Prove: $p \wedge (p \rightarrow r) \equiv p \wedge r$

Make a Truth Table and show:

$$(p \wedge (p \rightarrow r)) \leftrightarrow (p \wedge r) \equiv \mathbf{T}$$

p	r	$p \rightarrow r$	$(p \wedge (p \rightarrow r))$	$p \wedge r$	$(p \wedge (p \rightarrow r)) \leftrightarrow (p \wedge r)$
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	F	F	T

Prove these propositions are equivalent: Option 2

Prove: $p \wedge (p \rightarrow r) \equiv p \wedge r$

$$p \wedge (p \rightarrow r) \equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv p \wedge r$$

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Law of Implication

$$p \rightarrow q \equiv \neg p \vee q$$

Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Double Negation

$$p \equiv \neg \neg p$$

Prove these propositions are equivalent: Option 2

Prove: $p \wedge (p \rightarrow r) \equiv p \wedge r$

$p \wedge (p \rightarrow r) \equiv p \wedge (\neg p \vee r)$	Law of Implication
$\equiv (p \wedge \neg p) \vee (p \wedge r)$	Distributive
$\equiv \mathbf{F} \vee (p \wedge r)$	Negation
$\equiv (p \wedge r) \vee \mathbf{F}$	Commutative
$\equiv p \wedge r$	Identity

- **Identity**
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- **Domination**
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- **Idempotent**
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- **Commutative**
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- **Associative**
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive**
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- **Absorption**
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- **Negation**
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Law of Implication

$$p \rightarrow q \equiv \neg p \vee q$$

Contrapositive

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Double Negation

$$p \equiv \neg\neg p$$

Prove this is a Tautology: Option 1

$$(p \wedge r) \rightarrow (r \vee p)$$

Make a Truth Table and show:

$$(p \wedge r) \rightarrow (r \vee p) \equiv \mathbf{T}$$

$\boldsymbol{p}$	$\boldsymbol{r}$	$\boldsymbol{p} \wedge \boldsymbol{r}$	$\boldsymbol{r} \vee \boldsymbol{p}$	$(\boldsymbol{p} \wedge \boldsymbol{r}) \rightarrow (\boldsymbol{r} \vee \boldsymbol{p})$
T	**T**			
T	**F**			
F	**T**			
F	**F**			

Prove this is a Tautology: Option 1

$$(p \wedge r) \rightarrow (r \vee p)$$

Make a Truth Table and show:

$$(p \wedge r) \rightarrow (r \vee p) \equiv \mathbf{T}$$

$\boldsymbol{p}$	$\boldsymbol{r}$	$\boldsymbol{p \wedge r}$	$\boldsymbol{r \vee p}$	$\boldsymbol{(p \wedge r) \rightarrow (r \vee p)}$
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**

Prove this is a Tautology: Option 2

$$(p \wedge r) \rightarrow (r \vee p)$$

Use a series of equivalences like so:

$$(p \wedge r) \rightarrow (r \vee p) \equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv$$
$$\equiv \text{T}$$

Identity
- $p \wedge \text{T} \equiv p$
- $p \vee \text{F} \equiv p$

Domination
- $p \vee \text{T} \equiv \text{T}$
- $p \wedge \text{F} \equiv \text{F}$

Idempotent
- $p \vee p \equiv p$
- $p \wedge p \equiv p$

Commutative
- $p \vee q \equiv q \vee p$
- $p \wedge q \equiv q \wedge p$

Associative
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Distributive
- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption
- $p \vee (p \wedge q) \equiv p$
- $p \wedge (p \vee q) \equiv p$

Negation
- $p \vee \neg p \equiv \text{T}$
- $p \wedge \neg p \equiv \text{F}$

Prove this is a Tautology: Option 2

$$(p \wedge r) \rightarrow (r \vee p)$$

Use a series of equivalences like so:

$(p \wedge r) \rightarrow (r \vee p)$	$\equiv \neg(p \wedge r) \vee (r \vee p)$	Law of Implication
	$\equiv (\neg p \vee \neg r) \vee (r \vee p)$	De Morgan
	$\equiv \neg p \vee (\neg r \vee (r \vee p))$	Associative
	$\equiv \neg p \vee ((\neg r \vee r) \vee p)$	Associative
	$\equiv \neg p \vee (p \vee (\neg r \vee r))$	Commutative
	$\equiv (\neg p \vee p) \vee (\neg r \vee r)$	Associative
	$\equiv (p \vee \neg p) \vee (r \vee \neg r)$	Commutative (twice)
	$\equiv \mathbf{T} \vee \mathbf{T}$	Negation (twice)
	$\equiv \mathbf{T}$	Domination/Identity

Associative
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Distributive
- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption
- $p \vee (p \wedge q) \equiv p$
- $p \wedge (p \vee q) \equiv p$

Negation
- $p \vee \neg p \equiv \mathrm{T}$
- $p \wedge \neg p \equiv \mathrm{F}$

Identity
- $p \wedge \mathrm{T} \equiv p$
- $p \vee \mathrm{F} \equiv p$

Domination
- $p \vee \mathrm{T} \equiv \mathrm{T}$
- $p \wedge \mathrm{F} \equiv \mathrm{F}$

Idempotent
- $p \vee p \equiv p$
- $p \wedge p \equiv p$

Commutative
- $p \vee q \equiv q \vee p$
- $p \wedge q \equiv q \wedge p$

Logical Proofs of Equivalence/Tautology

- Not smaller than truth tables when there are only a few propositional variables...

- ...but usually ***much shorter*** than truth table proofs when there are many propositional variables

- A big advantage will be that we can extend them to a more in-depth understanding of logic for which truth tables don't apply.