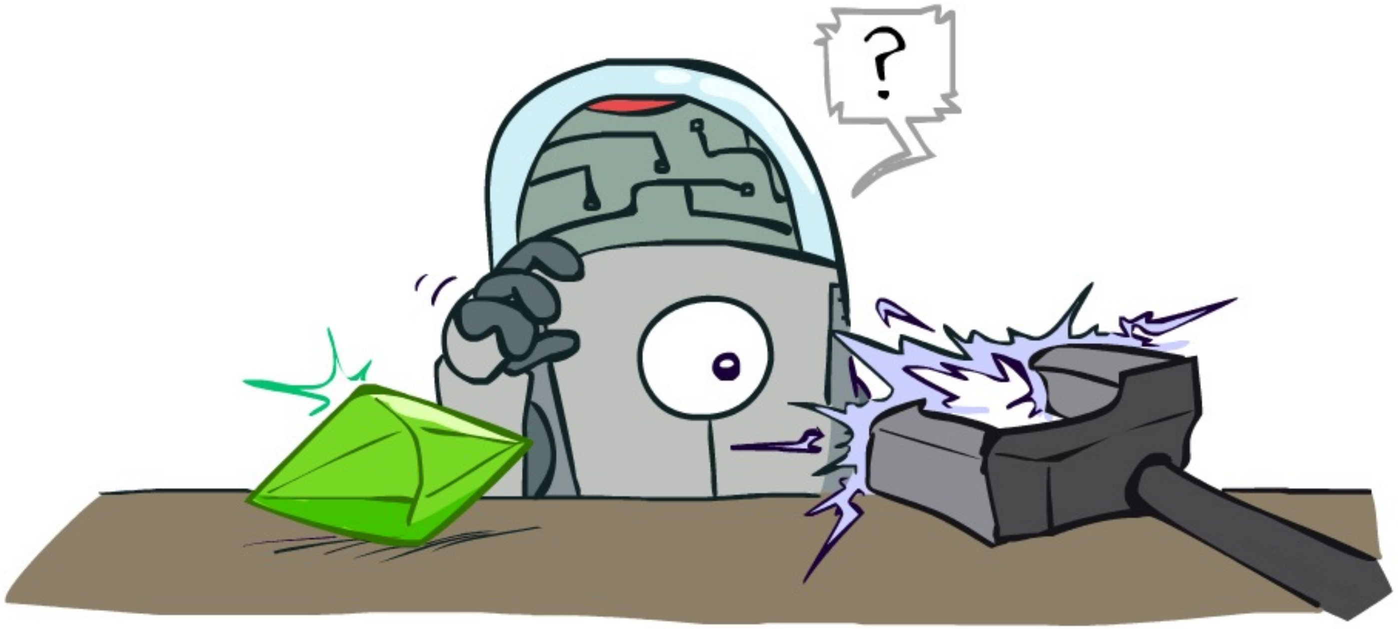
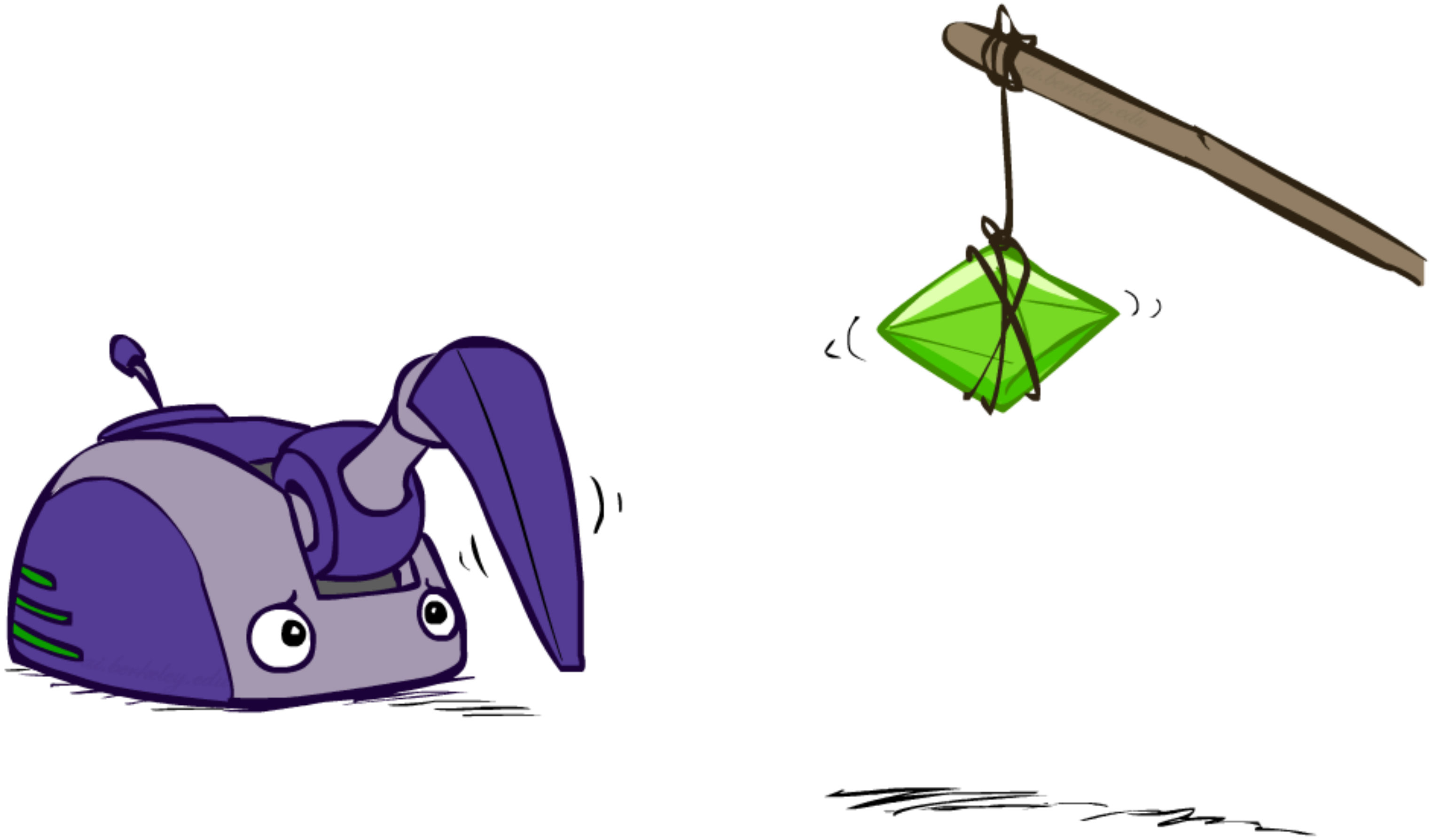
CSEP 573: Artificial Intelligence

Reinforcement Learning

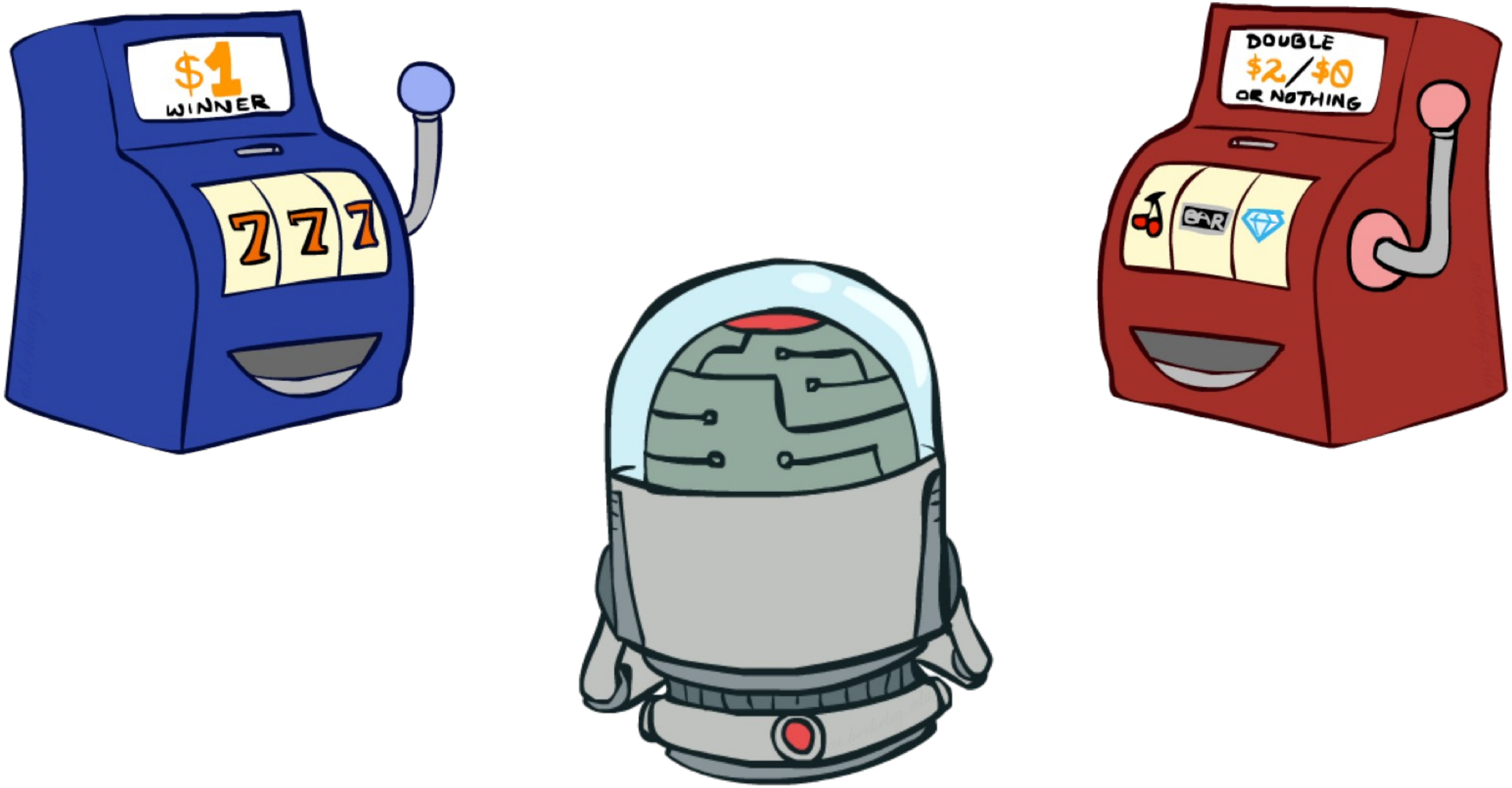


slides adapted from
Stuart Russel, Dan Klein, Pieter Abbeel from ai.berkeley.edu
And Hanna Hajishirzi, Jared Moore, Dan Weld

Reinforcement Learning

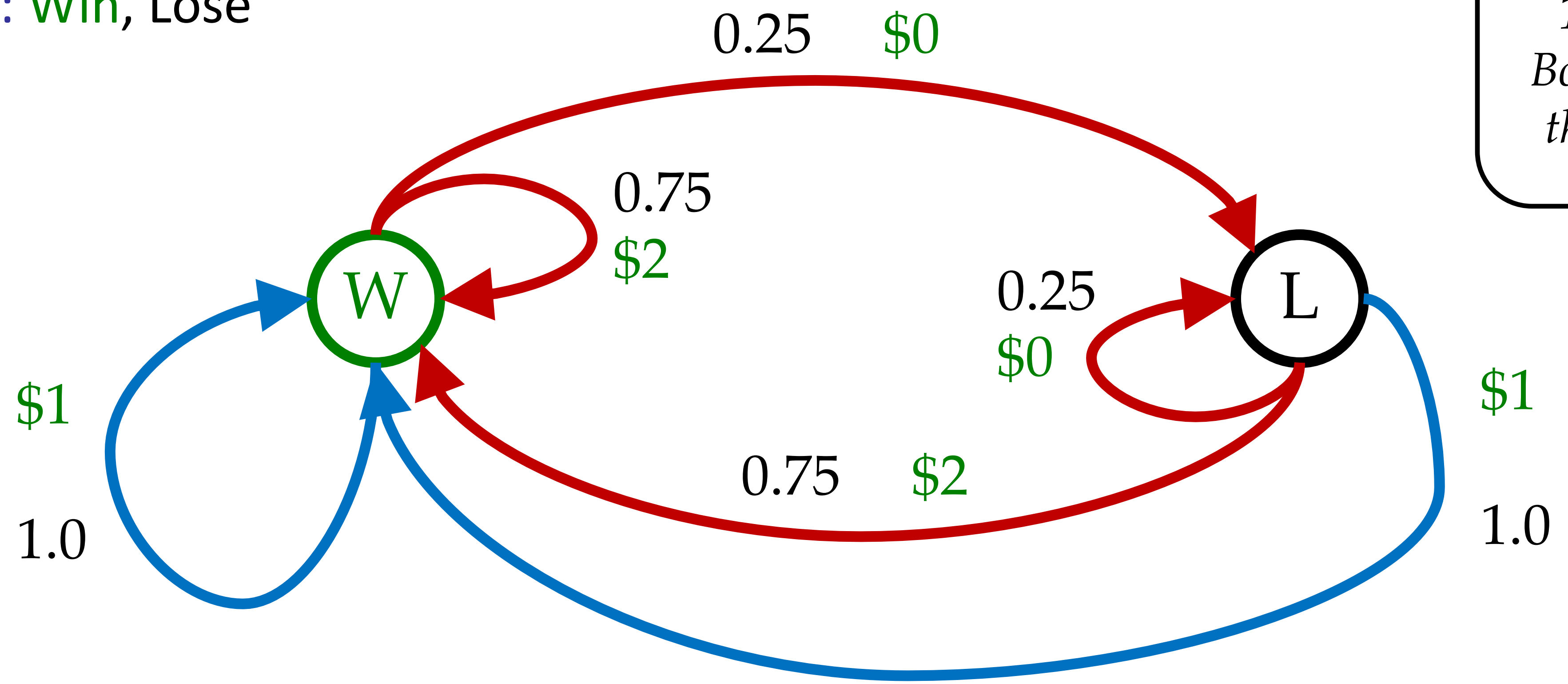


Double Bandits



Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose



No discount
10 time steps
Both states have the same value

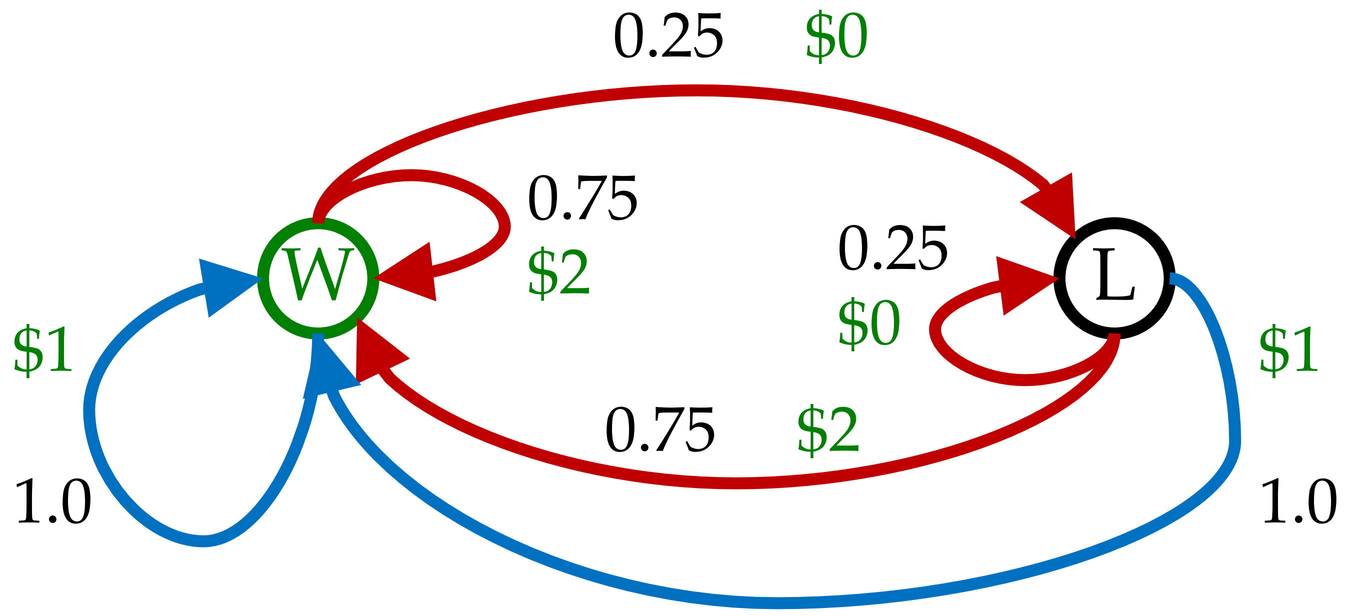
Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount

10 time steps

	Value
Play Red	15
Play Blue	10



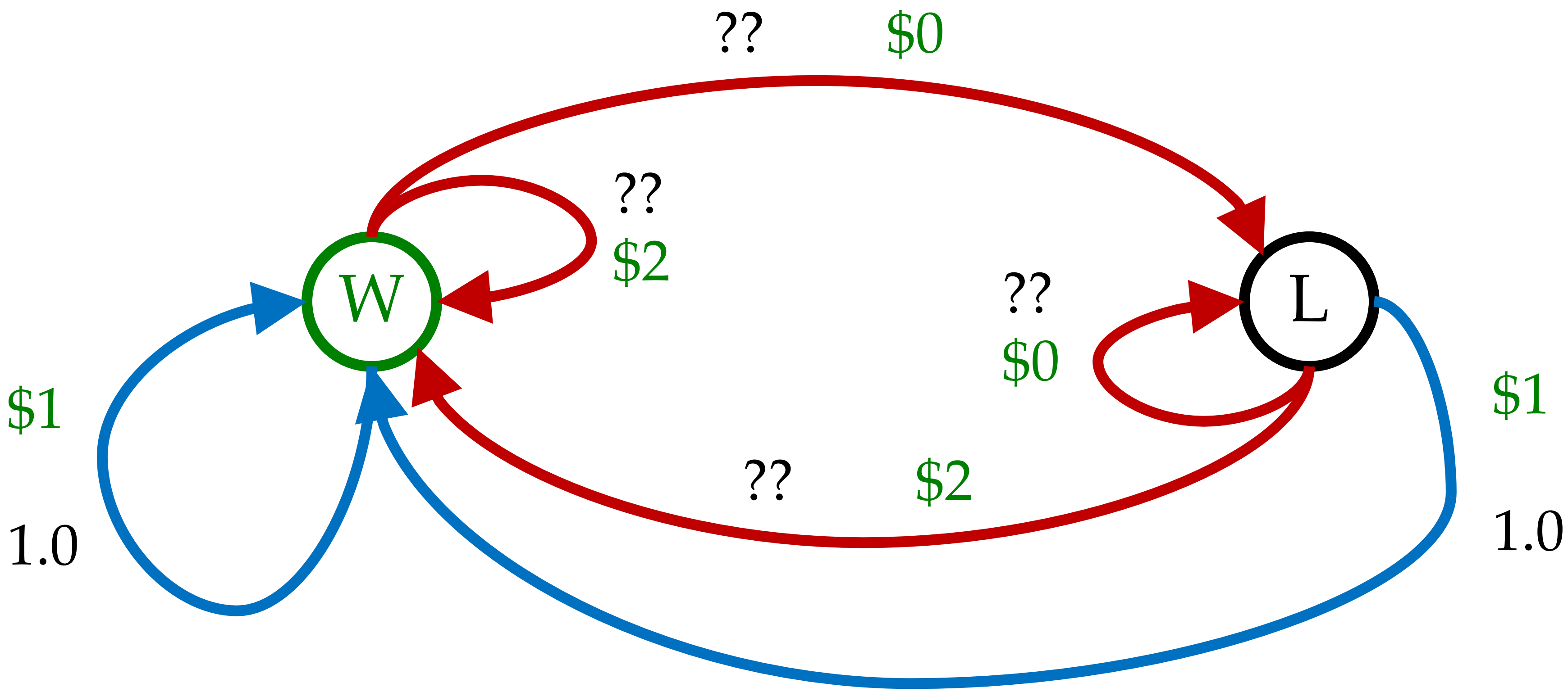
Let's Play!

$2 $2 $0 $2 $2

$2 $2 $0 $0 $0

Online Planning

- Rules changed! Red's win chance is different.



Let’s Play!

$0 $0 $2 $0

$0 $2 $2 $0 $0

$0

What Just Happened?

- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out

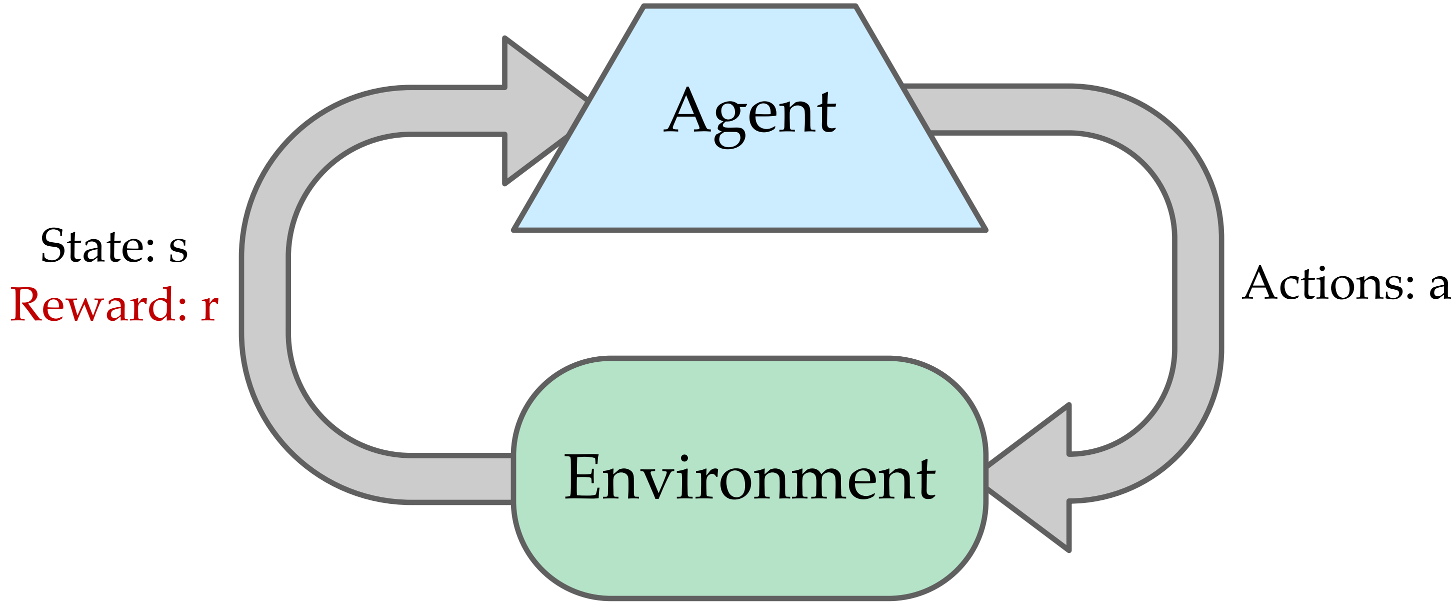
- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn

Reinforcement Learning



- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

Robotics Rubik Cube

- https://www.youtube.com/watch?v=x4O8pojMF0w

DeepMind Atari (©Two Minute Lectures)

Video of Demo Crawler Bot

Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
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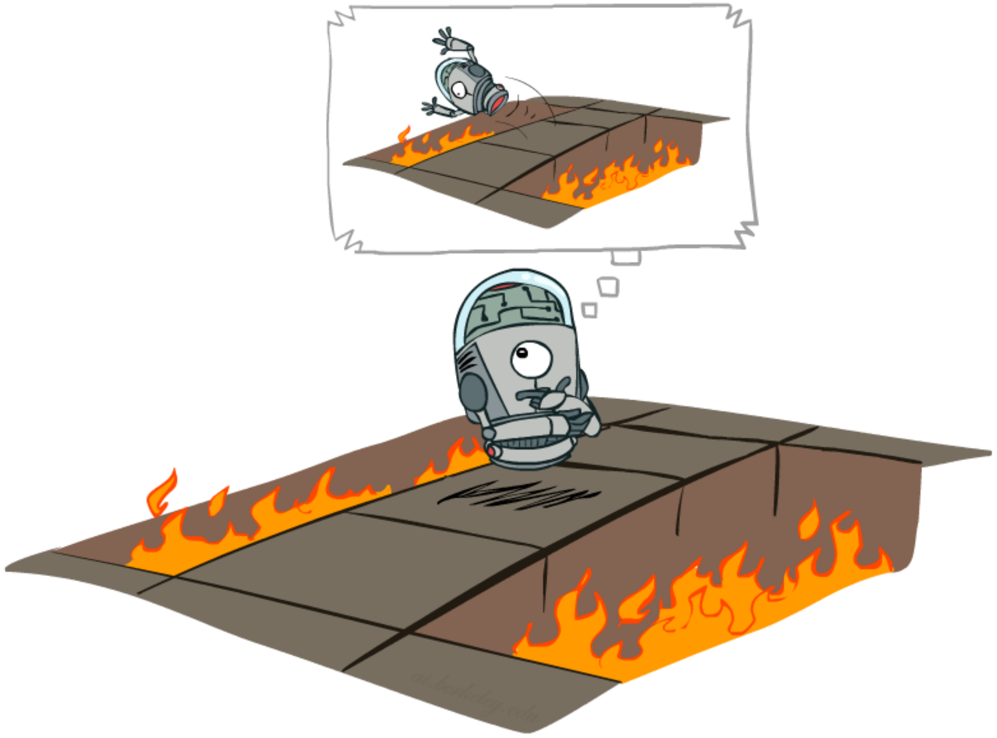
Offline (MDPs) vs. Online (RL)



Offline Solution

Online Learning

Analogy: Expected Age

Goal: Compute expected age of students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples [a_1, a_2, … a_N]

Unknown P(A): “Model Based”

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

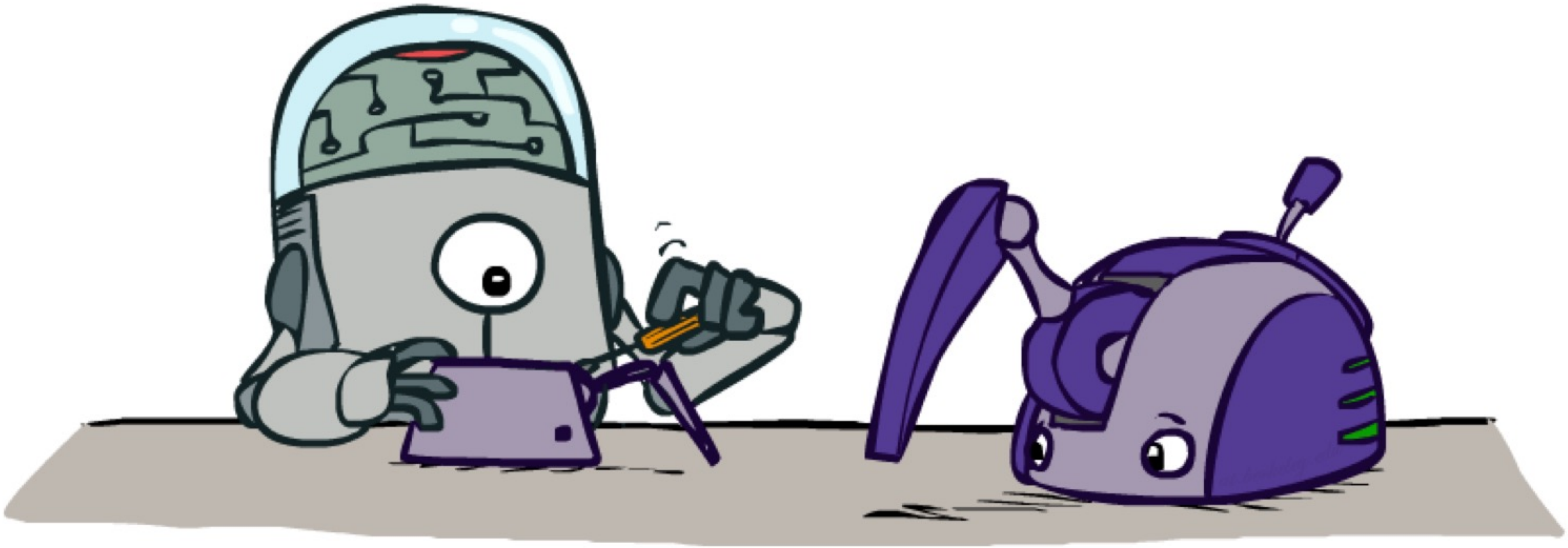
Why does this work? Because eventually you learn the right model.

Unknown P(A): “Model Free”

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Model-Based Learning

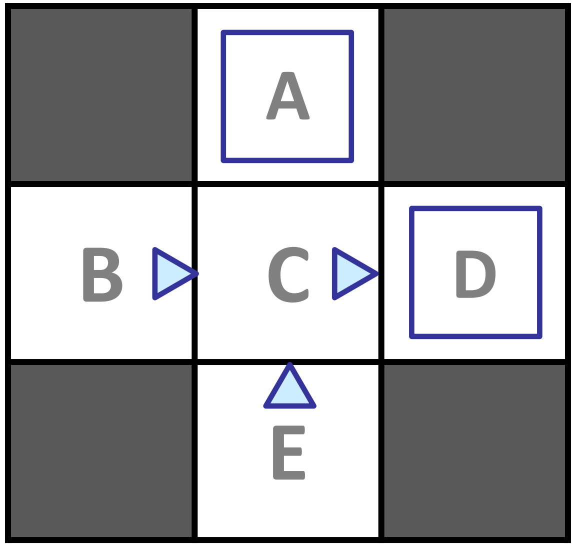


Model-Based Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
  - Count outcomes s’ for each s, a
  - Normalize to give an estimate of $\widehat{T}(s, a, s')$
  - Discover each $\widehat{R}(s, a, s')$ when we experience (s, a, s’)
- Step 2: Solve the learned MDP
  - For example, use value iteration, as before

Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

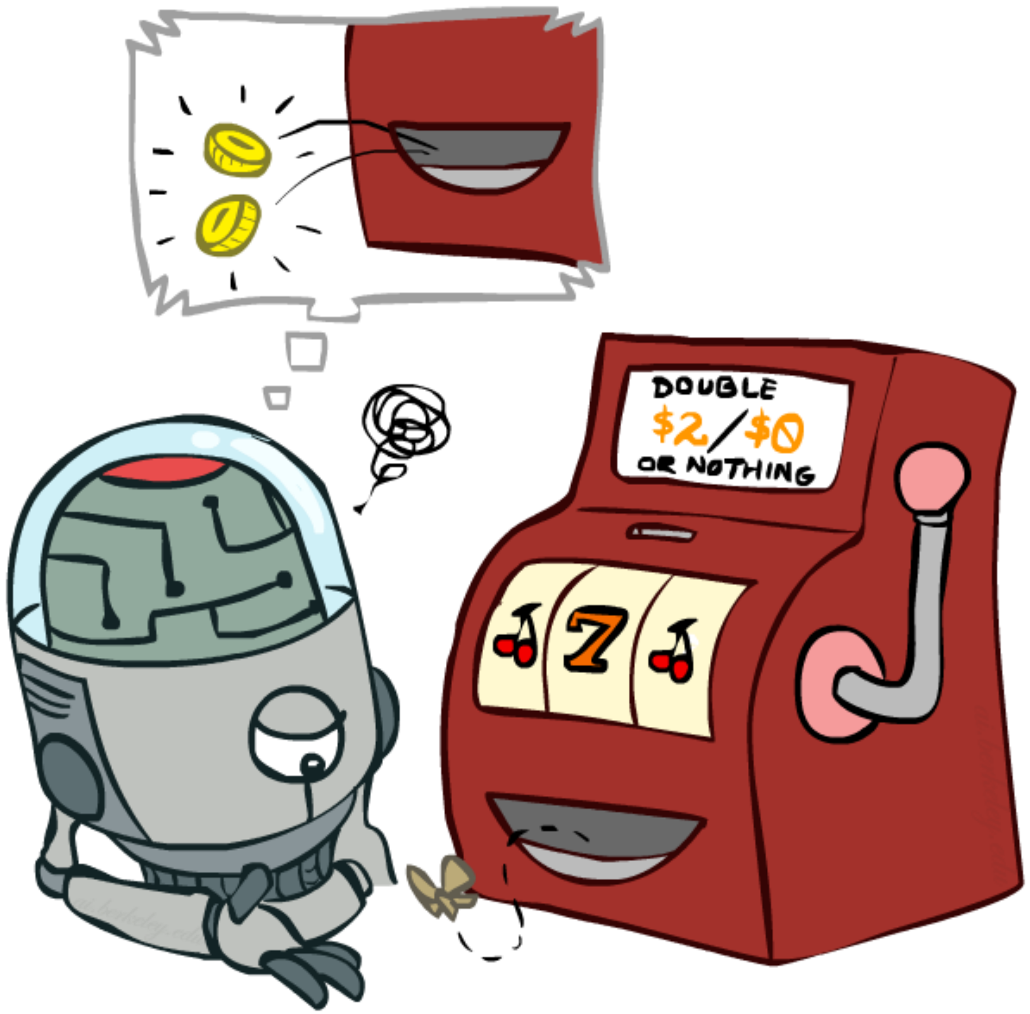
$\hat{T}(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

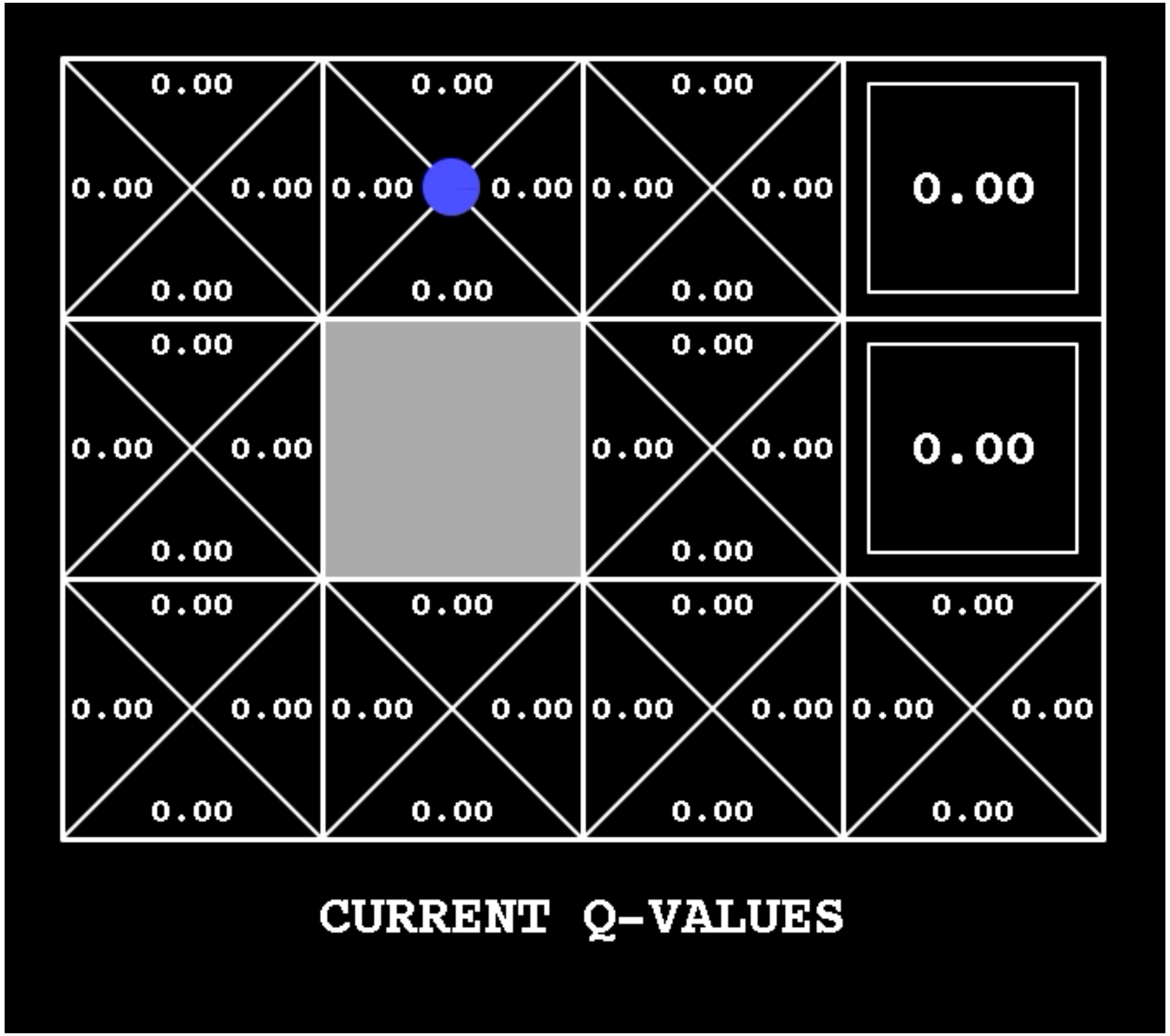
$\hat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Model-Free Learning



A Motivating Example Video



Direct Evaluation

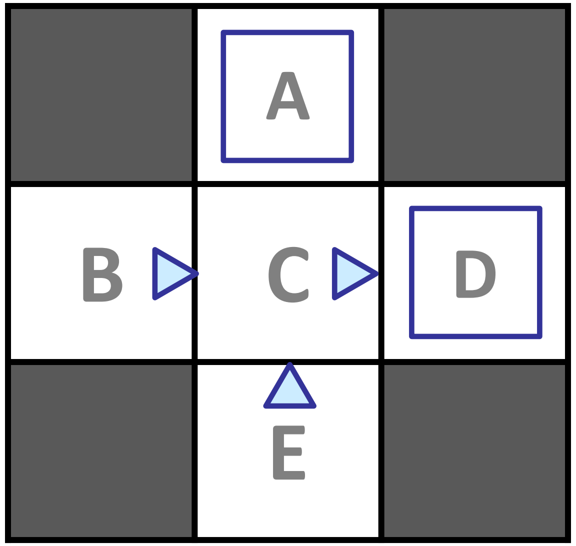
- Goal: Compute values for each state under π

- Idea: Average together observed sample values
  - Act according to π
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples

- This is called direct evaluation

Example: Direct Evaluation

Input Policy π



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Observed Episodes (Training)

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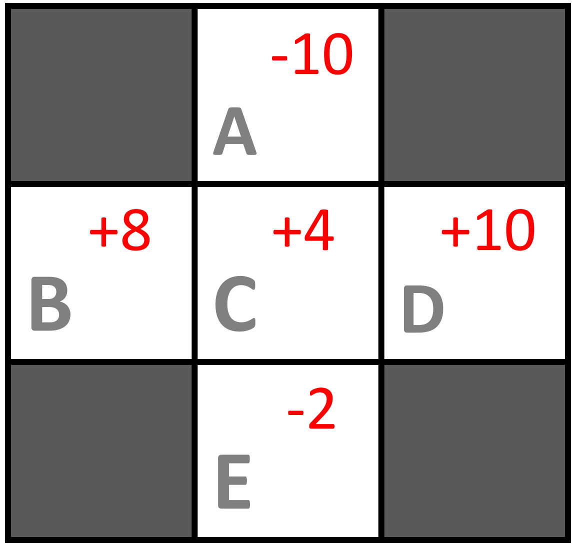
Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values



$U^{\pi}(D) = 3/3 * 10 = 10$

$U^{\pi}(A) = 1/1 * -10 = -10$

$U^{\pi}(B) = 2/2 * (-1 + -1 + 10) = 8$

$U^{\pi}(C) = 3/4 * (-1 + 10) +$
$1/4\ (-1 + -10) = 4$

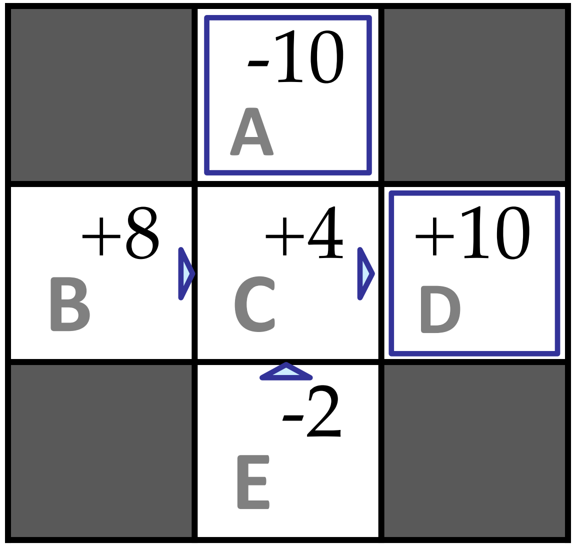
$U^{\pi}(E) = 1/2 * (-1 + -1 + 10) +$
$1/2\ (-1 + -1 + -10) = -2$

Problems with Direct Evaluation

- What's good about direct evaluation?
  - It's easy to understand
  - It doesn't require any knowledge of T, R
  - It eventually computes the correct average values, using just sample transitions

- What bad about it?
  - It wastes information about state connections
  - Each state must be learned separately
  - So, it takes a long time to learn

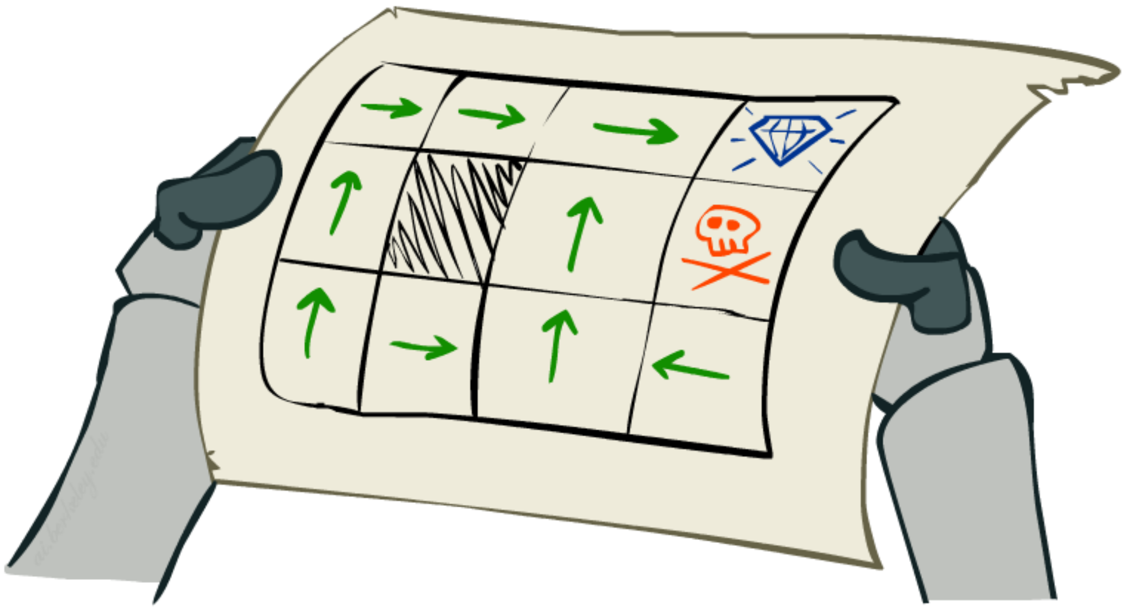
Output Values



If B and E both go to C under this policy, how can their values be different?

Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - Goal: learn the state values



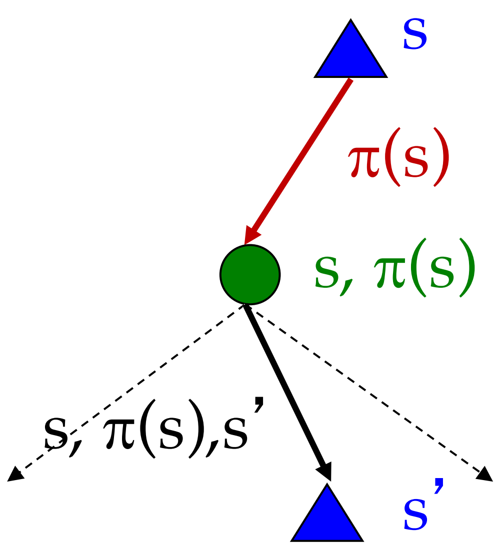
- In this case:
  - Learner is "along for the ride"
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - This is NOT offline planning! You actually take actions in the world.

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:
  - Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



  - This approach fully exploited the connections between the states
  - Unfortunately, we need T and R to do it!

- Key question: how can we do this update to V without knowing T and R?
  - In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

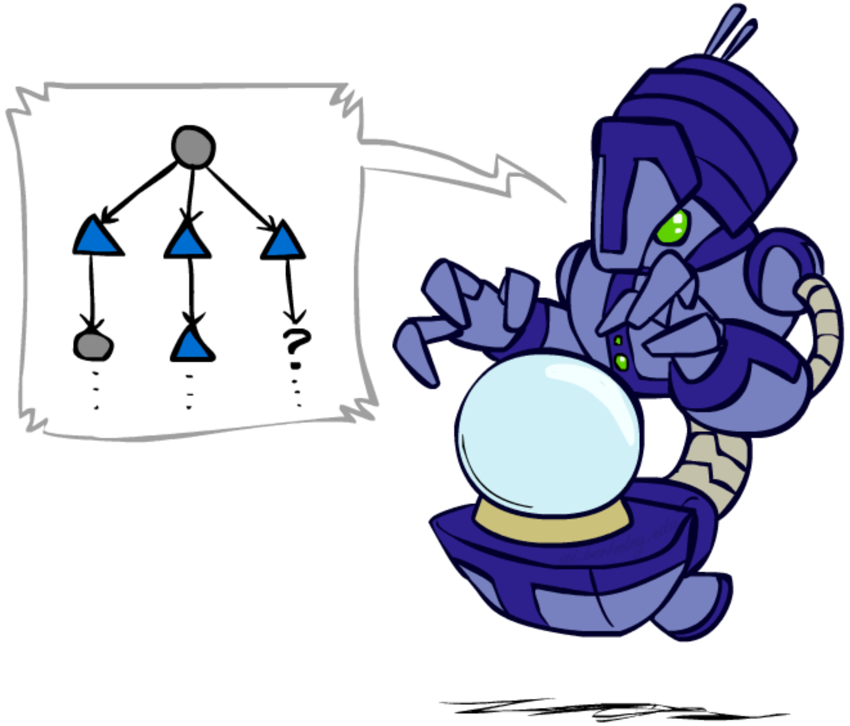
- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s’ (by doing the action!) and average

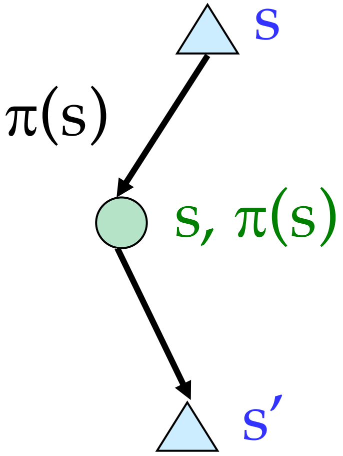
$$sample_1 = R(s, \pi(s), s_1') + \gamma V_k^{\pi}(s_1')$$
$$sample_2 = R(s, \pi(s), s_2') + \gamma V_k^{\pi}(s_2')$$
$$\ldots$$
$$sample_n = R(s, \pi(s), s_n') + \gamma V_k^{\pi}(s_n')$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i sample_i$$



Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often



- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha) sample$

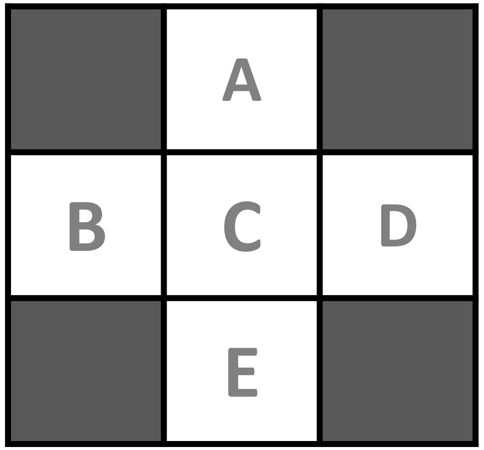
Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

Exponential Moving Average

- Exponential moving average
  - The running interpolation update: $\bar{x}_n = (1-\alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
  - Makes recent samples more important
  - Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States

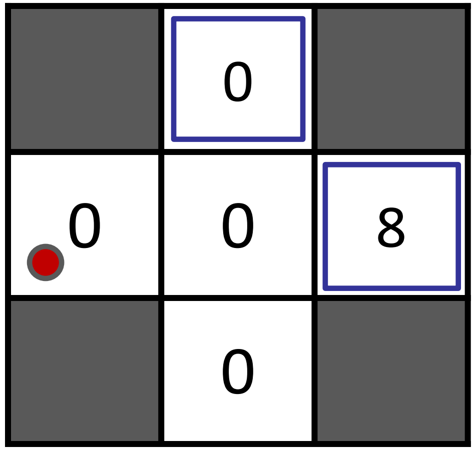


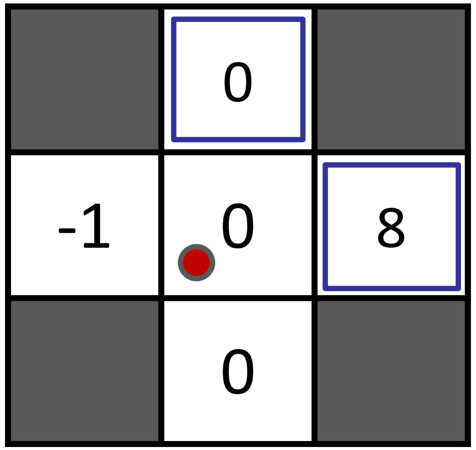
Assume: $\gamma = 1$, $\alpha = 1/2$

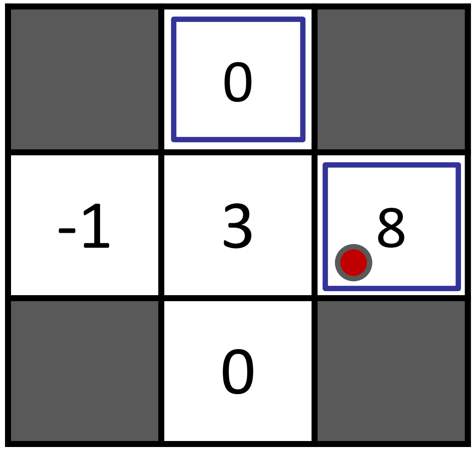
Observed Transitions

B, east, C, -2

C, east, D, -2







$$
\begin{aligned}
U^{\pi}(B) &\leftarrow (1/2)U^{\pi}(B) + \\
&\quad 1/2\,[-2 + U^{\pi}(C)] \\
&\leftarrow -1
\end{aligned}
$$

$$
\begin{aligned}
U^{\pi}(C) &\leftarrow (1/2)U^{\pi}(C) + \\
&\quad 1/2\,[-2 + U^{\pi}(D)] \\
&\leftarrow 3
\end{aligned}
$$

$$U^{\pi}(s) \leftarrow (1 - \alpha)U^{\pi}(s) + \alpha\,[R(s,\pi(s),s') + \gamma U^{\pi}(s')]$$

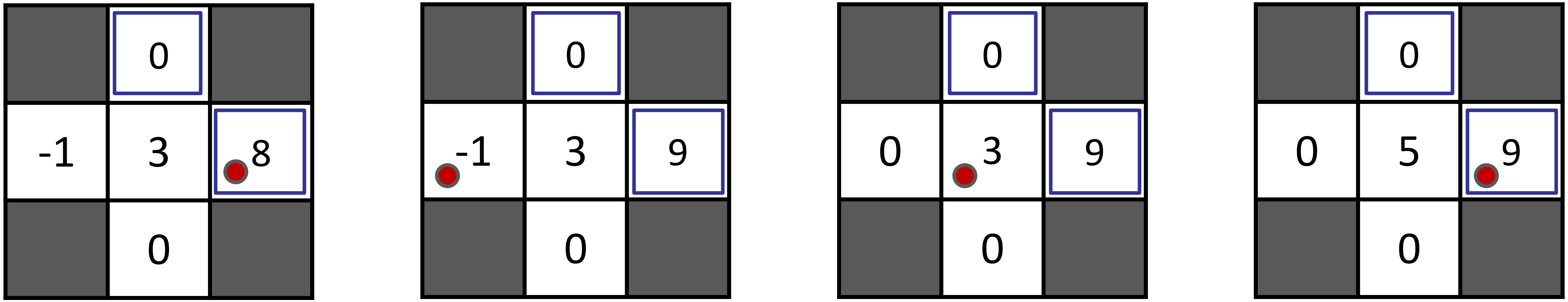
Example: Temporal Difference Learning

Observed Transitions

D, exit, , +10

B, east, C, -2

C, east, D, -2



$U^\pi(D) \leftarrow (1/2)U^\pi(D) + 1/2\,[+10]$
$\leftarrow 9$

$U^\pi(B) \leftarrow (1/2)U^\pi(B) + 1/2\,[-2 + U^\pi(C)]$
$\leftarrow -1/2+1.5 = 0$

$U^\pi(C) \leftarrow (1/2)U^\pi(C) + 1/2\,[-2 + U^\pi(D)]$
$\leftarrow 1.5 + 3.5 = 5$

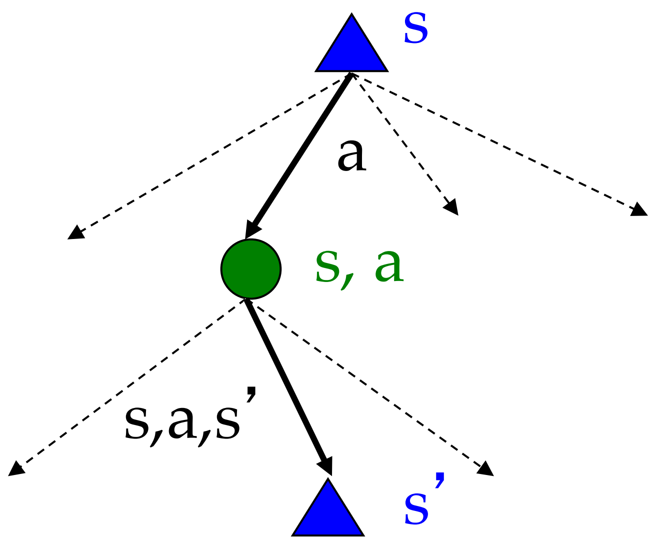
$$U^\pi(s) \leftarrow (1 - \alpha)U^\pi(s) + \alpha\,[R(s,\pi(s),s') + \gamma U^\pi(s')]$$

Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

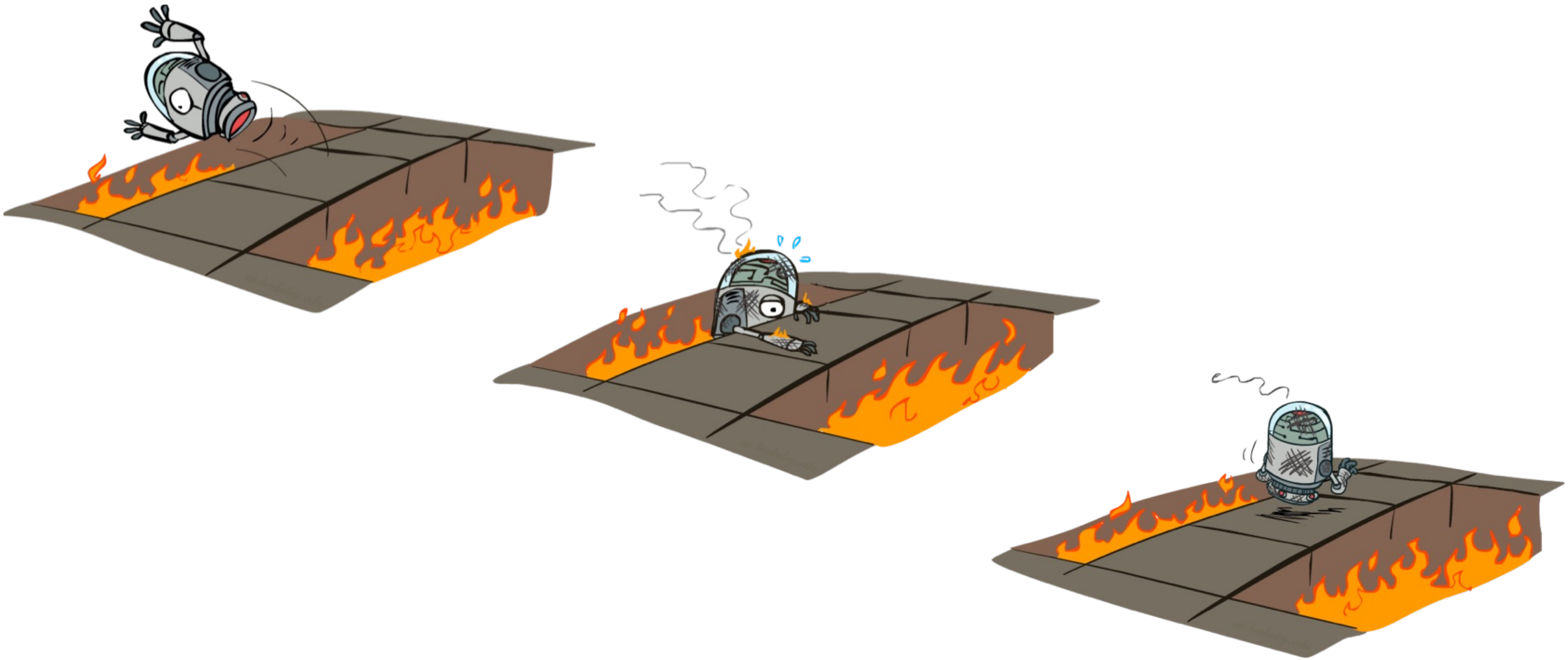
$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$



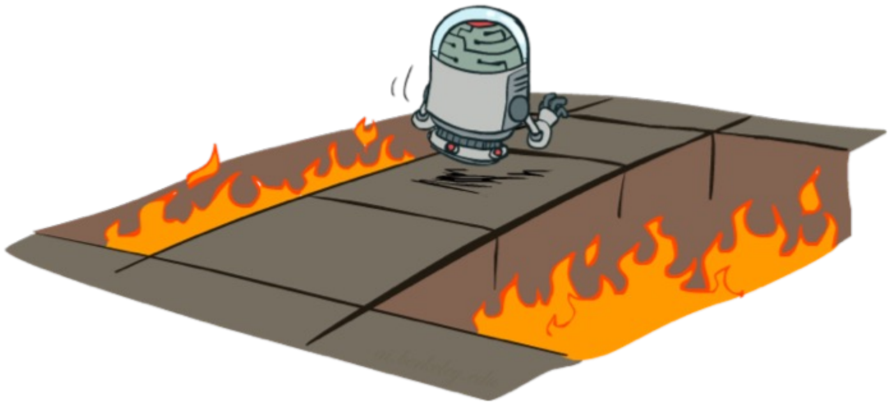
- Idea: learn Q-values, not values
- Makes action selection model-free too!

Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You choose the actions now
  - Goal: learn the optimal policy / values



- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Q_k, calculate the depth k+1 q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

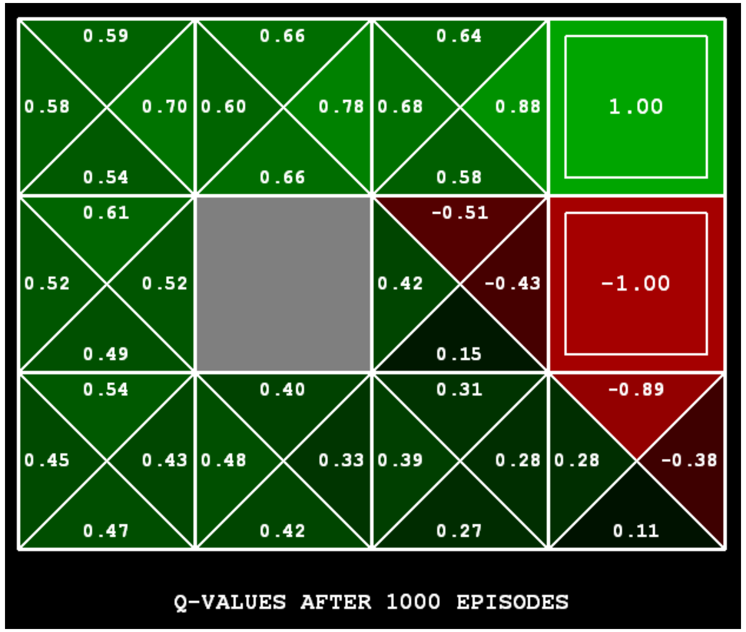
- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

no longer policy evaluation!

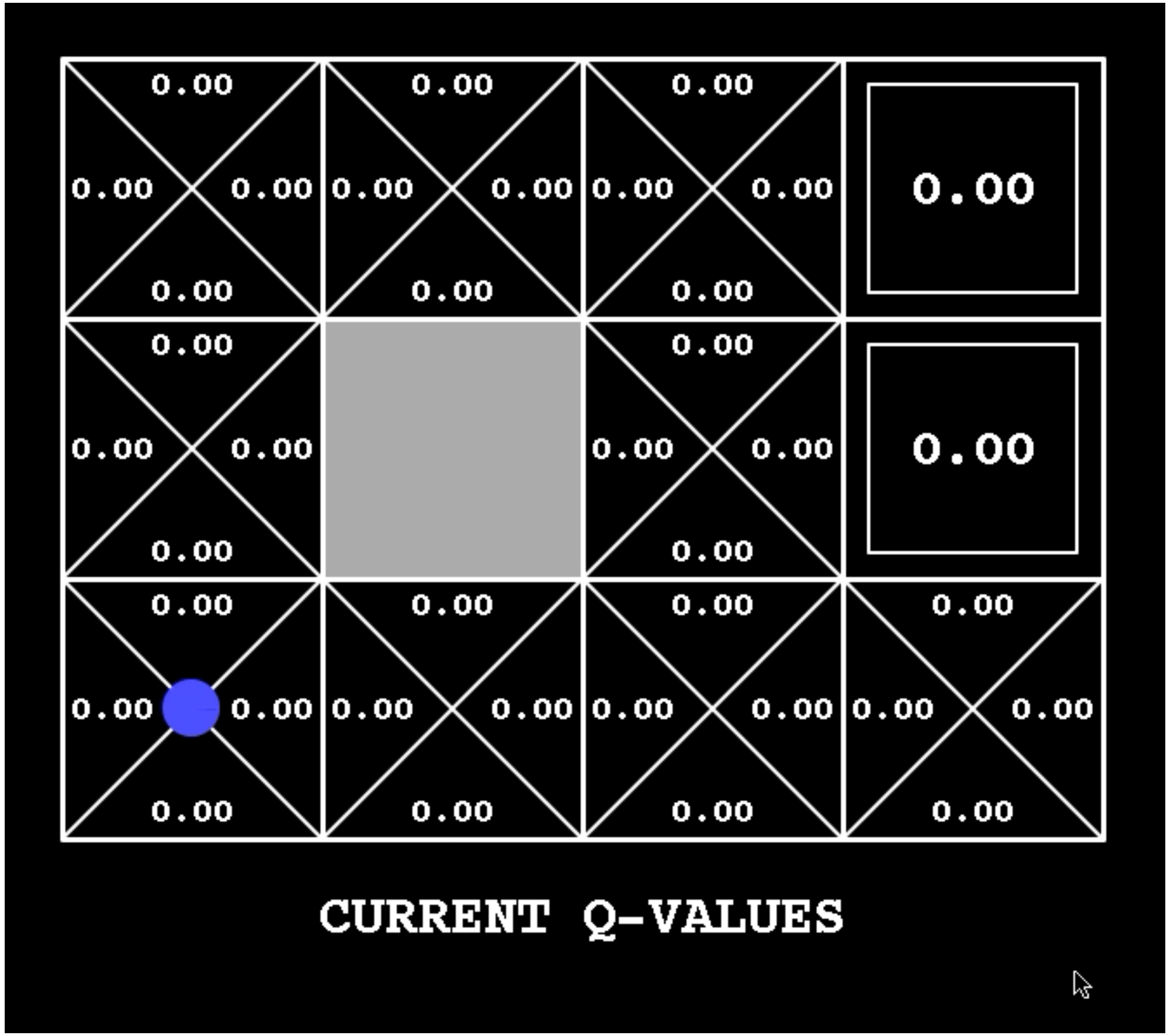
  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$

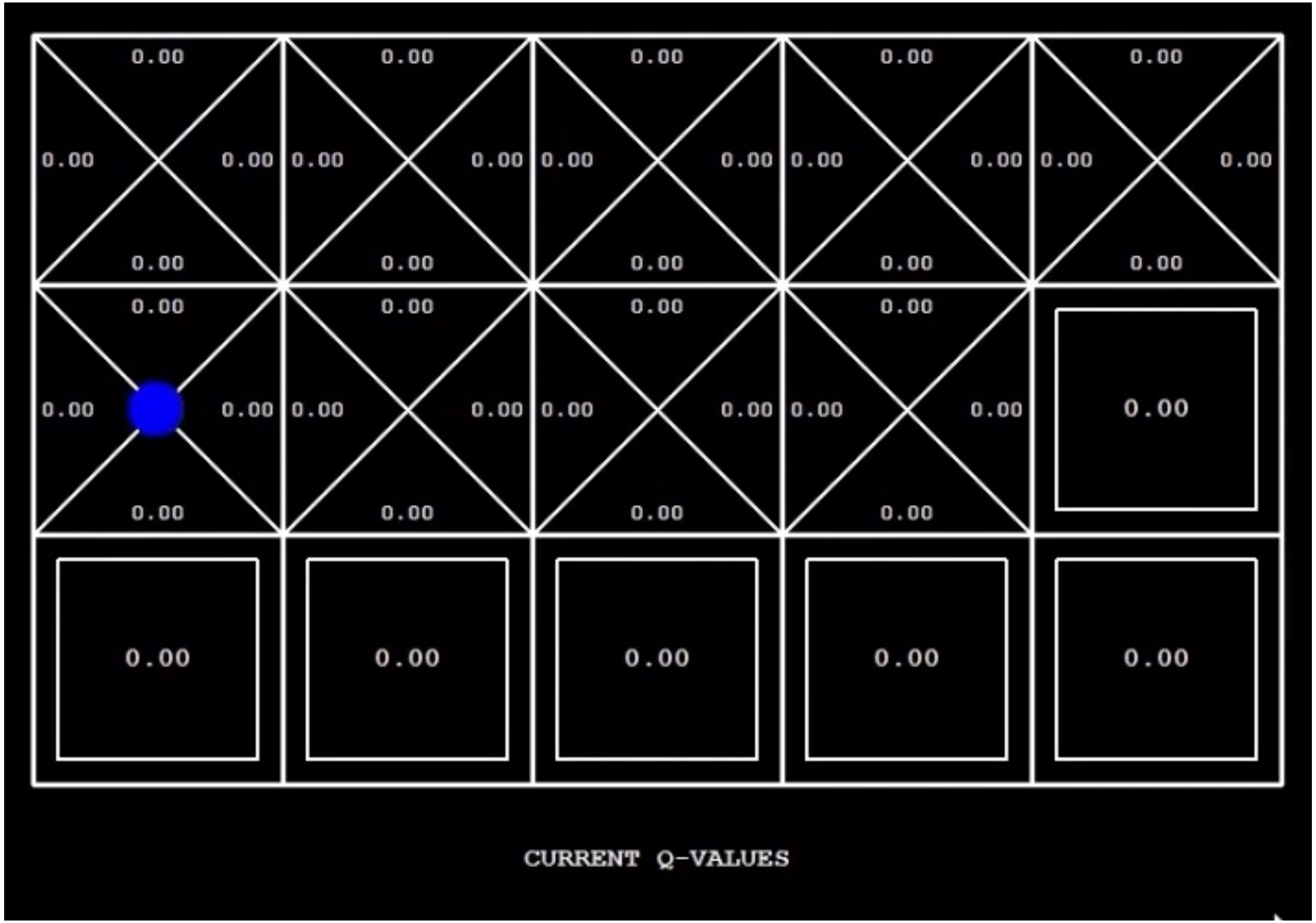


[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

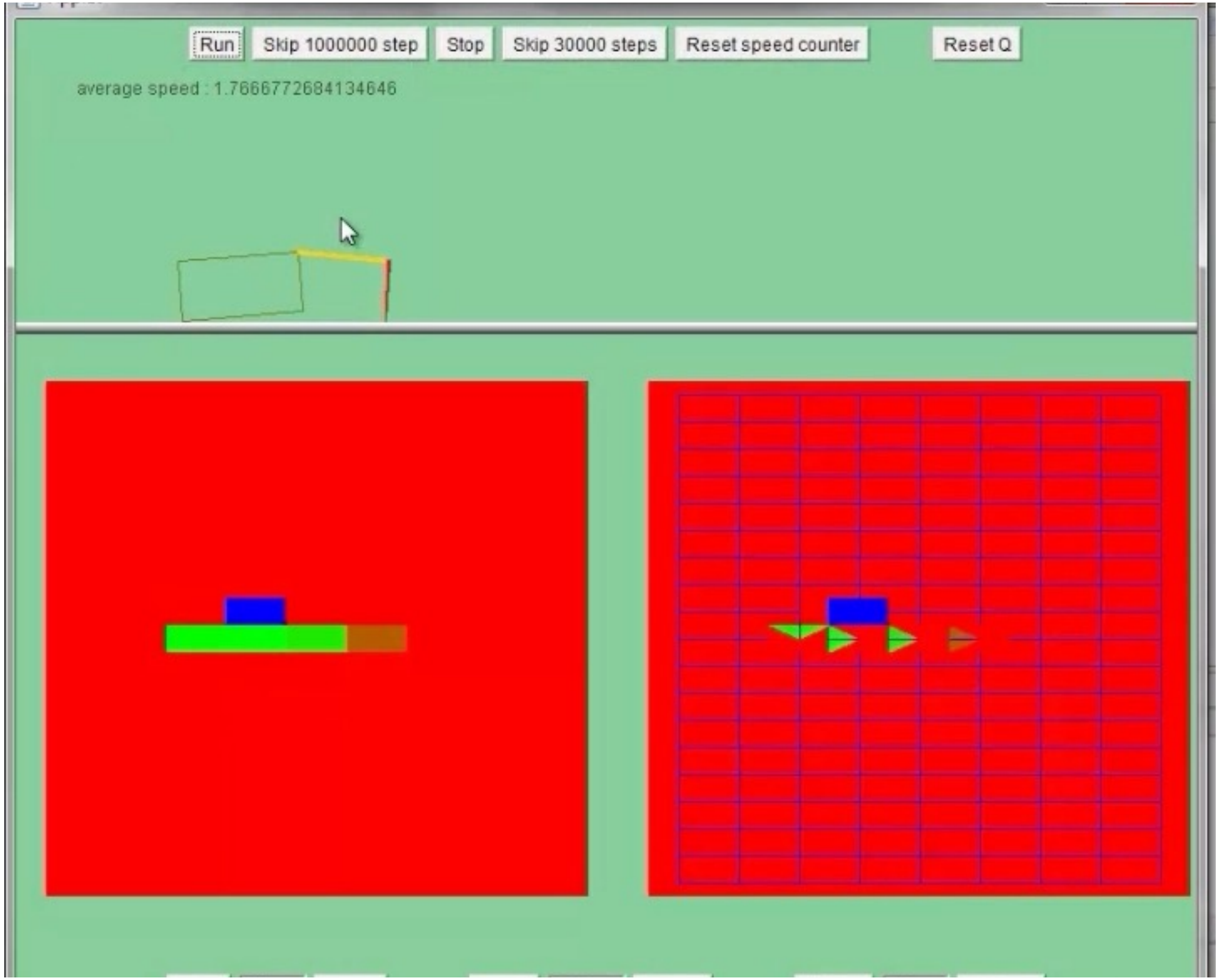
Q-Learning Demo



Video of Demo Q-Learning -- Gridworld

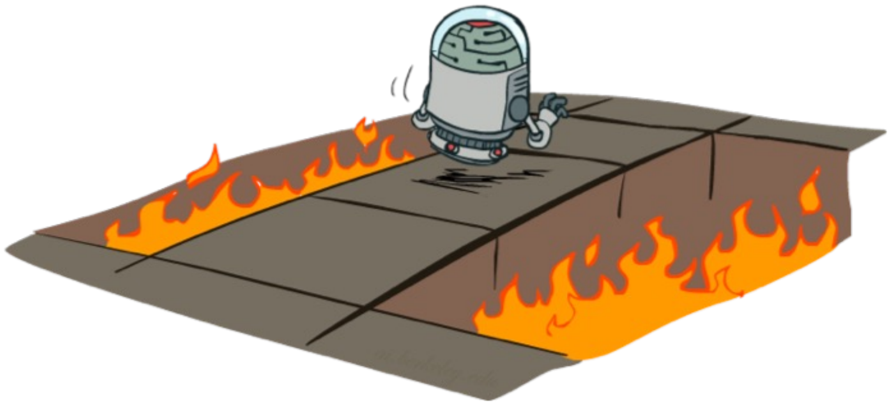


Video of Demo Q-Learning -- Crawler



Q-Learning: act according to current optimal (and also explore…)

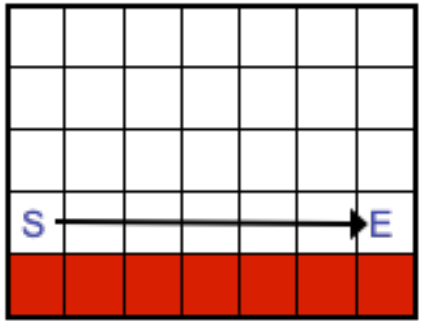
- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You choose the actions now
  - Goal: learn the optimal policy / values

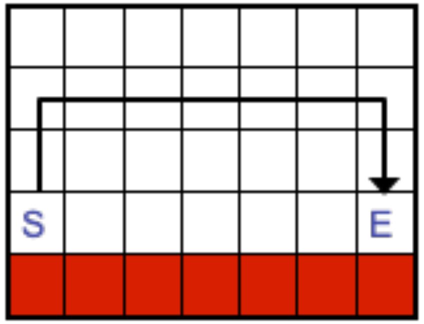


- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Q-Learning Properties

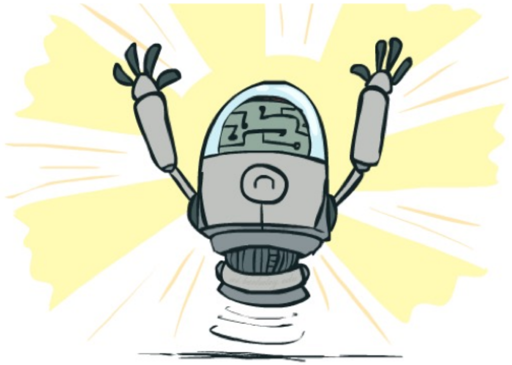
- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!



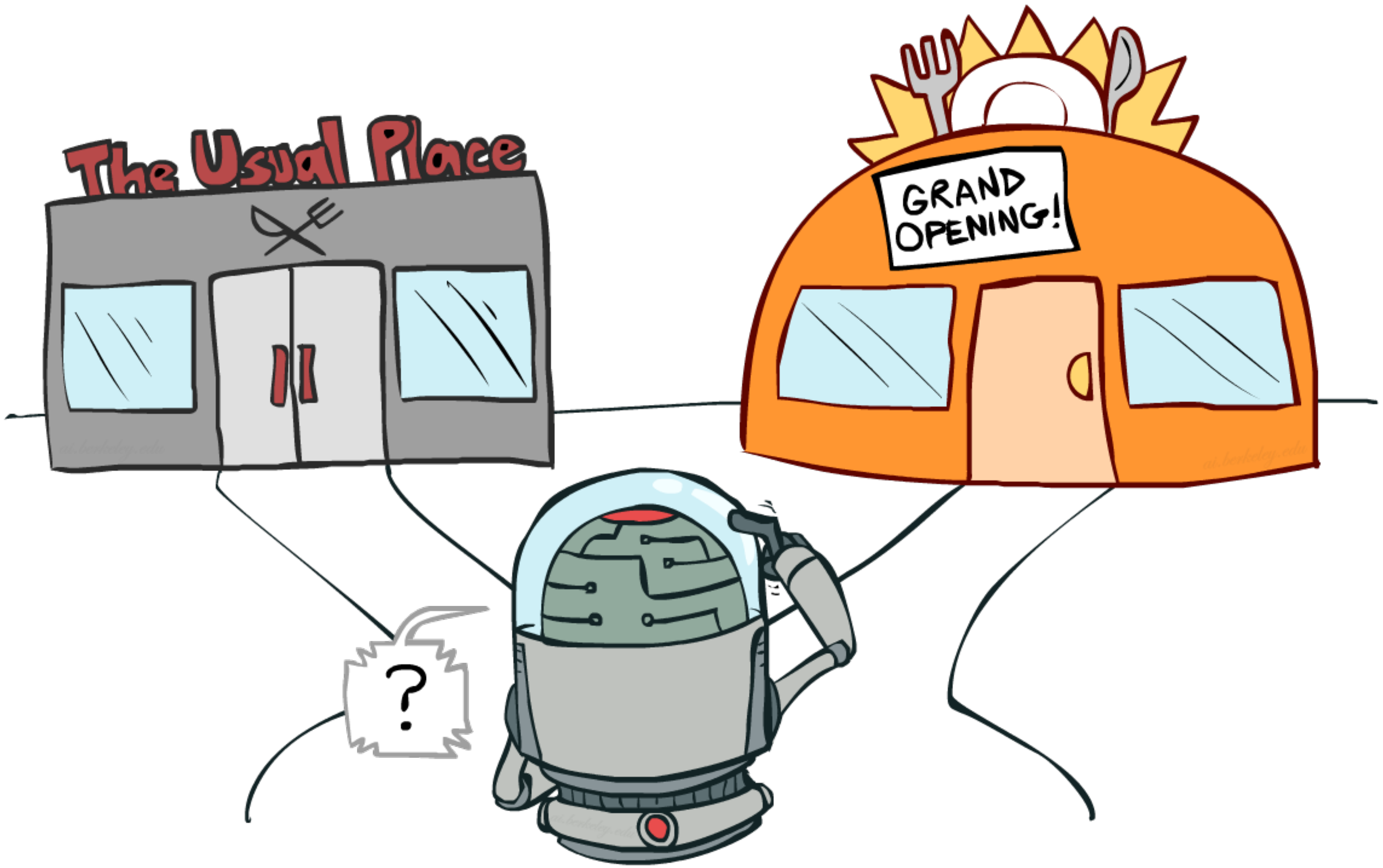


- This is called off-policy learning

- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)



Exploration vs. Exploitation



How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions (ε-greedy)
    - Every time step, flip a coin
    - With (small) probability ε, act randomly
    - With (large) probability 1-ε, act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

Exploration Functions

- When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

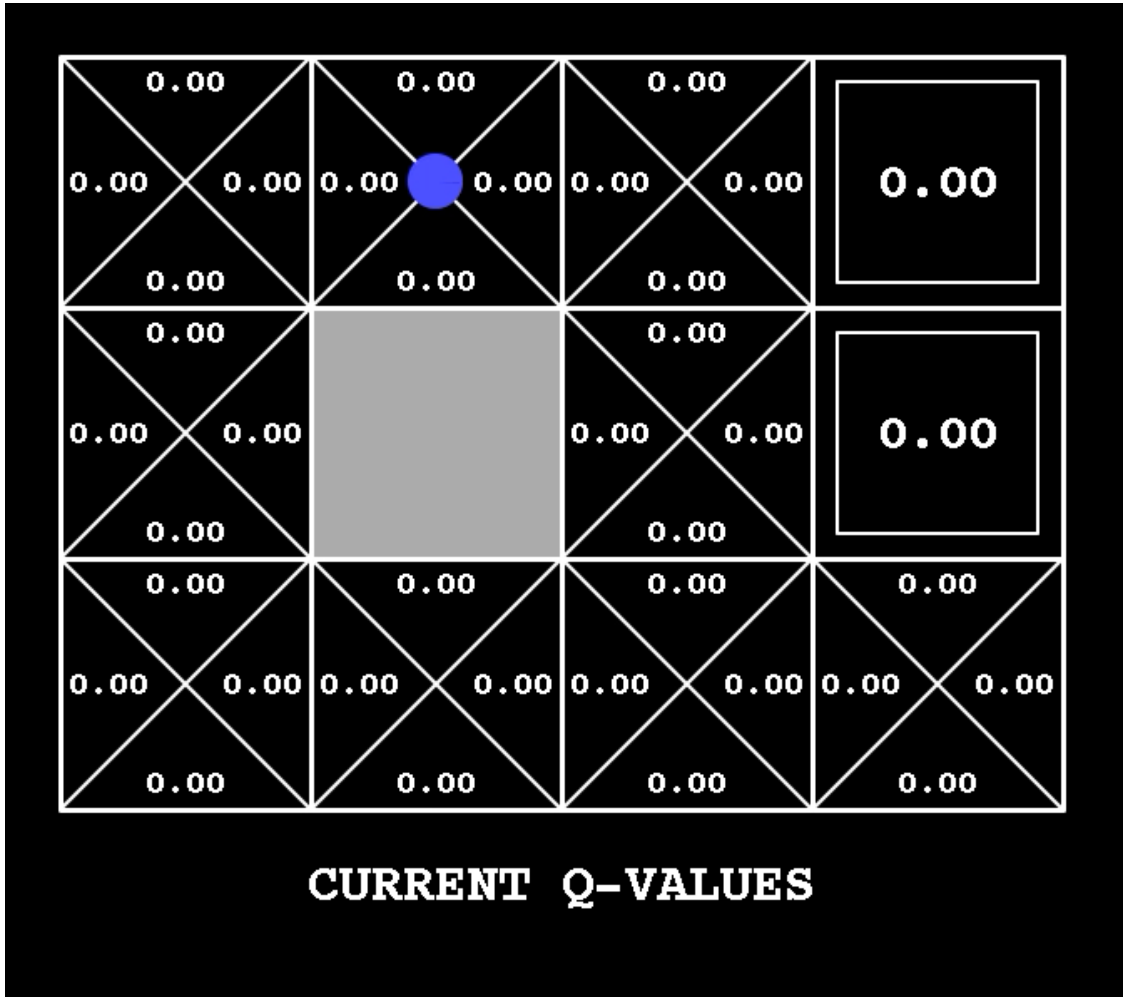
- Exploration function
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g. $f(u,n) = u + k/n$

    Regular Q-Update: $Q(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} Q(s',a')$

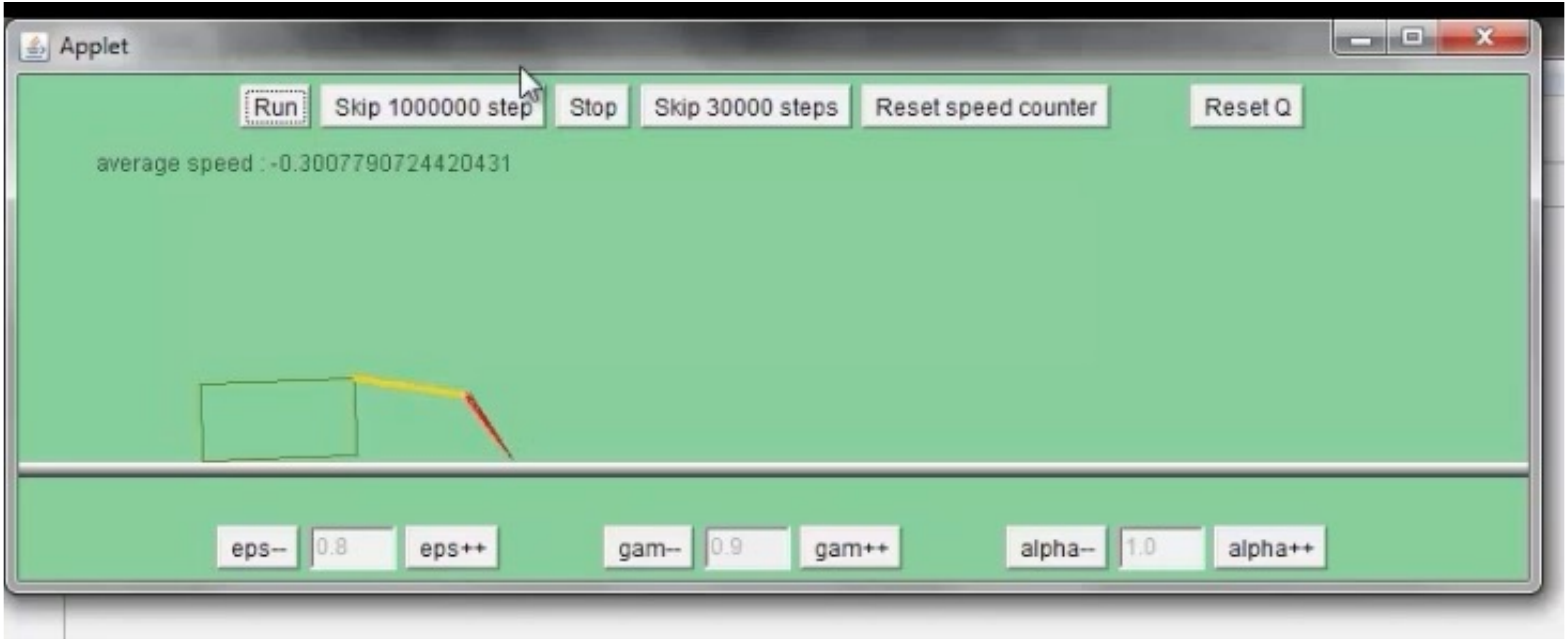
    Modified Q-Update: $Q(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} f(Q(s',a'),N(s',a'))$

  - Note: this propagates the "bonus" back to states that lead to unknown states as well!

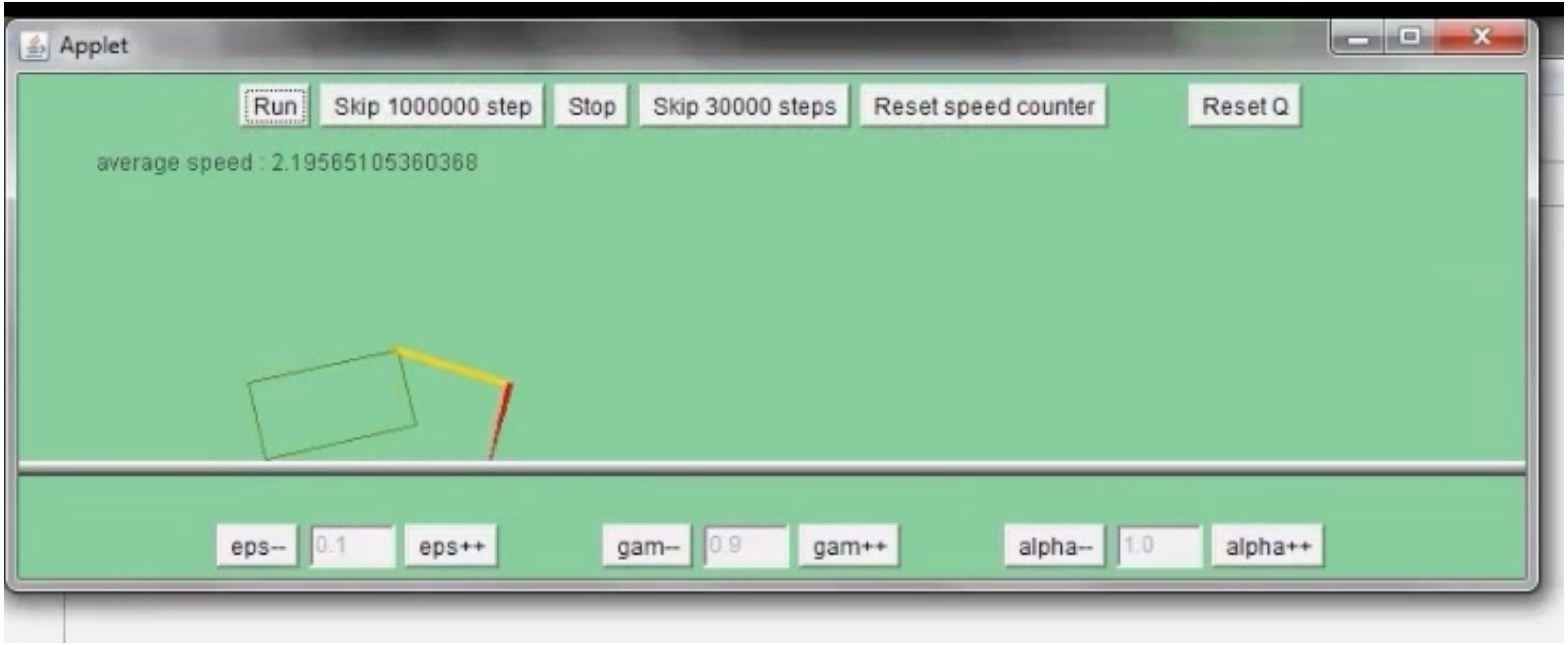
Q-Learn Epsilon Greedy



Video of Demo Q-learning – Epsilon-Greedy – Crawler

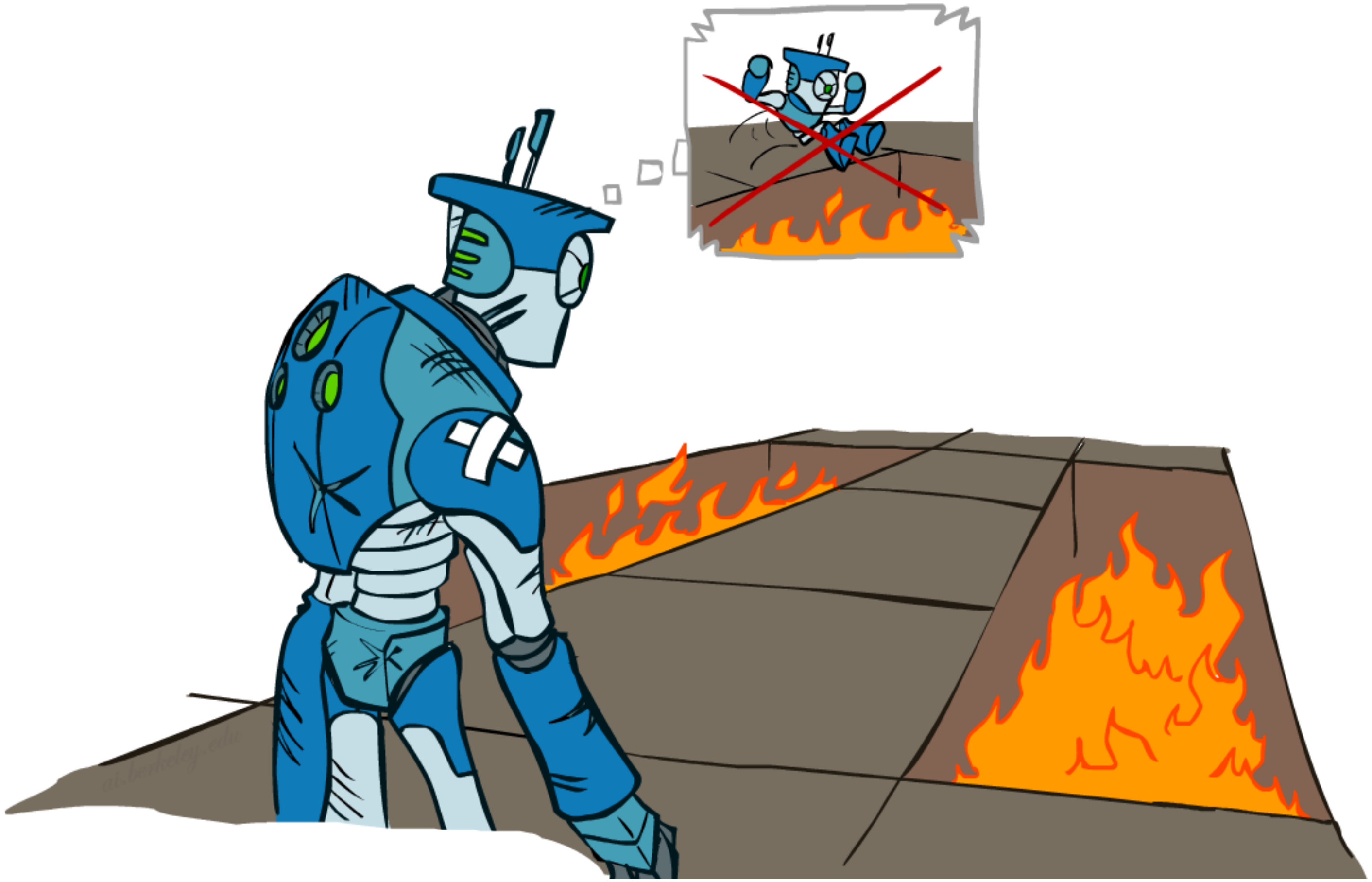


Video of Demo Q-learning – Exploration Function – Crawler

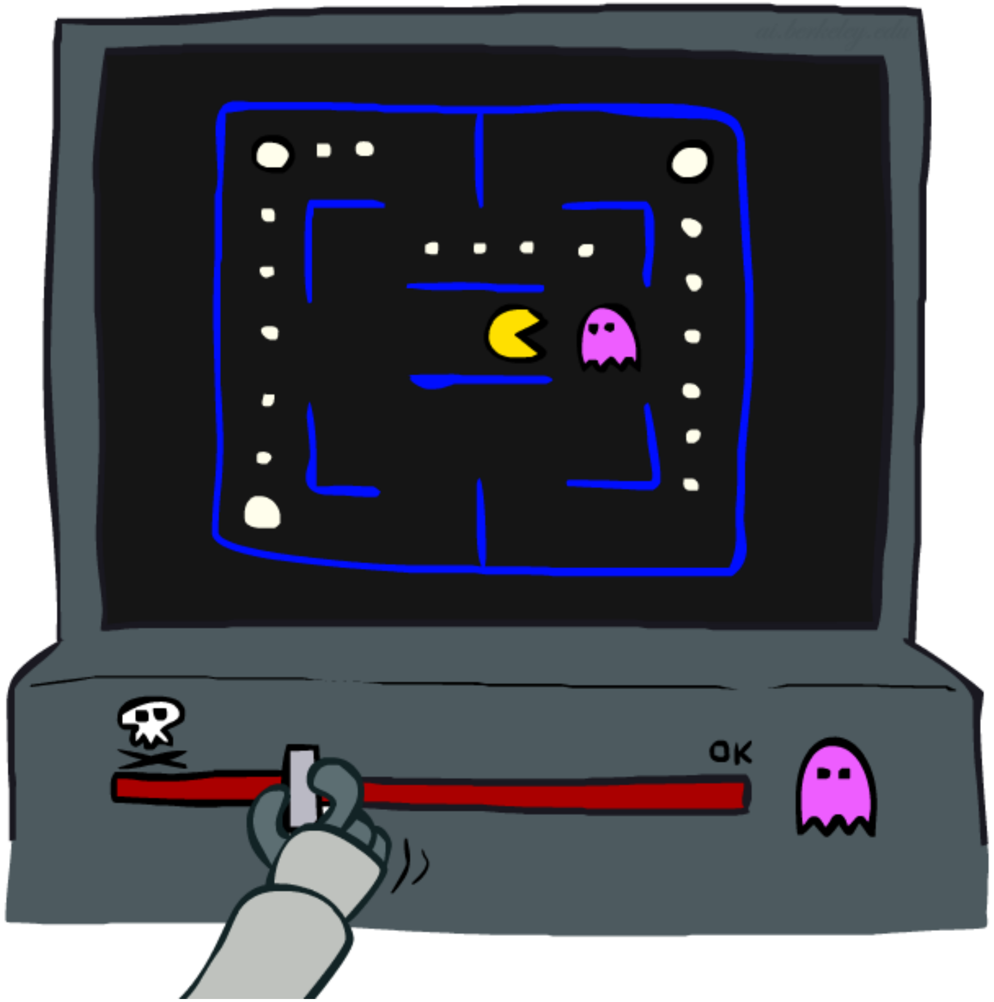


Regret

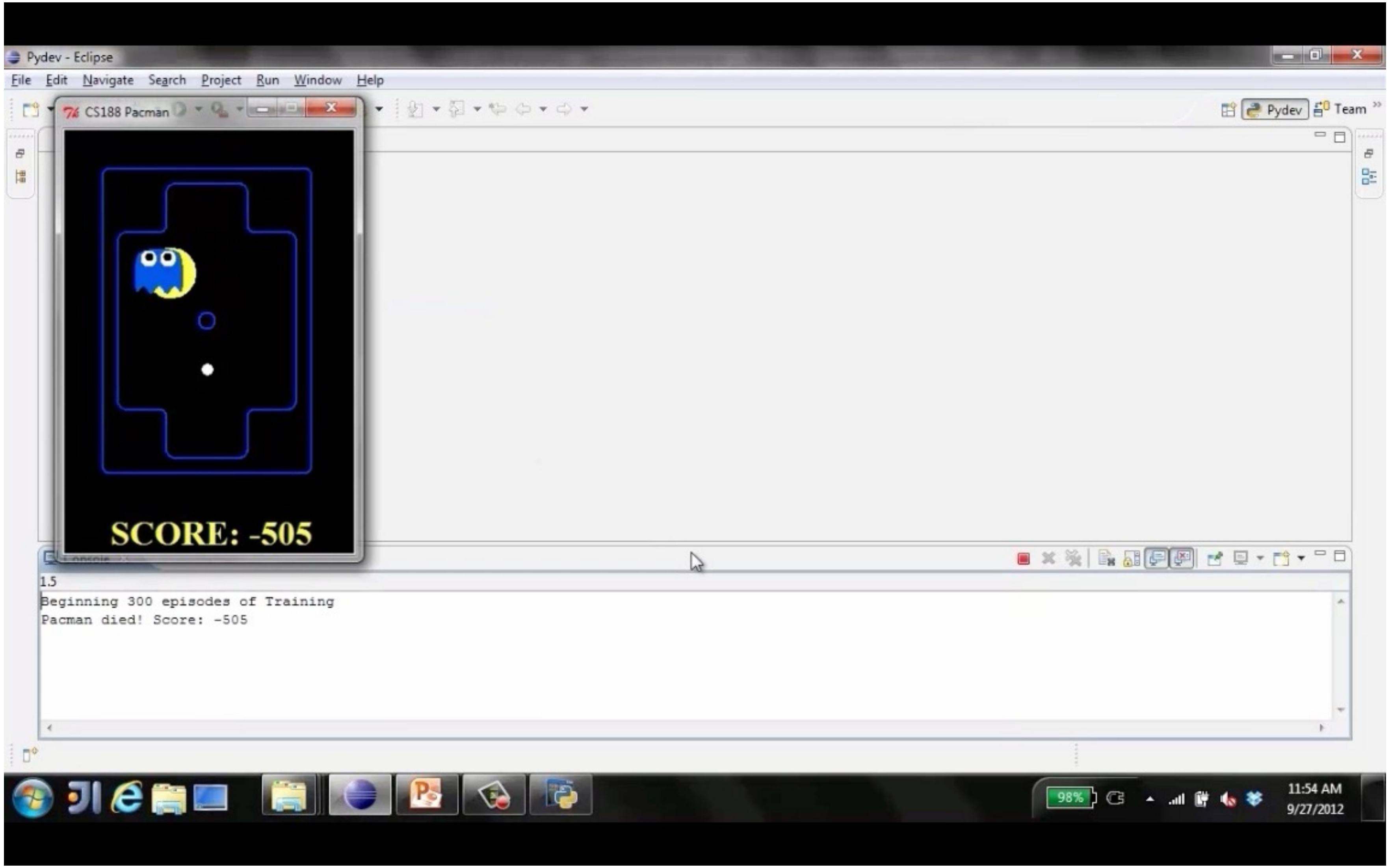
- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret



Approximate Q-Learning

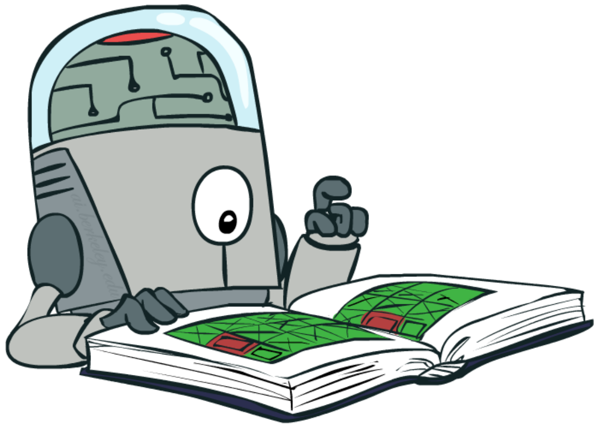


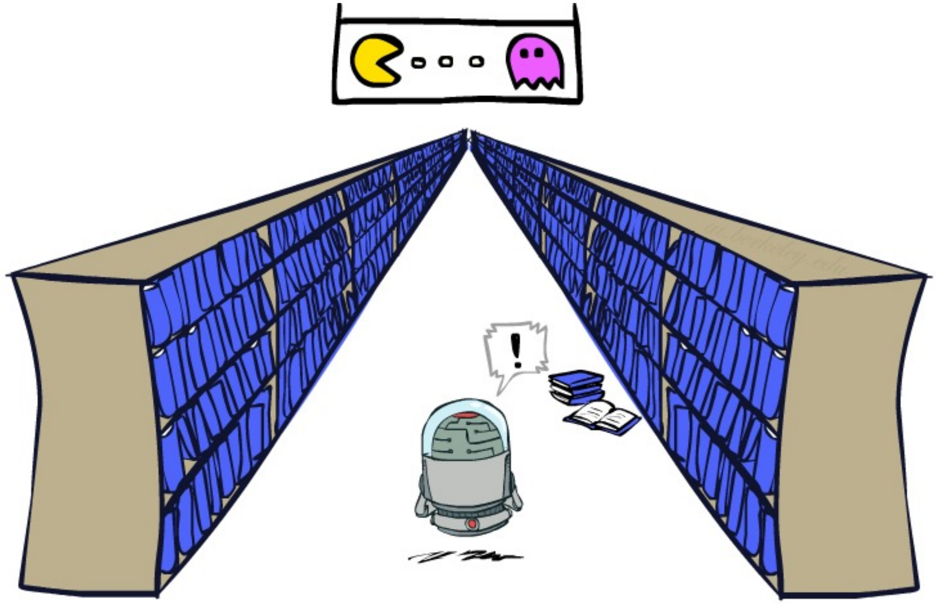
Video of Demo Q-Learning Pacman – Tricky – Watch All



Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

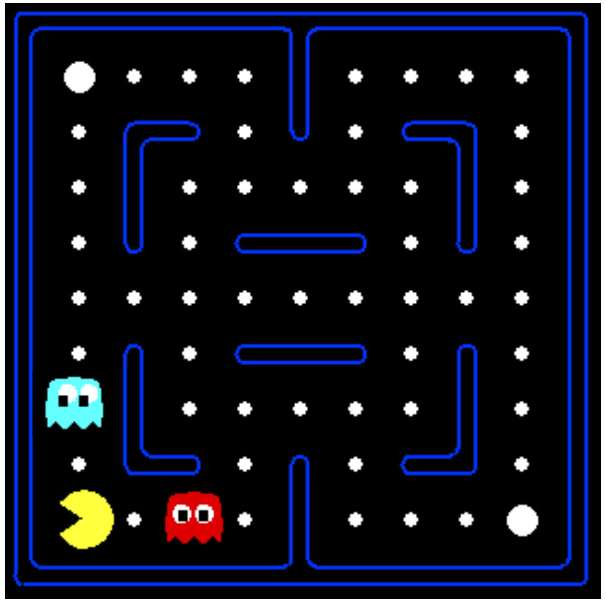




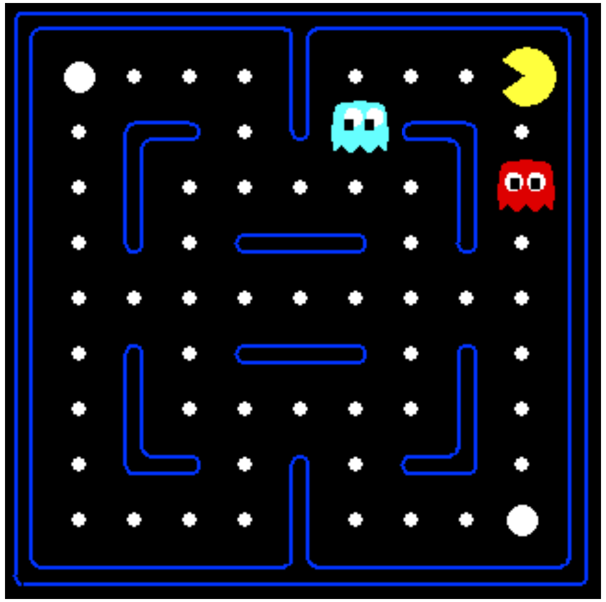
[demo – RL pacman]

Example: Pacman

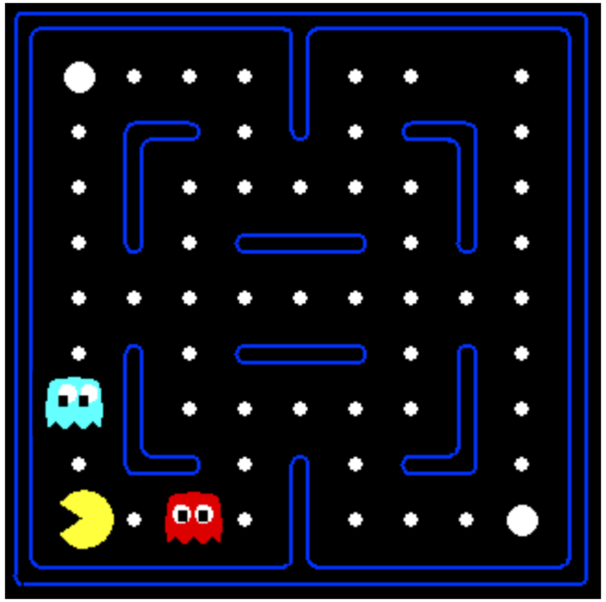
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:

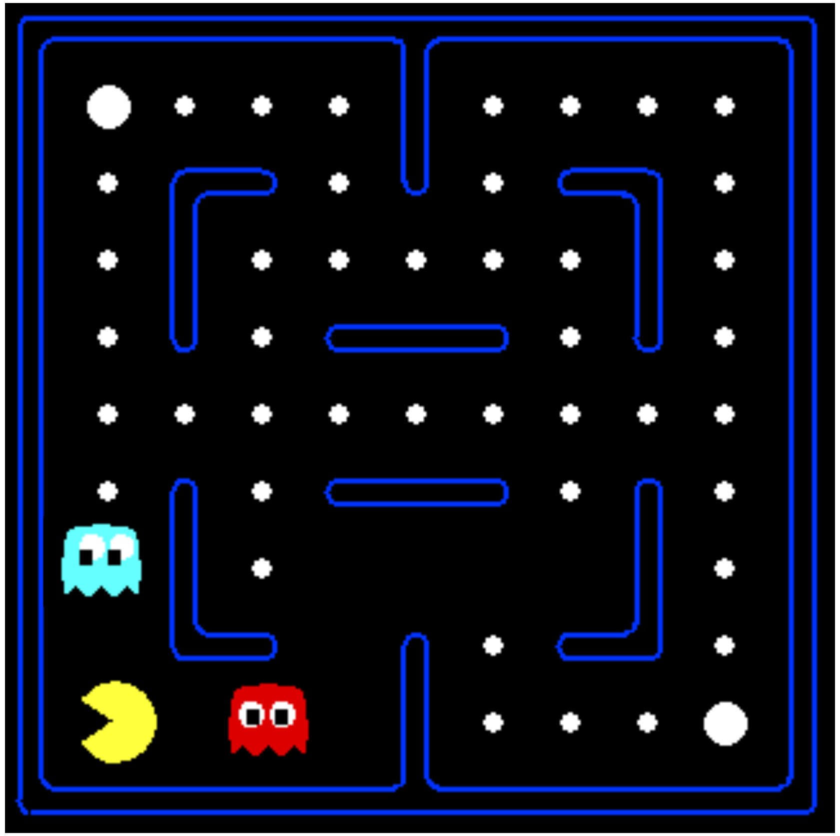


Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

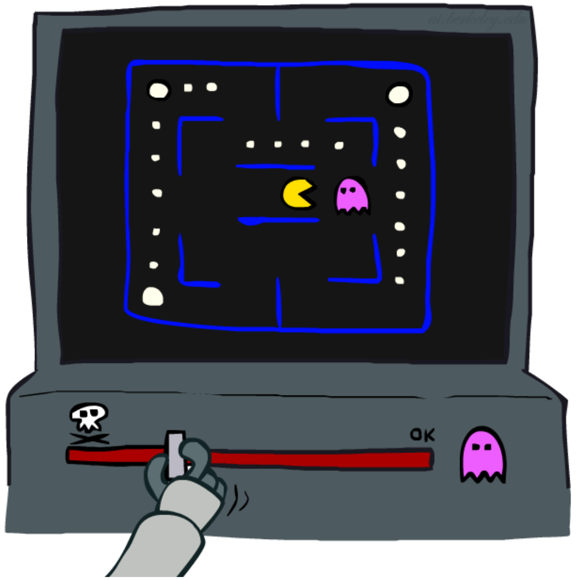
- Q-learning with linear Q-functions:

$$\text{transition } = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha \, [\text{difference}] \qquad \text{Exact Q's}$$

$$w_i \leftarrow w_i + \alpha \, [\text{difference}] \, f_i(s,a) \qquad \text{Approximate Q's}$$

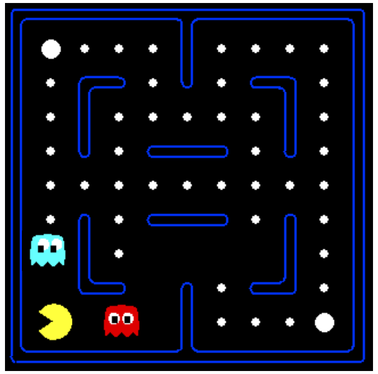


- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s

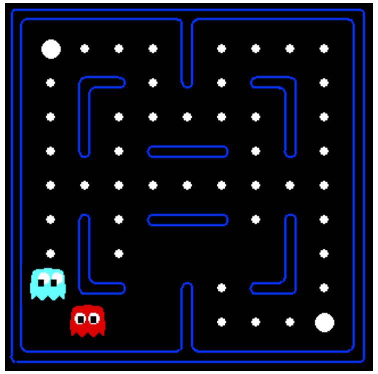


$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$a = \text{NORTH}$
$r = -500$

s'



$Q(s, \text{NORTH}) = +1$

$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

$Q(s', \cdot) = 0$

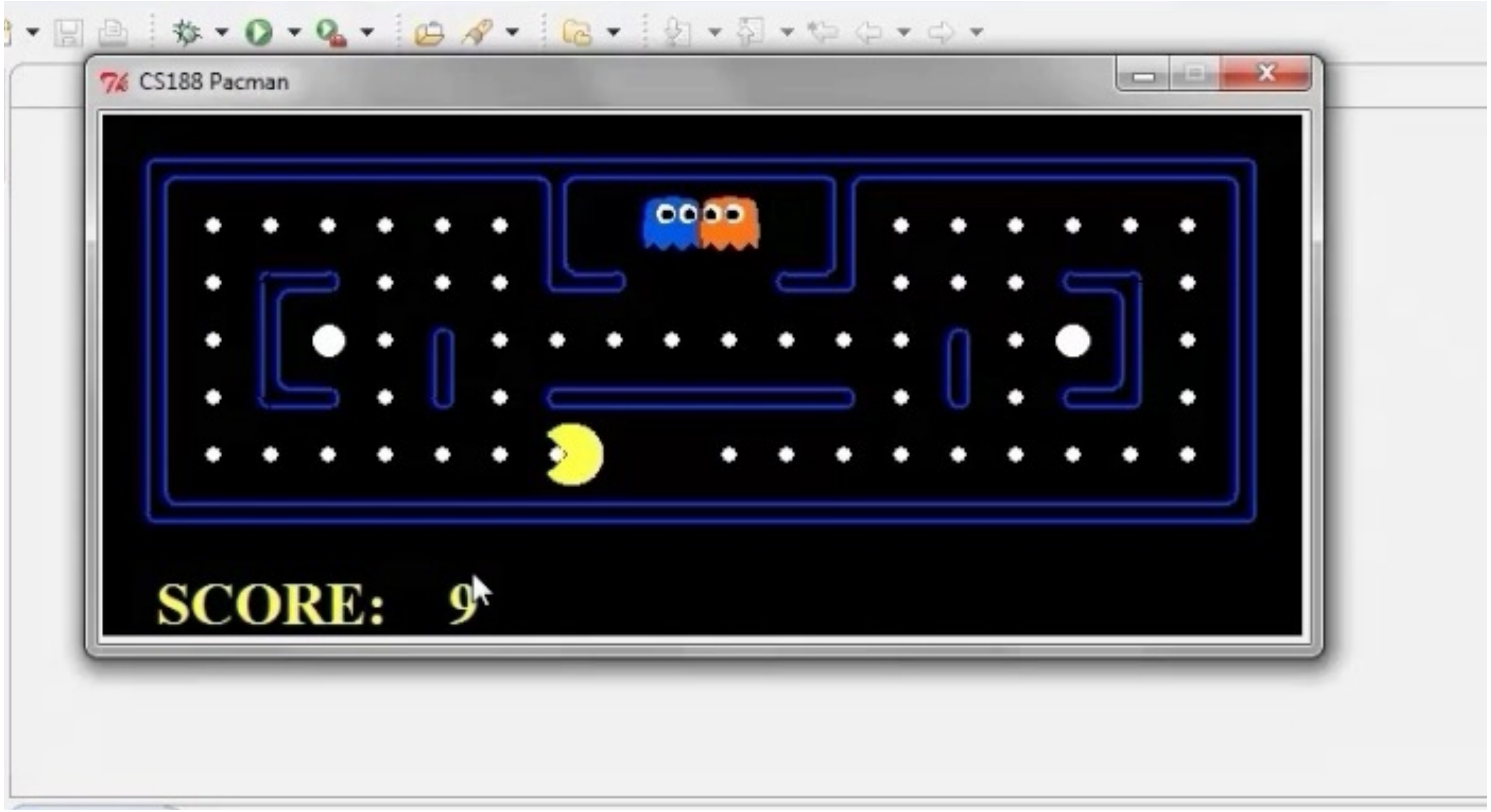
$\text{difference} = -501$

$w_{DOT} \leftarrow 4.0 + \alpha \left[-501\right] 0.5$

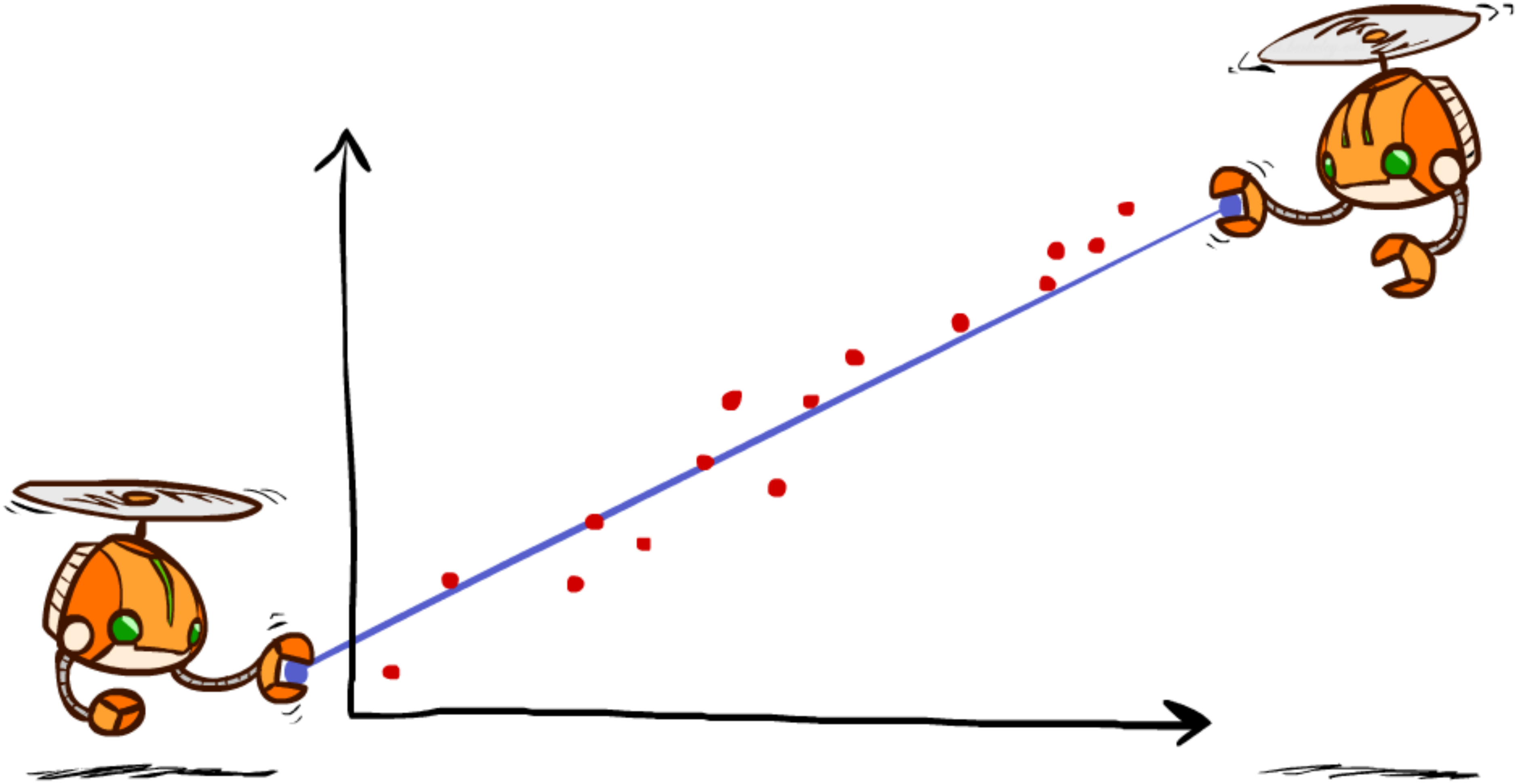
$w_{GST} \leftarrow -1.0 + \alpha \left[-501\right] 1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

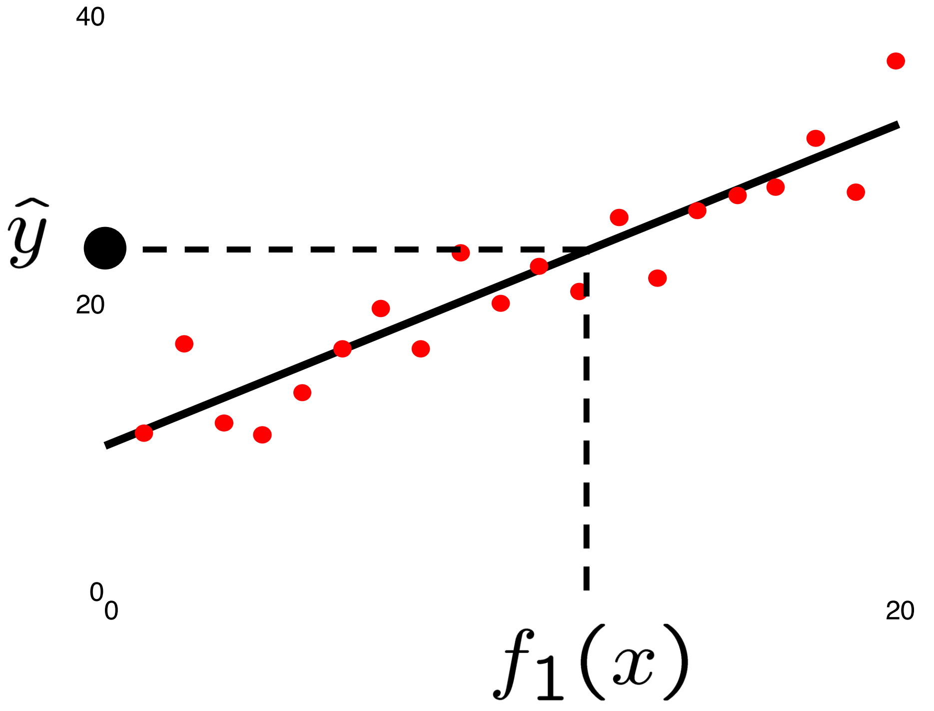
Video of Demo Approximate Q-Learning -- Pacman

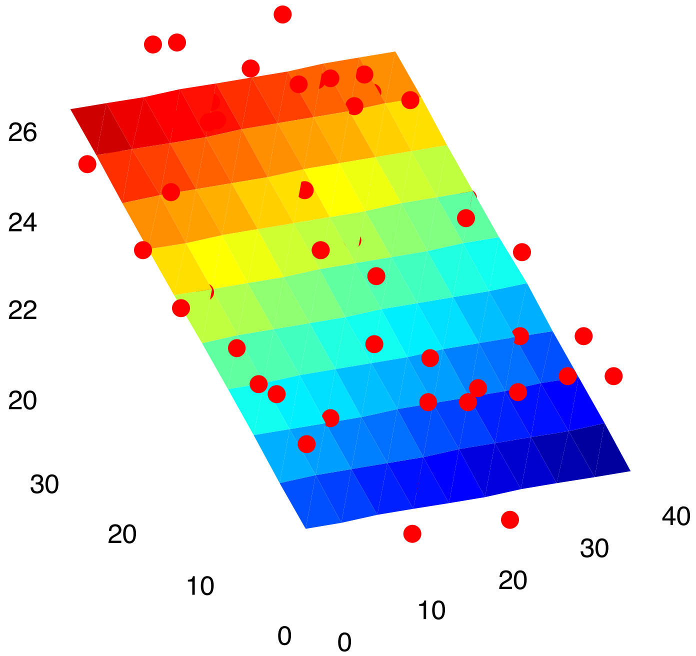


Bonus: Q-Learning and Least Squares*



Linear Approximation: Regression*





Prediction:

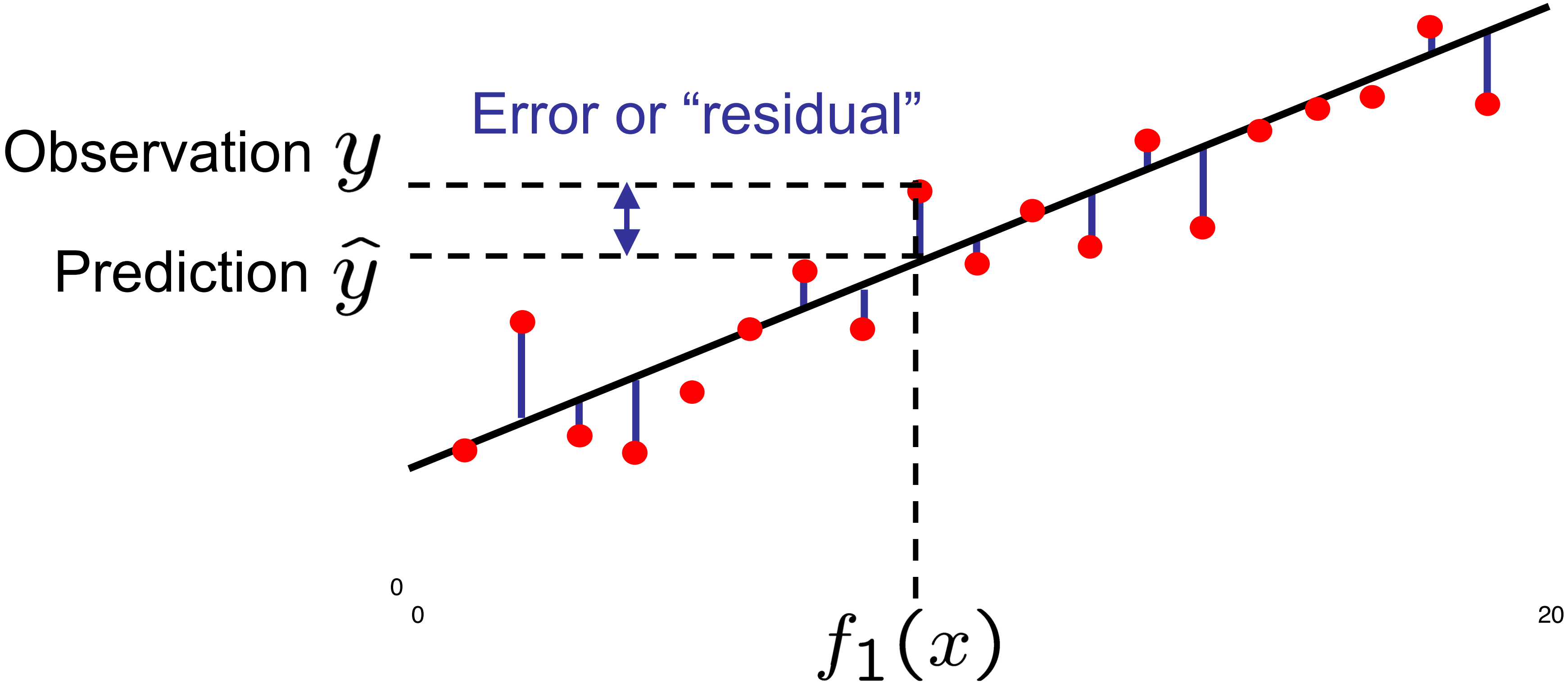
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares*

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



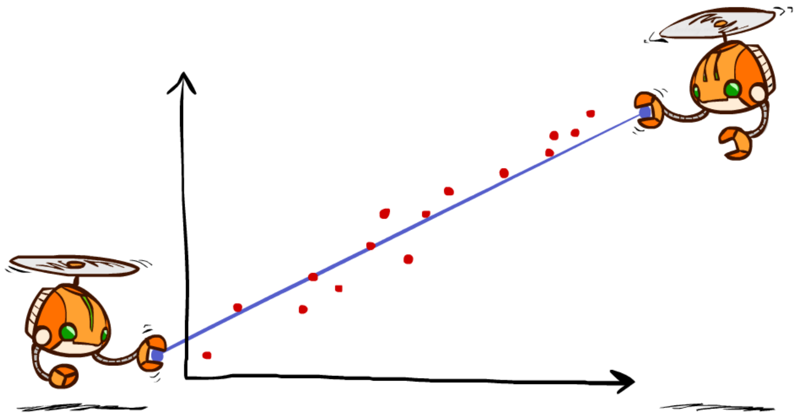
Minimizing Error*

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial \, \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

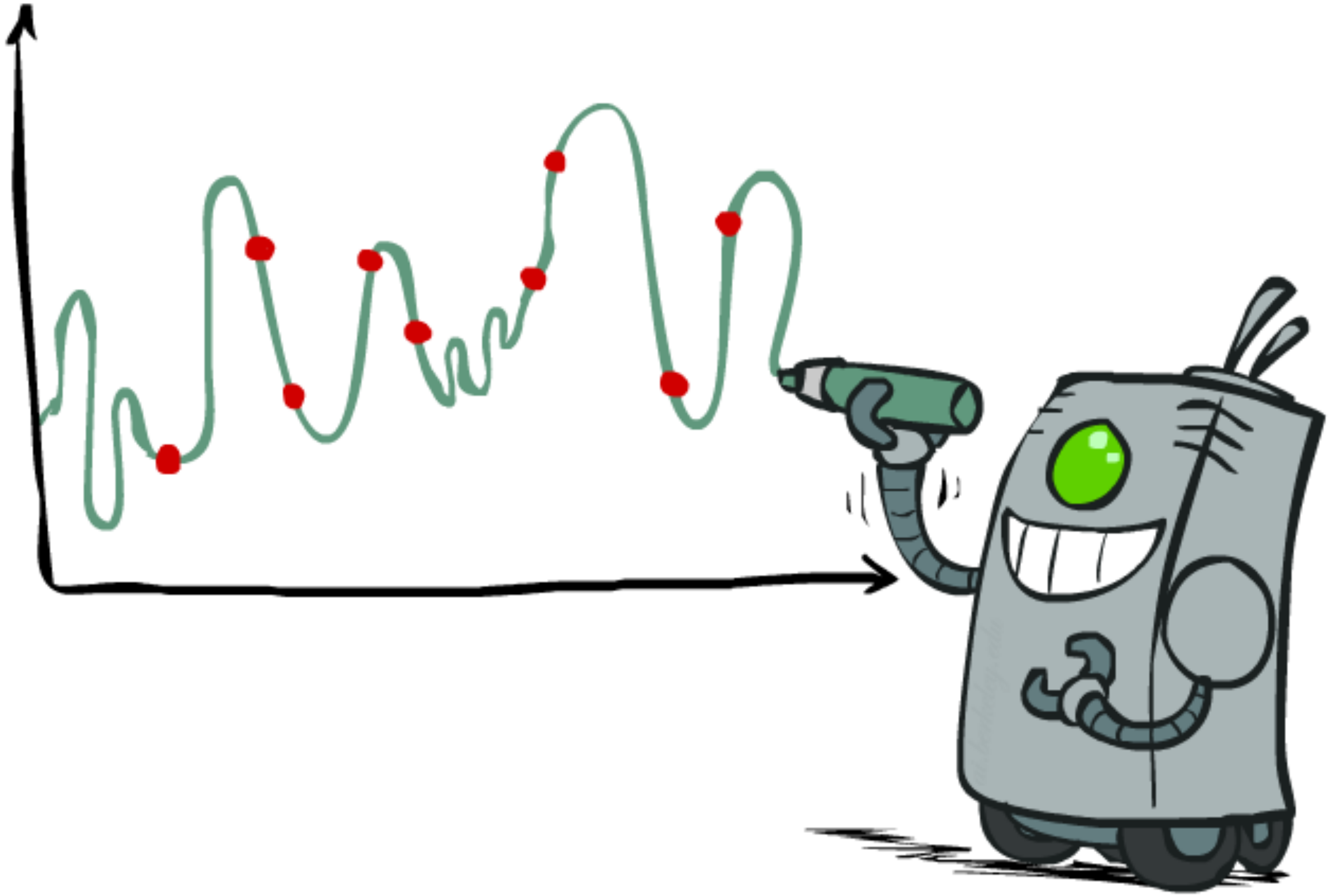


Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a)\right] f_m(s, a)$$

"target" "prediction"

Overfitting: Why Limiting Capacity Can Help



Summary: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

**use features to generalize*

Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

**use features to generalize*

Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	Value Learning

Conclusion

- We've seen how AI methods can solve problems in:
  - Search
  - Games
  - Markov Decision Problems
  - Reinforcement Learning

- Next up: Uncertainty and Learning!