

Section 01: Solutions

1. Warm-Up

Translate the English sentences below into symbolic logic.

- (a) If I am lifting weights this afternoon, then I do a warm-up exercise.

Solution:

Since we're in "if...then..." form, the sentence is an implication.

p : I am lifting weights

q : I do a warm-up exercise

$$p \rightarrow q$$

- (b) If I am cold and going to bed or I am two-years old, then I carry a blanket. **Solution:**

p : I am cold

q : I am going to bed

r : I am two-years old

s : I carry a blanket

$$[(p \wedge q) \vee r] \rightarrow s$$

How did we know the translation wasn't $[p \wedge (q \vee r)] \rightarrow s$? Two hints were available: first, omitted words ("going to bed" instead of "I am going to bed" indicates q should be closer to the "and" than the "or"), second the interpretation of the sentence – two-year olds more commonly carry blankets during the day than warm adults.

2. If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

- (a) whenever I walk my dog, I make new friends. **Solution:**

p : I walk my dog

q : I make new friends

$$p \rightarrow q$$

.

The promise is that we will definitely make new friends on the condition of walking our dog.

- (b) I will drink coffee, if Starbucks is open or my coffeemaker works.

Solution:

p : I will drink coffee

q : Starbucks is open

r : my coffeemaker works

$$(q \vee r) \rightarrow p$$

- (c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote. **Solution:**

p : One is a U.S. citizen q : One is over 18 r : One is eligible to vote

$$(p \wedge q) \rightarrow r$$

The original sentence omits a subject. We introduced a dummy subject “one” to the propositions, you might have said “someone” or “a person” instead (among other options).

- (d) I can go home only if I have finished my homework. **Solution:**

p : I can go home. q : I have finished my homework.

$$p \rightarrow q$$

The promise here is that if I can go home then I must have finished my homework. It can sometimes help to imagine when the sentence is broken. Is it broken if my homework is finished, but I cannot go home? No, perhaps I also have to say bye to my friends before I leave. But if I can go home with unfinished homework, then the promise is broken.

“Only if” is one of the more confusing arrangements – the consequence (“the then part”) is adjacent to the “only if.”

- (e) Having an internet connection is necessary to log onto zoom. **Solution:**

p : One has an internet connection

q : One can log onto zoom

$$q \rightarrow p$$

The internet connection is not enough (what if you don’t have the meeting link?) but certainly if you are in the meeting then you have a connection.

3. I can rewrite these formulas in English, only if..

Given propositions and a logical formula, write **two** potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

- (a) p : The sun is out
 q : We have class outside

$$p \rightarrow q$$

Solution:

If the sun is out, then we have class outside.
Whenever the sun is out, we have class outside.

- (b) p : the book has been out for a week.
 q : I don't have homework.
 r : I have finished reading the book.

$$(p \wedge q) \rightarrow r$$

Solution:

I have finished reading the book, if it has been out for a week and I don't have homework.
The book being out for a week and me not having homework is sufficient for me to have finished reading the book.

- (c) p : I have read the manual
 q : I operate the machine

$$q \rightarrow p$$

Solution:

I operate the machine only if I have read the manual.
Operating the machine implies that I have read the manual.

4. Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

- (a) I will remember to send you the address only if you send me an e-mail message. **Solution:**

p : I will remember to send you the address
 q : You send me an e-mail message

$$p \rightarrow q$$

- (b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area. **Solution:**

p : Berries are ripe along the trail
 q : Hiking is safe
 r : Grizzly bears have not been seen in the area

$$p \rightarrow (q \leftrightarrow r)$$

- (c) Unless I am trying to type something, my cat is either eating or sleeping. **Solution:**

p : My cat is eating
 q : My cat is sleeping
 r : I'm trying to type

$$\neg r \rightarrow (p \oplus q)$$

5. Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

- (a) Define propositional variables and translate the sentence into an expression in logical notation. **Solution:**

p : I am drinking tea
 q : I am eating a cookie

$$(p \rightarrow q) \vee (q \rightarrow p)$$

- (b) Fill out a truth table for your expression. **Solution:**

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

6. Exclusive Or

Exclusive or (\oplus) and inclusive or (\vee) both can be translated as “or” in English. For each of the following ambiguous phrases, decide which type of “or” is likely meant and why.

- (a) Experience with C or Java is required. **Solution:**

Inclusive or. Experience with both is usually not a bad thing.

- (b) Lunch includes soup or salad. **Solution:**

Exclusive or. Most restaurants charge you more for both.

- (c) Publish or perish. **Solution:**

This phrase is a common one among researchers – it means publish papers or your career will perish. Exclusive or is meant; i.e. if you do indeed publish you should avoid the loss of your career.

(d) To enter the country, you need a passport or voter registration card. **Solution:**

Inclusive or – if you have both, they won't kick you out!