Learned about: LSH/Similarity search & recommender systems

Search: “jaguar”

Uncertainty about the user’s information need
- Don’t put all eggs in one basket!

Relevance isn’t everything – need diversity!
Many applications need diversity!

- **Recommendation:**
  **NETFLIX**

- **Summarization:**
  “Robert Downey Jr.”
  **WIKIPEDIA**

- **News Media:**
Goal: Timeline should express their relationships to other people through events (personal, collaboration, mentorship, etc.)

Why timelines?
- Easier: Wikipedia article is 18 pages long
- Context: Through relationships & event descriptions
- Exploration: Can “jump” to other people/entities
Problem Definition

- **Given:**
  - Relevant *relationships*
  - *Events* that each cover some relationships

- **Goal:** Given a large set of *events*, pick a small subset that explains most known *relationships* (“the timeline”)
Example Timeline

“RDJr starred in Chaplin in 1992 together with Anthony Hopkins.”

Good overview
Why diversity?

- User studies: People hate redundancy!

Iron Man
US Release

Iron Man
Award
Ceremony

Iron Man
EU Release

Iron Man
US Release

Chaplin
Academy
Award N.

Rented Lips
US Release

- Want to see more diverse set of relationships
Diversity as Coverage
Encode Diversity as Coverage

- **Idea:** Encode diversity as coverage problem
- **Example:** Selecting events for timeline
  - Try to cover all important relationships
What is being covered?

- **Q:** What is being covered?
- **A:** Relationships

  Captain America  Anthony Hopkins  Gwyneth Paltrow  Susan Downey

Downey Jr. starred in *Chaplin* together with Anthony Hopkins

- **Q:** Who is doing the covering?
- **A:** Timeline Events
Suppose we are given a set of events $E$

- Each event $e$ covers a set $X_e \subseteq U$ of relationships

For a set of events $S \subseteq E$ we define:

$$F(S) = \left| \bigcup_{e \in S} X_e \right|$$

- Goal: We want to $\max_{|S| \leq k} F(S)$

- Note: $F(S)$ is a set function: $F(S) : 2^E \to \mathbb{N}$
Maximum Coverage Problem

- **Given universe of elements and sets**
  \[ U = \{u_1, \ldots, u_n\} \]
  \[ \{X_1, \ldots, X_m\} \subseteq U \]

- **Goal**: Find set of \( k \) events \( X_1 \ldots X_k \) covering most of \( U \)
  - More precisely: Find set of \( k \) events \( X_1 \ldots X_k \) whose size of the union is the largest
Simple Greedy Heuristic

**Simple Heuristic: Greedy Algorithm:**

- Start with $S_0 = \{\}$
- For $i = 1 \ldots k$
  - Take event $e$ that max $F(S_{i-1} \cup e)$
  - Let $S_i = S_{i-1} \cup \{e\}$

**Example:**

- Eval. $F(\{e_1\}), \ldots, F(\{e_m\})$, pick best (say $e_1$)
- Eval. $F(\{e_1\} \cup \{e_2\}), \ldots, F(\{e_1\} \cup \{e_m\})$, pick best (say $e_2$)
- Eval. $F(\{e_1, e_2\} \cup \{e_3\}), \ldots, F(\{e_1, e_2\} \cup \{e_m\})$, pick best
- And so on...

$$F(S) = \bigcup_{e \in S} X_e$$
Simple Greedy Heuristic

- Goal: Maximize the covered area
Simple Greedy Heuristic

- Goal: Maximize the covered area
Simple Greedy Heuristic

- Goal: Maximize the covered area
Simple Greedy Heuristic

- Goal: Maximize the covered area
Simple Greedy Heuristic

- Goal: Maximize the covered area
**Goal:** Maximize the size of the covered area with two sets

- Greedy first picks A and then C
- But the optimal way would be to pick B and C
Bad News & Good News

- **Bad news:** Maximum Coverage is NP-hard
  - Related to Set Cover Problem

- **Good news:** Good approximations exist
  - Problem has certain *structure* to it that even simple greedy algorithms perform reasonably well
  - Details in 2nd half of lecture

- **Now:** *Generalize* our objective for timeline generation
Issue 1: Not all relationships are created equal

- **Objective values all relationships equally**

\[ F(S) = \bigg| \bigcup_{e \in S} X_e \bigg| = \sum_{r \in R} 1 \text{ where } R = \bigcup_{e \in S} X_e \]

- **Unrealistic**: Some relationships are more important than others
  - Use **different weights** ("weighted coverage function")

\[ F(S) = \sum_{r \in R} w(r) \quad w : R \to \mathbb{R}^+ \]
Example weight function

- Use **global importance** weights
- How much interest is there?
- Could be measured as
  - \( w(X) = \# \text{ search queries} \) for person X
  - \( w(X) = \# \text{ Wikipedia article views} \) for X
  - \( w(X) = \# \text{ news article mentions} \) for X
Some relationships are **not (very) globally important but (not) highly relevant** to timeline

Need **relevant to timeline** instead of **globally relevant**

\[ w(\text{Susan Downey} \mid \text{RDJr}) > w(\text{Justin Bieber} \mid \text{RDJr}) \]
Capturing relevance to timeline

- Can use co-occurrence statistics
  \[ w(X \mid RDJr) = \frac{\#(X \text{ and } RDJr)}{\#(RDJr) \times \#(X)} \]
  - Similar: Pointwise mutual information (PMI)
  - How often do X and Y occur together compared to what you would expect if they were independent
  - Accounts for popular entities (e.g., Justin Bieber)
Issue 2: Differentiating between events

- How to differentiate between two events that cover the same relationships?

- **Example**: Robert and Susan Downey
  - **Event 1**: Wedding, August 27, 2005
  - **Event 2**: Minor charity event, Nov 11, 2006

- We need to be able to distinguish these!
Further improvement when we not only score relationships but also **score the event timestamp**

\[ F(S) = \sum_{r \in R} w_R(r) + \sum_{e \in S} w_T(t_e) \]

where \( R = \bigcup_{e \in S} X_e \)

- Relationship (as before)
- Timestamps

Again, use co-occurrences for weights \( w_T \)
Co-occurrences on Web Scale

- “Robert Downey Jr” and “May 4, 2012” occurs 173 times on 71 different webpages
- US Release date of *The Avengers*
- Use MapReduce on 10B web pages (10k+ machines)
Complete Optimization Problem

- Generalized earlier coverage function to linear combination of weighted coverage functions

\[ F(S) = \sum_{r \in R} w_R(r) + \sum_{e \in S} w_T(t_e) \]

- Goal: \( \max_{|S| \leq k} F(S) \)

- Still NP-hard (because generalization of NP-hard problem)
Next

- How can we **actually optimize** this function?
- What **structure** is there that will help us do this efficiently?

- Any questions so far?
For this optimization problem, Greedy produces a solution $S$ s.t. $F(S) \geq (1-1/e) \cdot OPT$  
($F(S) \geq 0.63 \cdot OPT$) 

[Nemhauser, Fisher, Wolsey ’78]

Claim holds for functions $F(\cdot)$ which are:

- Submodular, Monotone, Normal, Non-negative

(discussed next)
Submodularity: Definition 1

Definition:
- Set function $F(\cdot)$ is called submodular if:
  For all $P, Q \subseteq U$:
  \[ F(P) + F(Q) \geq F(P \cup Q) + F(P \cap Q) \]
Submodularity: Definition 2

- Checking the previous definition is not easy in practice
- Substitute $P = A \cup \{d\}$ and $Q = B$ where $A \subset B$ and $d \notin B$

  From before: $F(P) + F(Q) \geq F(P \cup Q) + F(P \cap Q)$

  $F(A \cup \{d\}) + F(B) \geq F(A \cup \{d\} \cup B) + F((A \cup \{d\}) \cap B)$

  $F(A \cup \{d\}) + F(B) \geq F(B \cup \{d\}) + F(A)$

  $F(A \cup \{d\}) - F(A) \geq F(B \cup \{d\}) - F(B)$

Common definition of Submodularity
Submodularity: Definition 2

- **Diminishing returns characterization**

\[
F(A \cup d) - F(A) \geq F(B \cup d) - F(B)
\]

Gain of adding \(d\) to a small set \(F(A \cup d) - F(A)\)

Gain of adding \(d\) to a large set \(F(B \cup d) - F(B)\)

- Large improvement
- Small improvement
Submodularity: Diminishing Returns

\[ F(A \cup d) - F(A) \geq F(B \cup d) - F(B) \]

Gain of adding \( d \) to a small set

Gain of adding \( d \) to a large set

Adding \( d \) to \( B \) helps less than adding it to \( A \)!

\( \forall A \subseteq B \)
Submodularity: An important property

Let $F_1 \ldots F_M$ be submodular functions and $\lambda_1 \ldots \lambda_M \geq 0$ and let $S$ denote some solution set, then the non-negative linear combination $F(S)$ (defined below) of these functions is also submodular.

$$F(S) = \sum_{i=1}^{M} \lambda_i F_i(S)$$
Submodularity: Approximation Guarantee

When maximizing a submodular function with cardinality constraints, Greedy produces a solution $S$ for which $F(S) \geq (1-1/e) \cdot OPT$

i.e., $(F(S) \geq 0.63 \cdot OPT)$

[Nemhauser, Fisher, Wolsey ’78]

Claim holds for functions $F(\cdot)$ which are:

- Monotone: if $A \subseteq B$ then $F(A) \leq F(B)$
- Normal: $F(\emptyset) = 0$
- Non-negative: For any $A$, $F(A) \geq 0$
- In addition to being submodular
Back to our Timeline Problem
Simple Coverage Model

- Suppose we are given a set of events $E$
  - Each event $e$ covers a set $X_e$ of relationships $U$
- For a set of events $S \subseteq E$ we define:

  $$F(S) = \left| \bigcup_{e \in S} X_e \right|$$

- Goal: We want to $\max_{|S| \leq k} F(S)$
  - Cardinality Constraint
- Note: $F(S)$ is a set function: $F(S) : 2^E \rightarrow \mathbb{N}$
Simple Coverage: Submodular?

- Claim: $F(S) = \bigcup_{e \in S} X_e$ is submodular.

Gain of adding $X_e$ to a smaller set

$$F(A \cup X_e) - F(A) \geq F(B \cup X_e) - F(B)$$

Gain of adding $X_e$ to a larger set

$\forall A \subseteq B$
Simple Coverage: Other Properties

- **Claim:** \( F(S) = \left| \bigcup_{e \in S} X_e \right| \) is normal & monotone

- **Normality:** When \( S \) is empty, \( \bigcup_{e \in S} X_e \) is empty.

- **Monotonicity:** Adding a new event to \( S \) can never decrease the number of relationships covered by \( S \).

- **What about non-negativity?**

**Monotone:** if \( A \subseteq B \) then \( F(A) \leq F(B) \)

**Normal:** \( F(\{\}) = 0 \)

**Non-negative:** For any \( A \), \( F(A) \geq 0 \)
# Summary so far

<table>
<thead>
<tr>
<th></th>
<th>Simple Coverage</th>
<th>Weighted Coverage (Relationships)</th>
<th>Weighted Coverage (Timestamps)</th>
<th>Complete Optimization Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Submodularity</td>
<td>✔</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monotonicity</td>
<td>✔</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality</td>
<td>✔</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Weighted Coverage (Relationships)

\[
F(S) = \sum_{r \in R} w(r) \quad w : R \to \mathbb{R}^+ \quad \text{where} \quad R = \bigcup_{e \in S} X_e
\]

- **Claim:** \( F(S) \) is submodular.
  - Consider two sets \( A \) and \( B \) s.t. \( A \subseteq B \subseteq S \) and let us consider an event \( e \notin B \)
  - Three possibilities when we add \( e \) to \( A \) or \( B \):
    - **Case 1:** \( e \) does not cover any new relationships w.r.t both \( A \) and \( B \)
      \[
      F(A \cup \{e\}) - F(A) = 0 = F(B \cup \{e\}) - F(B)
      \]
Weighted Coverage (Relationships)

\[ F(S) = \sum_{r \in R} w(r) \quad w : R \rightarrow \mathbb{R}^+ \]

- **Claim**: \( F(S) \) is submodular.

- Three possibilities when we add \( e \) to \( A \) or \( B \):
  - **Case 2**: \( e \) covers some new relationships w.r.t \( A \) but not w.r.t \( B \)
    - \( F(A \cup \{e\}) - F(A) = \nu \) where \( \nu \geq 0 \)
    - \( F(B \cup \{e\}) - F(B) = 0 \)
    - Therefore, \( F(A \cup \{e\}) - F(A) \geq F(B \cup \{e\}) - F(B) \)
Weighted Coverage (Relationships)

\[ F(S) = \sum_{r \in R} w(r) \quad w : R \rightarrow \mathbb{R}^+ \]

- **Claim:** \( F(S) \) is submodular.
  - Three possibilities when we add \( e \) to \( A \) or \( B \):
    - **Case 3:** \( e \) covers some new relationships w.r.t both \( A \) and \( B \)
      \[ F(A \cup \{e\}) - F(A) = \nu \quad \text{where} \quad \nu \geq 0 \]
      \[ F(B \cup \{e\}) - F(B) = \mu \quad \text{where} \quad \mu \geq 0 \]
      But, \( \nu \geq \mu \) because \( e \) will always cover fewer new relationships w.r.t \( B \) than w.r.t \( A \) because \( A \subset B \)
Weighted Coverage (Relationships)

\[ F(S) = \sum_{r \in R} w(r) \quad w : R \rightarrow \mathbb{R}^+ \]

where

\[ R = \bigcup_{e \in S} X_e \]

- **Claim:** \( F(S) \) is monotone and normal.

- **Normality:** When \( S \) is empty, \( R = \bigcup_{e \in S} X_e \) is empty.

- **Monotonicity:** Adding a new event to \( S \) can never decrease the number of relationships covered by \( S \).
## Summary so far

<table>
<thead>
<tr>
<th></th>
<th>Simple Coverage</th>
<th>Weighted Coverage (Relationships)</th>
<th>Weighted Coverage (Timestamps)</th>
<th>Complete Optimization Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Submodularity</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td><strong>Monotonicity</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td><strong>Normality</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>
**Weighted Coverage (Timestamps)**

\[ F(S) = \sum_{e \in S} w_T(t_e) \]

- **Claim:** \( F(S) \) is submodular, monotone and normal

- Analogous arguments to that of weighted coverage (relationships) are applicable
## Summary so far

<table>
<thead>
<tr>
<th></th>
<th>Simple Coverage</th>
<th>Weighted Coverage (Relationships)</th>
<th>Weighted Coverage (Timestamps)</th>
<th>Complete Optimization Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Submodularity</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Normality</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Complete Optimization Problem

- Generalized earlier coverage function to non-negative linear combination of weighted coverage functions

\[ F(S) = F_1(S) + F_2(S) \]

- Goal: \( \max_{|S| \leq k} F(S) \)

- Claim: \( F(A) \) is submodular, monotone and normal

\[ R = \bigcup_{e \in S} X_e \]
Complete Optimization Problem

- **Submodularity:** $F(S)$ is a non-negative linear combination of two submodular functions. Therefore, it is submodular too.

- **Normality:**
  \[
  F_1(\emptyset) = 0 = F_2(\emptyset) \\
  F_1(\emptyset) + F_2(\emptyset) = 0
  \]

- **Monotonicity:** Let $A \subseteq B \subseteq S$,
  \[
  F_1(A) \leq F_1(B) \text{ and } F_2(A) \leq F_2(B) \\
  F_1(A) + F_2(A) \leq F_1(B) + F_2(B)
  \]
# Summary so far

<table>
<thead>
<tr>
<th></th>
<th>Simple Coverage</th>
<th>Weighted Coverage (Relationships)</th>
<th>Weighted Coverage (Timestamps)</th>
<th>Complete Optimization Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Submodularity</strong></td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td><strong>Monotonicity</strong></td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td><strong>Normality</strong></td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>
Lazy Optimization of Submodular Functions
Greedy Solution

Greedy
Marginal gain: 
\[ F(S \cup x) - F(S) \]

- **Greedy Algorithm is Slow!**
- At each iteration, we need to evaluate marginal gains of all the remaining elements
- Runtime \( O(|U| \times K) \) for selecting \( K \) elements out of the set \( U \)

Add element with highest marginal gain
Speeding up Greedy

- **In round** $i$:
  - So far we have $S_{i-1} = \{e_1 \ldots e_{i-1}\}$
  - Now we pick an element $e \not\in S_{i-1}$ which maximizes the marginal benefit $\Delta_i = F(S_{i-1} \cup \{e\}) - F(S_{i-1})$

- **Key observation**:
  - *Marginal gain of any element $e$ can never increase!*
  - For every element $e$: $\Delta_i(e) \geq \Delta_j(e)$ for all iterations $i < j$
Lazy Greedy

- **Idea:**
  - Use $\Delta_i$ as upper-bound on $\Delta_j$ ($j > i$)

- **Lazy Greedy:**
  - Keep an ordered list of marginal benefits $\Delta_i$ from previous iteration
  - Re-evaluate $\Delta_i$ only for top node
  - Re-sort and prune

\[
F(A \cup \{d\}) - F(A) \geq F(B \cup \{d\}) - F(B) \quad A \subseteq B
\]
Lazy Greedy

- **Idea:**
  - Use $\Delta_i$ as upper-bound on $\Delta_j$ ($j > i$)

- **Lazy Greedy:**
  - Keep an ordered list of marginal benefits $\Delta_i$ from previous iteration
  - Re-evaluate $\Delta_i$ only for top node
  - Re-sort and prune

$$F(A \cup \{d\}) - F(A) \geq F(B \cup \{d\}) - F(B) \quad A \subseteq B$$

[Leskovec et al., KDD '07]
Lazy Greedy

- **Idea:**
  - Use $\Delta_i$ as upper-bound on $\Delta_j$ ($j > i$)

- **Lazy Greedy:**
  - Keep an ordered list of marginal benefits $\Delta_i$ from previous iteration
  - Re-evaluate $\Delta_i$ only for top node
  - Re-sort and prune

$$F(A \cup \{d\}) - F(A) \geq F(B \cup \{d\}) - F(B) \quad A \subseteq B$$
Lazy greedy offers significant speed-up over traditional greedy implementations in practice.

[Leskovec et al., KDD ‘07]
References

- Althoff et. al., TimeMachine: Timeline Generation for Knowledge-Base Entities, KDD 2015
- Leskovec et. al., Cost-effective Outbreak Detection in Networks, KDD 2007
- Andreas Krause, Daniel Golovin, Submodular Function Maximization
- UW Research by Jeff Bilmes (ECE)