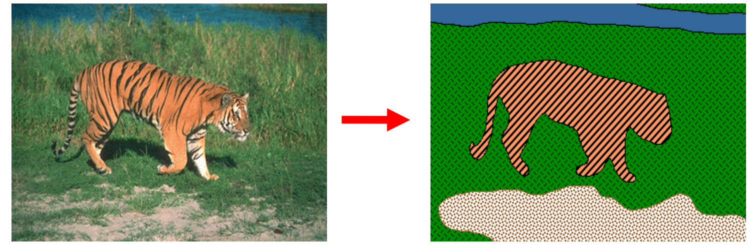
Image Segmentation

From Sandlot Science

Today's Readings

- Szesliski Chapter 5

From images to objects



What Defines an Object?

- Subjective problem, but has been well-studied
- Gestalt Laws seek to formalize this
  - proximity, similarity, continuation, closure, common fate

Image Segmentation

We will consider different methods

- K-means clustering (color-based)
- Normalized Cuts (region-based)

Image histograms

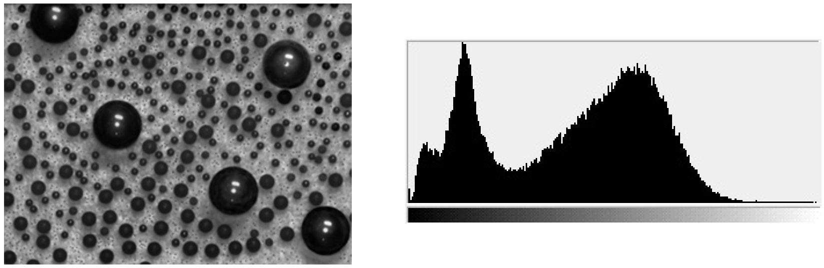
How many "orange" pixels are in this image?

- This type of question answered by looking at the *histogram*
- A histogram counts the number of occurrences of each color
  - Given an image $F[x,y] \rightarrow RGB$
  - The histogram is $H_F[c] = |\{(x,y) \mid F[x,y] = c\}|$
    - » i.e., for each color value c (x-axis), plot # of pixels with that color (y-axis)
  - What is the dimension of the histogram of an NxN RGB image?

Photoshop demo

What do histograms look like?

Photoshop demo



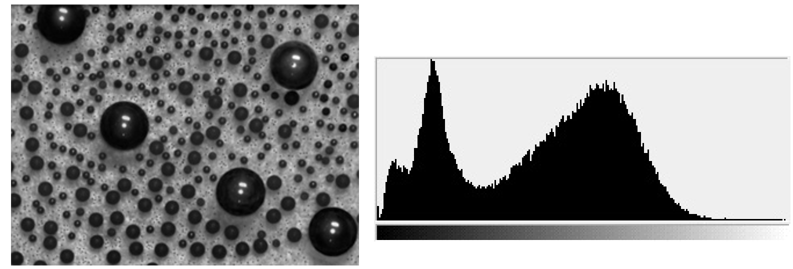
How Many Modes Are There?

- Easy to see, hard to compute

Histogram-based segmentation

Goal

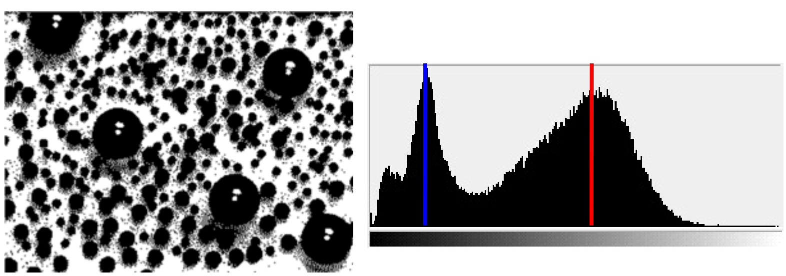
- Break the image into K regions (segments)
- Solve this by reducing the number of colors to K and mapping each pixel to the closest color
  - photoshop demo



Histogram-based segmentation

Goal

- Break the image into K regions (segments)
- Solve this by reducing the number of colors to K and mapping each pixel to the closest color
  - photoshop demo

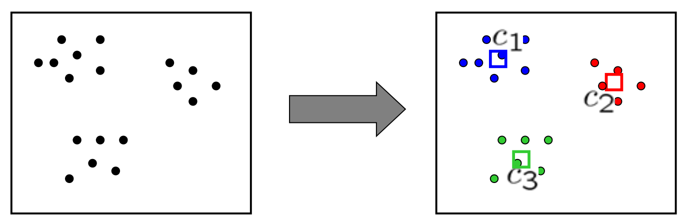


Here's what it looks like if we use two colors

Clustering

How to choose the representative colors?

- This is a clustering problem!



Objective

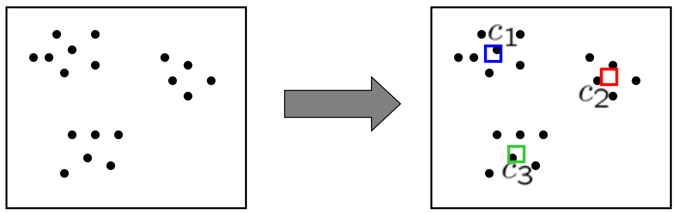
- Each point should be as close as possible to a cluster center
– Minimize sum squared distance of each point to closest center

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

Break it down into subproblems

Suppose I tell you the cluster centers c_i

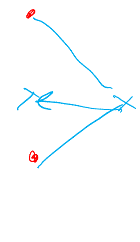
- Q: how to determine which points to associate with each c_i?



Suppose I tell you the points in each cluster

- Q: how to determine the cluster centers?

Examples



K-means clustering

K-means clustering algorithm

1. Randomly initialize the cluster centers, $c_1, ..., c_K$
2. Given cluster centers, determine points in each cluster
   - For each point p, find the closest c_i. Put p into cluster i
3. Given points in each cluster, solve for c_i
   - Set c_i to be the mean of points in cluster i
4. If c_i have changed, repeat Step 2

Javascript demo: https://miguelmota.com/blog/k-means-clustering-in-javascript/demo/

Properties

- Will always converge to *some* solution
- Can be a "local minimum"
  - does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

Cleaning up the result

Problem:

- Histogram-based segmentation can produce messy regions
  - segments do not have to be connected
  - may contain holes

How can these be fixed?

photoshop demo

Dilation operator: $G = H \oplus F$

Assume: binary image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	0	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

1	1	1
1	1	1
1	1	1

$H[u, v]$

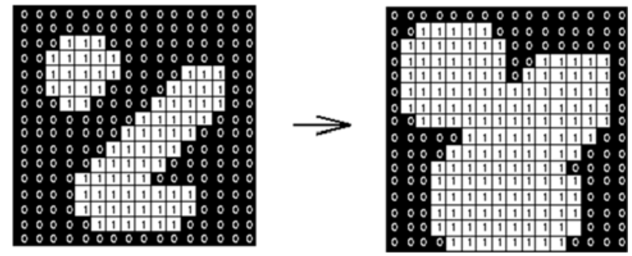
Dilation: does H "overlap" F around [x,y]?

- G[x,y] = 1 if H[u,v] and F[x+u-1,y+v-1] are both 1 **somewhere**
  0 otherwise
- Written $G = H \oplus F$

Dilation operator

Demo

- http://www.cs.bris.ac.uk/~majid/mengine/morph.html



Erosion operator: $G = H \ominus F$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	0	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

1	1	1
1	1	1
1	1	1

$H[u, v]$

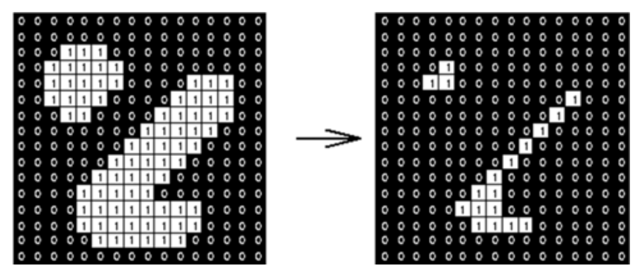
Erosion: is H "contained in" F around [x,y]

- G[x,y] = 1 if F[x+u-1,y+v-1] is 1 **everywhere** that H[u,v] is 1
  0 otherwise
- Written $G = H \ominus F$

Erosion operator

Demo

- http://www.cs.bris.ac.uk/~majid/mengine/morph.html



Nested dilations and erosions

What does this operation do?

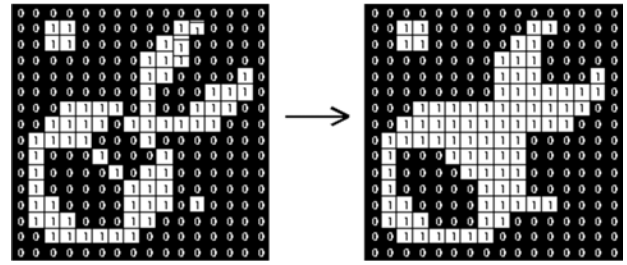
$$G = H \ominus (H \oplus F)$$

- this is called a **closing** operation

Nested dilations and erosions

What does this operation do?

$$G = H \ominus (H \oplus F)$$



- this is called a **closing** operation

Is this the same thing as the following?

$$G = H \oplus (H \ominus F)$$

Nested dilations and erosions

What does this operation do?

$$G = H \oplus (H \ominus F)$$

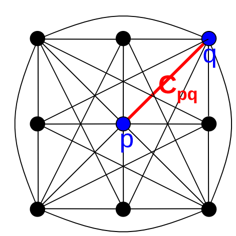
- this is called an **opening** operation
- http://www.dai.ed.ac.uk/HIPR2/open.htm

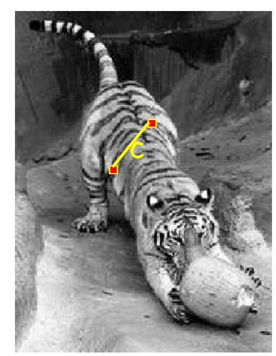
You can clean up binary pictures by applying combinations of dilations and erosions

Dilations, erosions, opening, and closing operations are known as **morphological operations**

- see http://www.dai.ed.ac.uk/HIPR2/morops.htm

Images as graphs

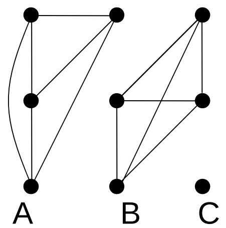




Fully-connected graph

- node for every pixel
- link between *every* pair of pixels, **p**,**q**
- cost $\mathbf{c_{pq}}$ for each link
  - $\mathbf{c_{pq}}$ measures *similarity*
    - » similarity is *inversely proportional* to difference in color and position

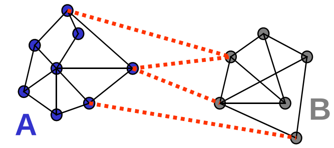
Segmentation by Graph Cuts



Break Graph into Segments

- Delete links that cross between segments
- Easiest to break links that have low cost (low similarity)
  - similar pixels should be in the same segments
  - dissimilar pixels should be in different segments

Cuts in a graph



Link Cut

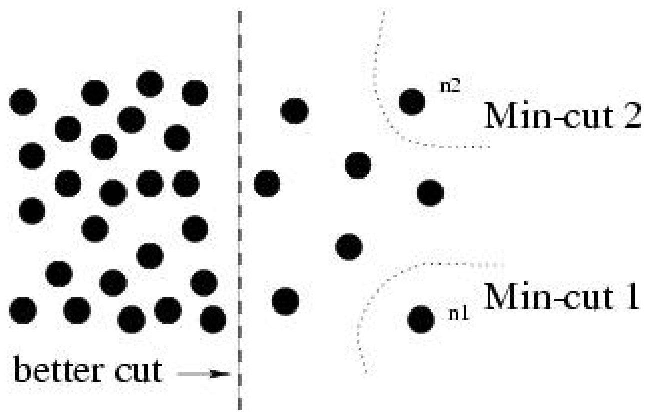
- set of links whose removal makes a graph disconnected
- cost of a cut:

$$cut(A,B) = \sum_{p \in A, q \in B} c_{p,q}$$

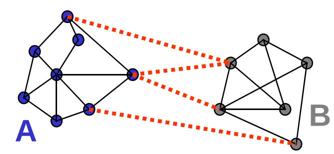
Find minimum cut

- gives you a segmentation
- fast algorithms exist for doing this

But min cut is not always the best cut...



Cuts in a graph



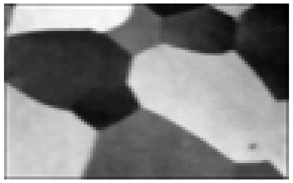
Normalized Cut

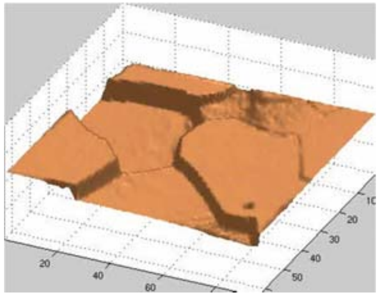
- a cut penalizes large segments
- fix by normalizing for size of segments

$$Ncut(A,B) = \frac{cut(A,B)}{volume(A)} + \frac{cut(A,B)}{volume(B)}$$

- volume(A) = sum of costs of all edges that touch A

Interpretation as a Dynamical System

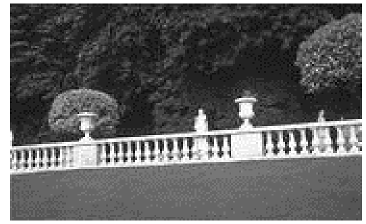


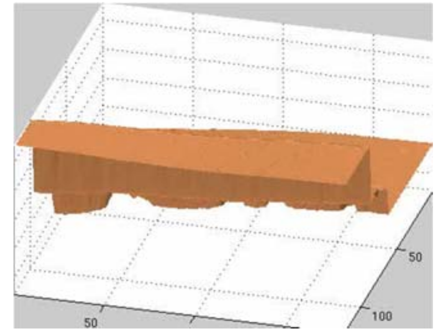


Treat the links as springs and shake the system

- elasticity proportional to cost
- vibration "modes" correspond to segments
  - can compute these by solving an eigenvector problem
  - for more details, see
    - » J. Shi and J. Malik, Normalized Cuts and Image Segmentation, CVPR, 1997

Interpretation as a Dynamical System





Color Image Segmentation