1 Mass Action Kinetics

Part a (3 points)

\[ \emptyset \overset{k_1}{\rightarrow} A \]
\[ A \overset{k_2}{\rightarrow} \emptyset \]

The mass action kinetics for this reaction network are,
\[ \frac{d}{dt}[A] = k_1 - k_2[A]. \]

Part b (3 points)

\begin{align*}
\text{Rabbit} & \overset{\alpha}{\rightarrow} 2 \text{ Rabbit} \\
\text{Rabbit} + \text{ Fox} & \overset{\beta}{\rightarrow} 2 \text{ Fox} \\
\text{ Fox} & \overset{\gamma}{\rightarrow} \emptyset
\end{align*}

The mass action kinetics for Rabbits are,
\[ \frac{d}{dt}[\text{Rabbit}] = \alpha[\text{Rabbit}](2 - 1) \quad \text{(from the first reaction)} \]
\[ - \beta[\text{Rabbit}][\text{Fox}] \quad \text{(from the second reaction)}. \]

The mass action kinetics for Foxes are,
\[ \frac{d}{dt}[\text{Fox}] = \beta[\text{Rabbit}][\text{Fox}](2 - 1) \quad \text{(from the second reaction)} \]
\[ - \gamma[\text{Fox}] \quad \text{(from the third reaction)}. \]

All together,
\[ \frac{d}{dt}[\text{Rabbit}] = \alpha[\text{Rabbit}] - \beta[\text{Rabbit}][\text{Fox}] \]
\[ \frac{d}{dt}[\text{Fox}] = \beta[\text{Rabbit}][\text{Fox}] - \gamma[\text{Fox}]. \]
Part c (5 points)

The equations are,
\[
\frac{d}{dt}[X] = k_5 - [X](k_1[Y] + k_4)
\]
\[
\frac{d}{dt}[Y] = -[Y](k_1[X] + k_2[Z])
\]
\[
\frac{d}{dt}[Z] = k_1[X][Y] - k_3[Z]^2.
\]

Rewritten, these equations are,
\[
\frac{d}{dt}[X] = -k_1[X][Y] - k_4[X] + k_5
\]
\[
\frac{d}{dt}[Y] = -k_1[X][Y] - k_2[Y][Z]
\]
\[
\frac{d}{dt}[Z] = k_1[X][Y] - k_3[Z]^2
\]
\[
= k_1[X][Y] + k_2[Y][Z](1 - 1) + k_3[Z]^2(2 - 1).
\]

These equations are the mass action kinetics for the following chemical reaction network,
\[
X + Y \xrightarrow{k_{1\Delta}} Z
\]
\[
Y + Z \xrightarrow{k_{2\Delta}} Z
\]
\[
Z + Z \xrightarrow{k_{3\Delta}} Z
\]
\[
X \xrightarrow{k_{4\Delta}} \emptyset
\]
\[
\emptyset \xrightarrow{k_{5\Delta}} X.
\]

2 Visual DSD

Part a (5 points)

The following VisualDSD code encodes the reaction network from the class slides.

```python
directive duration 10000.0 points 1000
directive scale 10.0
def Input () = <t^1 t^> 
def Gate1 () = {t^*}[1 t^]<2 t^> 
def Gate2 () = {t^*}[2 t^]<3 t^> 
( 5 * Input () 
| 10 * Gate1 () 
| 10 * Gate2 () 
)
```

Note that the code above specifies the initial conditions where there are 10 * 5 = 50 copies of Input and 10 * 10 = 100 copies of Gate1 and Gate2. A stochastic simulation of this code generated the trajectory in Figure 1. Changing the scale directive to 100 encodes the initial conditions where there are 500 copies of Input and 1000 copies of Gate1 and Gate2. A trajectory using these initial conditions is shown in Figure 2.
Figure 2.1: Initial species counts: Input=50, Gate=100.

Figure 2.2: Initial species counts: Input=500, Gate=1000.
3 Nano Crafter game (4 points)

Thanks for your feedback! Your comments are being passed on to the development team.