Section 09: CFGs, Relations, DFAs, NFAs, and Minimization

1. CFGs

(a) All binary strings that end in 00.

(b) All binary strings that contain at least three 1's.

(c) All binary strings with an equal number of 1's and 0's.

2. Relations

(a) Draw the transitive-reflexive closure of $\{(1,2),(2,3),(3,4)\}$.

(b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.

(c) Consider the relation $R = \{(x,y) \ : \ x = y+1\}$ on $\mathbb{N}$. Is R reflexive? Transitive? Symmetric? Anti-symmetric?

(d) Consider the relation $S = \{(x,y) \ : \ x^2 = y^2\}$ on $\mathbb{R}$. Prove that S is reflexive, transitive, and symmetric.

3. DFAs, Stage 1

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0,1,2,3\}$.

(a) All binary strings.

(b) All strings whose digits sum to an even number.

(c) All strings whose digits sum to an odd number.

4. DFAs, Stage 2

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0,1\}$.

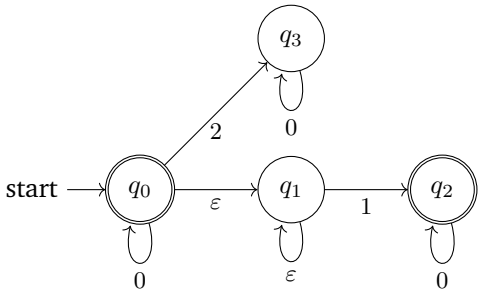
(a) All strings which do not contain the substring 101.

(b) All strings containing at least two 0's and at most one 1.

(c) All strings containing an even number of 1's and an odd number of 0's and not containing the substring 10.

5. NFAs

(a) What language does the following NFA accept?



(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

6. DFAs & Minimization

Note: We will not test you on minimization, although you may optionally read the extra slides and do this problem for fun

(a) Convert the NFA from 1a to a DFA, then minimize it.

(b) Minimize the following DFA: