Plan for Today

• Continue with auctions
  • Sponsored Search
  • The VCG auction
  • The FCC Incentive Auction
Last time

$v_i, v_2, \ldots$ are private values

1\textsuperscript{st} price auction: winner is highest bidder
winner pays her bid

2\textsuperscript{nd} price auction: winner is highest bidder
winner pays 2\textsuperscript{nd} highest bid

How to bid?
Model: each bidder bids to maximize (expected) utility, given beliefs about others

$$u_i = v_i \cdot \text{revenue} - p_i \cdot \text{payment}$$

Vickrey auction

2\textsuperscript{nd} price auction truthful
also called "strategy proof"
"dominant strategy incentive compatible" DSIC

$\implies$ it is a dominant strategy to bid your value
also "individually rational" IR
\[ \text{bidder's utility guaranteed to be nonnegative} \]
maximizes "welfare"
also called "surplus"
\[ \text{total happiness of all participants.} \]
\[ \text{sum of utilities} \]
\[ = \sum_{i=1}^{n} \left( v_i \cdot \mathbf{1}_{i \text{ wins}} - p_i \right) + \sum_{i=1}^{n} p_i \]
utility of bidder \( i \)
utility of auctioneer
\[ = \sum_{i=1}^{n} v_i \cdot \mathbf{1}_{i \text{ wins}} \]
1st price auction: no dominant strategies analyzed by assuming each player's value was drawn from distribution $F$.

Showed that if $F$ is Uniform $[0, 100]$ then $\beta(v) = \frac{v}{2}$ is a Bayes-Nash equilibrium for $n=2$.

$$E[\text{utility of bidding } \beta(v) \mid \text{other player bids } \beta(v_a)]$$

$$> E[\text{utility of bidding } v \mid \text{other player bids } \beta(v_a)]$$

Revenue Equivalence Thm

If $V_i \sim F \forall i \text{ s.t. in equilibrium item allocated to bidder of highest value and payment of a player with value } 0 \text{ is } 0$ has same expected revenue.
Advertising – how it used to be

Posters
Billboards

newspapers

magazines
television
Pay-per-impression

Price depends on how many people your advertisement is shown to.

(whether or not they look at it, or care about it)

“Half the money I spend on advertising is wasted; the trouble is, I don’t know which half”

Andrew Wanamaker, advertising pioneer
How is the price determined?

Complicated negotiations with high monthly premiums,

forms a barrier to entry for small advertisers.
Modern Advertising

On the web,
Many different kinds of ads ...
Sponsored Search Ads

Ad · www.nationalmesotheliomaclaims.com/
Mesothelioma Claim Center | Get Asbestos Cancer Payments
Learn how the $30B Asbestos Trust Fund may pay for your asbestos-caused cancer.

Ad · www.mesotheliomabook.com/ (888) 637-6234
Family Hurt by Mesothelioma? | We’re Here to Help You
We are the largest firm devoted only to Mesothelioma in the US, visit us today. We come to...

Ad · www.mesothelioma-survivor.com/mesothelioma/treatment (800) 336-0086
I Survived Mesothelioma Cancer - Learn How She Beat the Odds
Learn About Heather’s Successful Surgery from Preparation to the Procedure & Recovery.

Ad · www.navy-veterans-mesothelioma.org/Financial/Benefits
Mesothelioma & Asbestos Risk | Navy Vets Asbestos Claims
Important info for Navy Vets. Learn About Mesothelioma Claims

Mesothelioma is a type of cancer that develops from the thin layer of tissue that covers many of the internal organs (known as the mesothelium). The most
Price determined by auction – per keyword

“Most people don’t realize that all that money comes in pennies at a time”
Hal Varian, Google Chief Economist
How are these ads different than the ads in the offline media?

**In many cases: Pay-per-click**

**Model:** per keyword auction.

**Bidders:** advertisers who have a standing bid on keyword.

Auction often wins # \( k > 1 \) of slots.

**Slots not identical**

**CTR** click-through rates

\[ c_{ij} : \text{Prob of a click on the } j^{th} \text{ slot. } j = 1, \ldots, k \]

\[ v_i : \text{advertiser's value for a click.} \]

\[ q_i : \text{CTR rate due to quality ad.} \]
overall: if put bidder i's ad in slot j
Prob of click = c j . q i
Value to bidder i if gets his ad placed in slot j
exp v i . c j
exp. utility v i . c j - p i . c j

Generalized Second Price (GSP)
- allocate to slots in order decreasing bid
- on a click to slot j, payment is next highest bid.
Model

Click-through rates

Value per click

<table>
<thead>
<tr>
<th>Slot</th>
<th>Click-through rate</th>
<th>Value per click</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>0.4</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Value (in bids) = PPC utility

<table>
<thead>
<tr>
<th>Slot</th>
<th>PPC utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>(6 - 1) * 0.4</td>
</tr>
</tbody>
</table>

Slots

Advertisers
Example
\[
\begin{align*}
\text{Bidder 1: } & 1 \\ \text{Bidder 2: } & 2 \\ \text{Bidder 3: } & 3 \\
\end{align*}
\]
\[
\begin{align*}
\text{Value: } & \begin{bmatrix}
10 \\
4 \\
0.05
\end{bmatrix} \\
\end{align*}
\]
In this example, if bidder 1 bids 9 or below, then his utility will be higher than it was bidding truthfully.

Truthful auction: VCG auction
chooses outcome to maximize social welfare.

\[
V_1 > V_2 > \ldots
\]

Social welfare of an allocation
\[
S = \sum_{i=1}^{k} V_i c_{s(i)}
\]
slot i is assigned to

Social welfare maximizing allocation

\[
\begin{align*}
\text{Bidder 1: } & 1 \\
\text{Bidder 2: } & 2 \\
\text{Bidder 3: } & 3 \\
\end{align*}
\]

[change each bidder the "externality" her presence imposes on others]

What would she be willing to bid for in each

\[
\begin{align*}
\text{Bidder 1: } & 1 \\
\text{Bidder 2: } & 2
\end{align*}
\]

[her]
VCG payment for bidder $i$
charge bidder $i$ loses when winner because $j$ has
$$
\sum_{i=2}^{K+i} (c_{i+1}-c_i) v_i
$$
VCG payment for bidder $j$
$$
\sum_{i=j+1}^{K+i} (c_{i+1}-c_i) v_i
$$

VCG auction for sponsored search
Get a bid from each bidder
Related bids so $b_1 > b_2 > \ldots > b_n$
For $i = 1 ... K$ assign bidder $i$ to $i$-slot.
For $i = 1 ... K$, for each click, charge bidder $i$
$$
p_i = \frac{1}{c_i} \sum_{j=1}^{K+i} b_j (c_{j+1} - c_j)
$$
Then VCG is truthful, IR, and welfare maximizing

Proof of truthfulness
Fix other bids $b_j$ except $j$-highest
bid except me

<table>
<thead>
<tr>
<th>Slot $k$</th>
<th>$c_{k+1} b_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k-1$</td>
<td>$(c_{k+1} - c_k) b_{k-1} + c_{k+1} b_k$</td>
</tr>
<tr>
<td>$i$</td>
<td>$\sum_{j=i}^{K+i} (c_{j+1} - c_{j+1}) b_j$</td>
</tr>
</tbody>
</table>

$\Delta$ value as move from $i$ to slot $i-1$
$\Delta$ price
want to do this if $v > b_{i-1}$
b_i > v > b_{i+1} \quad \uparrow \quad \text{other bids} \quad b_i > b_{22} \ldots

valuations & CTRs, the GSP auction has an equilibrium of same allocation & payments as VCG.
VCG pays:

\begin{align*}
p_1 & \quad b_1 \\
p_2 & \quad b_2 \\
p_3 & \quad b_3 \\
p_k & \quad b_k \\ \vdots & \quad b_{k+1}
\end{align*}

no sweeping revenue comparison between GSP & VCG
Facebook

- what bidders can bid on.
  - various "events"
  - placement, size, format, goal
  - evaluation of quality goal

V: \epsilon \in \{0.05, \ldots, 0.07\}

more appropriate model,
  - bidder has value for each event type
  - difficult for advertisers to figure out CTRs
  - high quality outcomes
  - advertisers happy, bid easy

VCG (fully general)

- n participants
- finite set of outcomes \( \Omega \subseteq \mathbb{R} \)
- each agent \( i \) has private value
  \[ v_i(w) \quad \forall w \in \Omega \]
- \[ v_i(w) = \text{(value of some event) } \text{Pr(\text{event occurs in auction } w)} \]

\[ \text{VCG - choose outcome } w^* \in \{ \text{argmax } \sum_{i} b_i(w) \} \]

\[ \text{Let this outcome be } w^* \]
\[ p_i = \max_{w \in \mathcal{N}} \sum_{j : i \neq j} b_j(w) - \sum_{j : i \neq j} b_j(w) \]

\[ u_i = v_i(w^*) - p_i = v_i(w^*) + \sum_{j : i \neq j} b_j(w^*) - \max_{w \in \mathcal{N}} \sum_{j : i \neq j} b_j(w) \]

Choose \( b_i(w) \) wisely to maximize

\[
\sum_{j : i \neq j} b_j(w^*)
\]

\[ \Rightarrow \quad \sum_{j : i \neq j} b_j(w) \]

User interface for bidding:
- Allow bidders to bid on events they
- Allow bidders to specify budget

Computational requirements
\[ P_a = \text{total value others get if } a \text{ not here} - \text{value } a \text{ gets} = 5 \]
\[ P_b = 4 - 3 = 1 \]

**Figure 16.3.** The label on the edge from \( i \) on the left to \( j \) on the right is the value \( v_{ij} \) that employee \( i \) has for a yearly lease of house \( j \) (say in thousands of dollars). The VCG mechanism allocates according to purple shaded edges. The payment of bidder \( a \) is 0 since in his absence house 3 is still allocated to bidder \( b \). The payment of bidder \( b \) is 1 since in his absence the allocation is as follows: house 2 to bidder \( a \) and house 3 to bidder \( c \), and therefore the externality he imposes is 1.