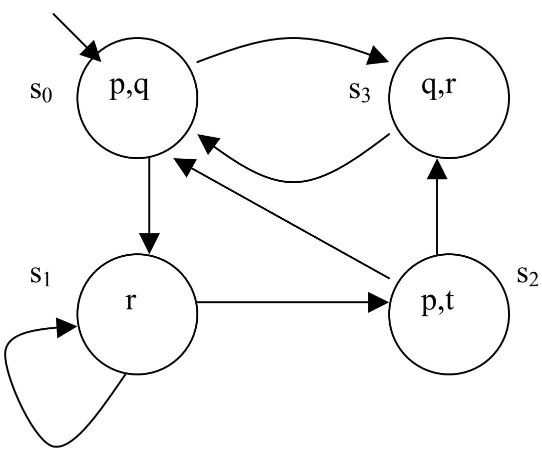
CSEP590 – Problem Set 3
Summer 2003
Due: Monday, July 21 by midnight.
Directions: Answer each of the following questions. Email your solutions in doc, pdf, or html format to evan@cs.washington.edu

1. This question will hopefully help you to gain a better understanding of the semantics of CTL. For each of the following pairs of formulas, decide whether or not the two formulae are semantically equivalent. If so, give an informal argument why based on the semantics of CTL. If not, give an example of a structure where they are not equivalent.

   a. $EFp \wedge EGq$, $EF(p \wedge EGq)$
   b. $AFp \wedge AGq$, $AF(p \wedge AGq)$
   c. $AFp \wedge AGq$, $AG(AFp \wedge q)$
   d. $AFAGp \wedge AFAGq$, $AF(AGp \wedge AGq)$
   e. $E[pUq] \wedge E[qUr]$, $E[pUr]$
   f. $A[pUq] \wedge A[qUr]$, $A[pUr]$

2. Consider the transition system M given below:



For each of the following CTL formulas ϕ, determine whether or not $M,s_0 \models \phi$ holds. If so, give an informal argument explaining why. If not, give a counter example execution of M showing why $M,s_0 \,!\!\models \phi$. States are labeled as s_0, s_1, s_2, and s_3.

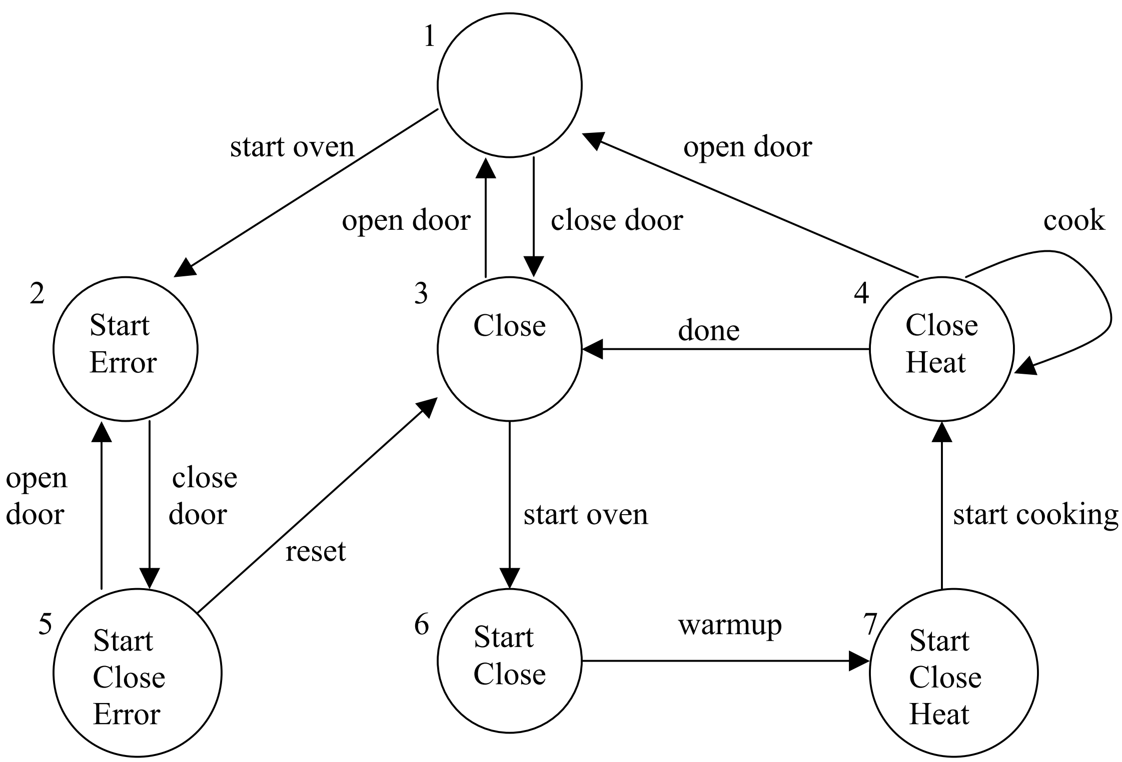
   a. AFq
   b. $AG(EF(p \vee q))$
   c. $EX(EXr))$
   d. $AG(AFq))$
   e. $AGEXE[(p \vee r)Uq]$
   f. $AF(A[(p \rightarrow r)Uq]$

3. Determine a CTL formula that expresses the following English properties on all computation paths of some system M.

   a. "The event p always precedes the event q."
   b. "After p, q is never true."
   c. "Between the events q and r, p is never true."

4. Give pseudo-code for a recursive function TRANSLATE which takes as input an arbitrarily, well-formed CTL formula ϕ and returns as output a semantically equivalent CTL formula ϕ' whose only operators are from the set $\{T, \neg, \wedge, AU, EU, EX\}$. The existence of such a routine necessarily means that $\{T, \neg, \wedge, AU, EU, EX\}$ is an adequate set for CTL.

5. For the final problem, we consider a model M of a microwave oven. M has atomic propositions {Start, Close, Heat, Error} specifying whether the oven is starting, whether the door is closed, whether the food in the oven is hot, and whether there is an error in operation. The model M is given below as a transition system where transitions are labeled according to an action that causes the transition (such as "open door" or "start oven"). The atomic propositions from among {Start, Close, Heat, Error} that are true in a given state are shown. Any propositions not shown to be true in a state are assumed to be false. States are labeled from 1 to 7.



In this problem, we'll check the CTL formula AG(Start $\rightarrow$ AF Heat) for the initial state 1.

a. What does this formula mean in English? What property of the microwave are we checking (for initial state 1)?
b. Using the equivalences shown in class between CTL formulas and operators, show that this formula is equivalent to $\neg$EF(Start $\wedge$ EG$\neg$Heat).
c. Using the equivalent formula ϕ = $\neg$EF(Start $\wedge$ EG$\neg$Heat) derived in part b, list all of the subformulas of ϕ, and for each subformula, give the set of

states of M that satisfy the subformula. Having done this, you can identify the set of states satisfying ϕ. Do this. Does $M,1 \models \phi$ hold?