CSE 311: Foundations of Computing I

Practice Final Exam

Name:

ID #:

TA: **Section:**

INSTRUCTIONS:

- You have **110 minutes** to complete the exam.
- The exam is closed book. You may not use cell phones or calculators.
- All answers you want graded should be written on the exam paper.
- If you need extra space, use the back of a page. Make sure to mention that you did though.
- The problems are of varying difficulty.
- If you get stuck on a problem, move on and come back to it later.

Problem	Points	Score	Problem	Points	Score
1	15		5	15	
2	15		6	15	
3	15		7	15	
4	15		8	15	
			Σ	120	

Basic Techniques.

This part will test your ability to apply techniques that have been explicitly identified in lecture and reinforced through sections and homeworks. Remember to show your work and justify your claims.

1. Regularly Irregular [15 points]

Let $\Sigma = \{0, 1\}$. Prove that the language $L = \{x \in \Sigma^* : \#_0(x) < \#_1(x)\}$ is irregular.

2. Recurrences, Recurrences [15 points]

Define

$$T(n) = \begin{cases} n & \text{if } n = 0, 1 \\ 4T\left(\lfloor \frac{n}{2} \rfloor\right) + n & \text{otherwise} \end{cases}$$

Prove that $T(n) \leq n^3$ for $n \geq 3$.

3. All The Machines! [15 points]

Let $\Sigma = \{0, 1, 2\}$.

Consider $L = \{w \in \Sigma^*$: Every 1 in the string has at least one 0 before and after it$\}$.

(a) (5 points) Give a regular expression that represents A.

(b) (5 points) Give a DFA that recognizes A.

(c) (5 points) Give a CFG that generates A.

4. Structural CFGs [15 points]

Consider the following CFG: **S** $\rightarrow \varepsilon$ | **SS** | **S**1 | **S**01. Another way of writing the recursive definition of this set, Q, is as follows:

- $\varepsilon \in Q$
- If $S \in Q$, then $S1 \in Q$ and $S01 \in Q$
- If $S, T \in Q$, then $ST \in Q$.

Prove, by structural induction that if $w \in Q$, then w has at least as many 1's as 0's.

5. Tralse! [15 points]
For each of the following answer True or False and give a short explanation of your answer.

(a) (3 points)

True **or** False	Any subset of a regular language is also regular.

(b) (3 points)

True **or** False	The set of programs that loop forever on at least one input is decidable.

(c) (3 points)

True **or** False	If $\mathbb{R} \subseteq A$ for some set A, then A is uncountable.

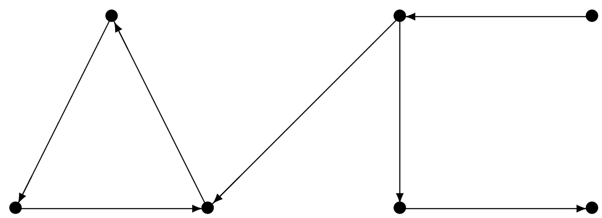
(d) (3 points)

True **or** False	If the domain of discourse is people, the logical statement $$\exists x\ (P(x) \to \forall y\ (x \neq y \to \neg P(y))$$ can be correctly translated as "There exists a unique person who has property P".

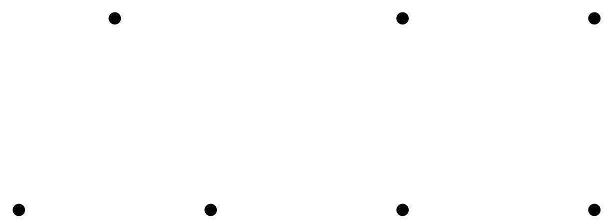
(e) (3 points)

True **or** False	$\exists x\ (\forall y\ P(x, y)) \to \forall y\ (\exists x\ P(x, y))$ is true regardless of what predicate P is.

6. Relationships! [15 points]
The following is the graph of a binary relation R.

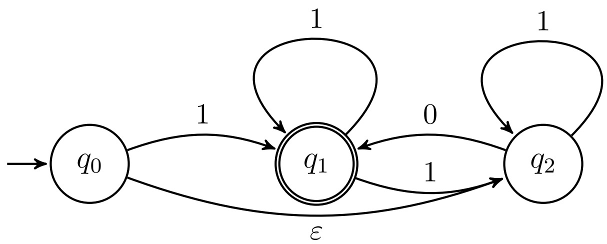


(a) (5 points) Draw the transitive-reflexive closure of R.



(b) (10 points) Let $S = \{(X, Y) : X, Y \in \mathcal{P}(\mathbb{N}) \wedge X \subseteq Y\}$.
Recall that R is antisymmetric iff $((a, b) \in R \wedge a \neq b) \rightarrow (b, a) \notin R$.
Prove that S is antisymmetric.

7. Construction Paper! [15 points]
Convert the following NFA into a DFA using the algorithm from lecture.



8. Modern DFAs [15 points]

Let $\Sigma = \{0, 1, 2\}$. Construct a DFA that recognizes exactly strings with a 0 in all positions i where i mod $3 = 0$.