

3.1

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

↖  
3x3

3.2

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 1 & 4 \\ 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

3.3

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \end{bmatrix}$$

affine

similarity

3.4

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$s \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$s=1 \Rightarrow$  euclidean

4 param

2 param

scaling + rotation

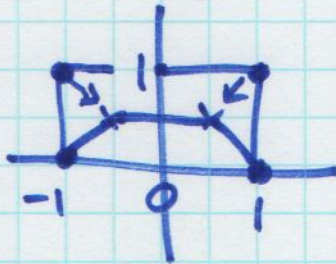
||

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

3.5

$$s \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

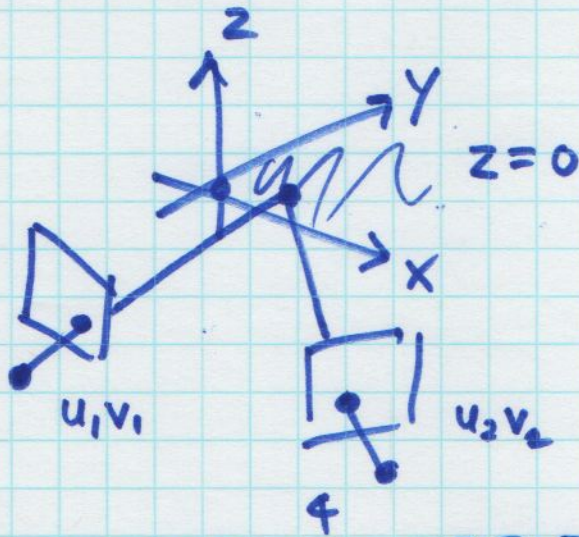
$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -1 & -0.5 & 1 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}$$

3.6



$${}^0 \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = {}_3 \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ | & | & | & | \\ | & | & | & | \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad z=0$$

$$= {}_3 \begin{bmatrix} p_1 & p_2 & p_4 \\ | & | & | \\ | & | & | \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$${}^0 \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \hat{u}_1 = \underset{3 \times 3}{H_1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\hat{u}_2 = H_2 \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\underline{\underline{\hat{u}_2 = H_2 H_1^{-1} \hat{u}_1}}$$

3.7

$$s \begin{bmatrix} u \\ v \\ i \end{bmatrix} = \begin{bmatrix} \delta & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ i \end{bmatrix}$$

$$t=0$$

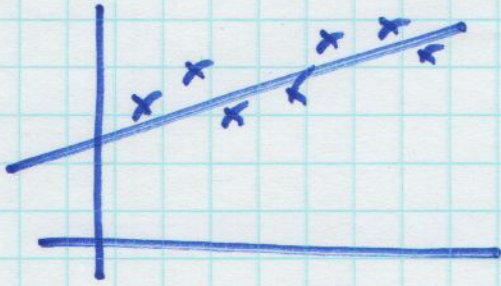
$$u_1 = s \begin{bmatrix} u \\ v \\ i \end{bmatrix} = K [R_1 | 0] \begin{bmatrix} x \\ y \\ z \\ i \end{bmatrix} = KR_1 \begin{bmatrix} x \\ y \\ z \\ i \end{bmatrix}$$

$$u_2 = KR_2 \begin{bmatrix} x \\ y \\ z \\ i \end{bmatrix}$$

$$\underline{u_2 = KR_2 R_1^{-1} K^{-1} u_1}$$

3.8

x



$$y = ax + b$$

set  $(x_i, y_i)$  solve for  $a, b$

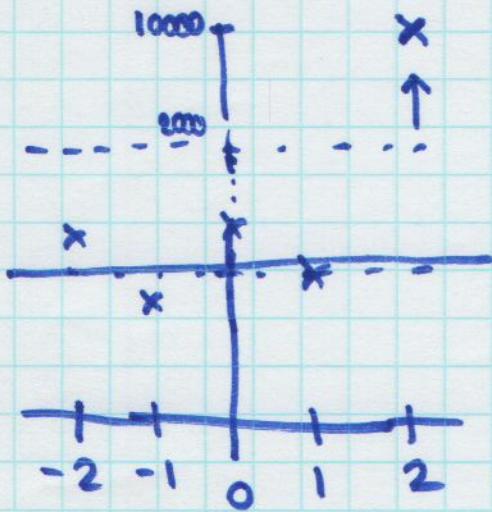
$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \end{bmatrix}$$

$$M\theta = y$$

$$\hat{\theta} = \arg \min_{\theta} |M\theta - y|^2$$

let  $a = 0$ , and  $y_i = \{ 3.1, 2.9, 3.2, 3.0, 10000 \}$

$x_i = \{ -2, -1, 0, 1, 2 \}$



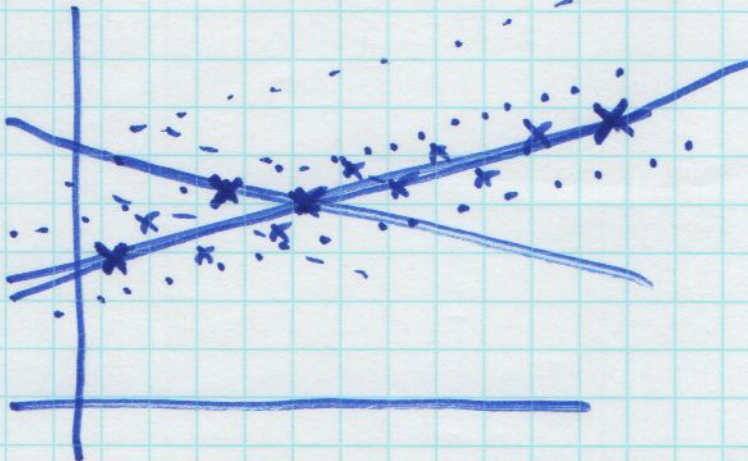
$$\hat{b} = \arg \min_b \sum_i (y_i - b)^2$$

$$\hat{b} = \frac{1}{N} \sum_i y_i$$

$$\hat{b} = (3.1 + 2.9 + 3.2 + 3.0 + 10000) / 5$$

$$\approx 2000$$

x



select minimal subset

solve for  $\theta$

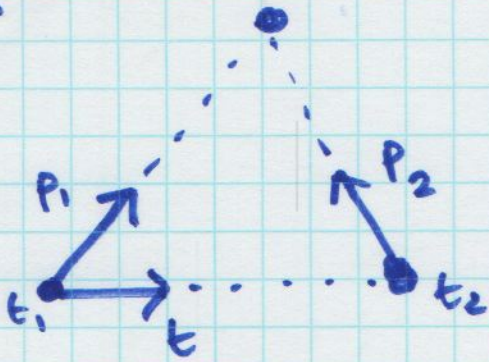
check consensus

repeat to maximise # inliers

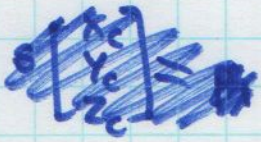
Random Sample Consensus

= RANSAC

3.10



$$P_2^T (t \times P_1) = 0$$



$$P_1 = R_1 K_1^{-1} u_1$$

$$P_2 = R_2 K_2^{-1} u_2$$

$$t = t_2 - t_1$$

$$P_1 = R_1 X_{c1}$$

$$P_2 = R_2 X_{c2}$$

$$u_2^T K_2^{-T} R_2^T (t_2 - t_1) \times R_1 K_1^{-1} u_1 = 0$$

$$\underbrace{\quad \quad \quad}_{\substack{\text{[t]}_x \\ \text{F is } 3 \times 3}}$$

$$\tilde{u}_2^T F \tilde{u}_1 = 0$$

$$0 = [u \ v \ 1] \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$