Quantum Computing Problem Set 3

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Problem 1: GHZ Measurements

The GHZ wave function (named after Greenberger, Horne, and Zeilinger) is a three qubit entangled wave function, which in the computational basis is written as

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{1}$$

In this problem we will discuss some of the properties of $|GHZ\rangle$.

(a) Suppose that we measure the first qubit in the computational basis ($|0\rangle$, $|1\rangle$). What are the probabilities of the two outcomes and what are the resulting three qubit wave functions for each of these two outcomes?

(b) Let $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Suppose that we measure the first qubit in this $|+\rangle$, $|-\rangle$ basis. What are the probabilities of these two outcomes, and what are the resulting three qubit wave functions for each of these two outcomes? Notice that, as opposed to your answer in part (a), the second and third qubits are entangled following this measurement.

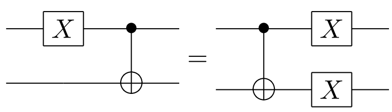
(c) Suppose that we measure all three qubits in the $|+\rangle$, $|-\rangle$ basis. What are the probabilities of all eight of the possible outcomes?

(d) Write down a circuit for creating the GHZ wave function which starts in the $|000\rangle$ state and uses only Hadamard and Controlled-NOT operations.

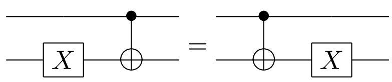
Problem 2: Circuit Identities

Prove (by explicit calculation of the matrix multiplications, for example), the following circuit identities. These circuit identities will be important when we discuss quantum error correction.

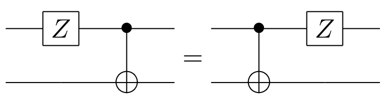
(a)



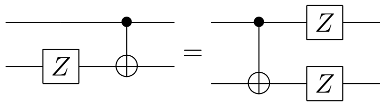
(b)



(c)



(d)



Problem 3: Teleporting an Entangled State

In this problem we will run through the problem of teleporting one half of an entangled quantum state. Alice will be teleporting to Bob. Suppose that Alice has two qubits whose wave function is given by

$$|\psi\rangle = \alpha|00\rangle + \beta|11\rangle \tag{2}$$

and that Alice and Bob share the two qubit entangled state

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \tag{3}$$

To be clear, Alice has three qubits and Bob has one. Recall that the Bell basis is given by the four basis states

$$\begin{aligned} |\Psi_+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad |\Psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle), \\ |\Phi_+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad |\Phi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \end{aligned} \tag{4}$$

If we label the qubits one through four, then Alice has the first three qubits and Bob has the final qubit and the initial state is $|\psi\rangle|\Phi_+\rangle$.

(a) Suppose that Alice measures her second and third qubits in the Bell basis. What are the probabilities of these four outcomes?

(b) What is the full quantum wave function for all four qubits after Alice measures in the Bell basis for each of these four outcomes?

(c) Show that if Alice gets the measurement outcome corresponding to $|\Phi_+\rangle$, then the state of the first qubit and the fourth qubit is $|\psi\rangle$.

(d) Show that if Alice gets the measurement outcome corresponding to $|\Psi_+\rangle$, then the state of the first qubit and the fourth qubit is $(I \otimes X)|\psi\rangle$.

(e) Describe a procedure for teleporting one half of Alice's entangled state $|\psi\rangle$. That is, show how Alice can send two classical bits of communication to Bob and Bob can apply an appropriate unitary operation, such that at the end of the procedure, the first and fourth qubits are in the state $|\psi\rangle$. (Notice that we have half solved this problem using the results in part (c) and part (d).)