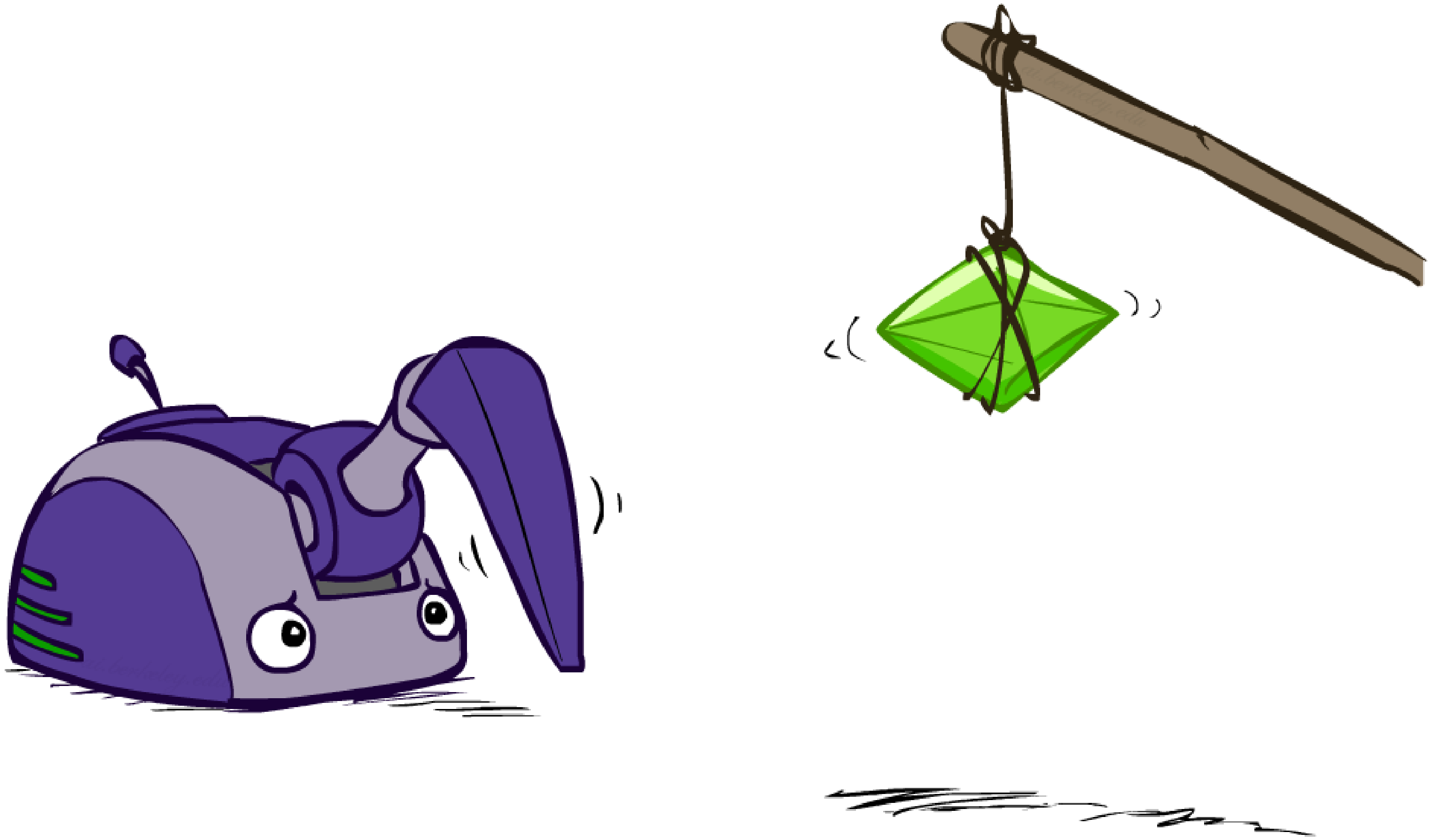
CSE 573 PMP: Artificial Intelligence

Hanna Hajishirzi

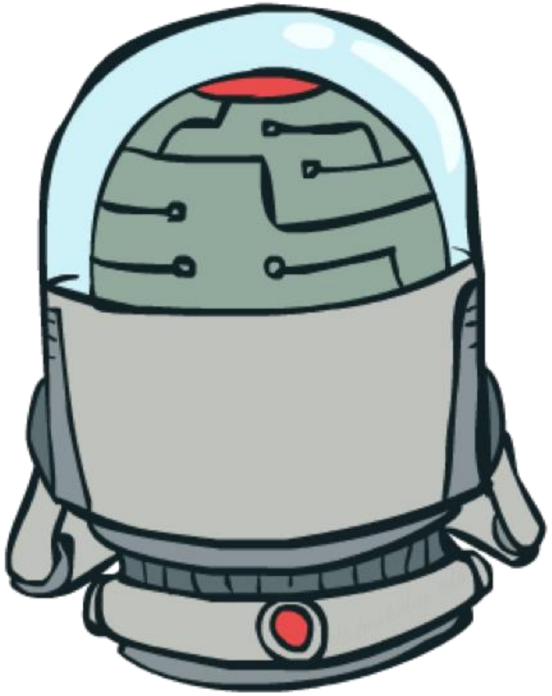
Reinforcement Learning

slides adapted from
Dan Klein, Pieter Abbeel ai.berkeley.edu
And Dan Weld, Luke Zettlemoyer

Reinforcement Learning



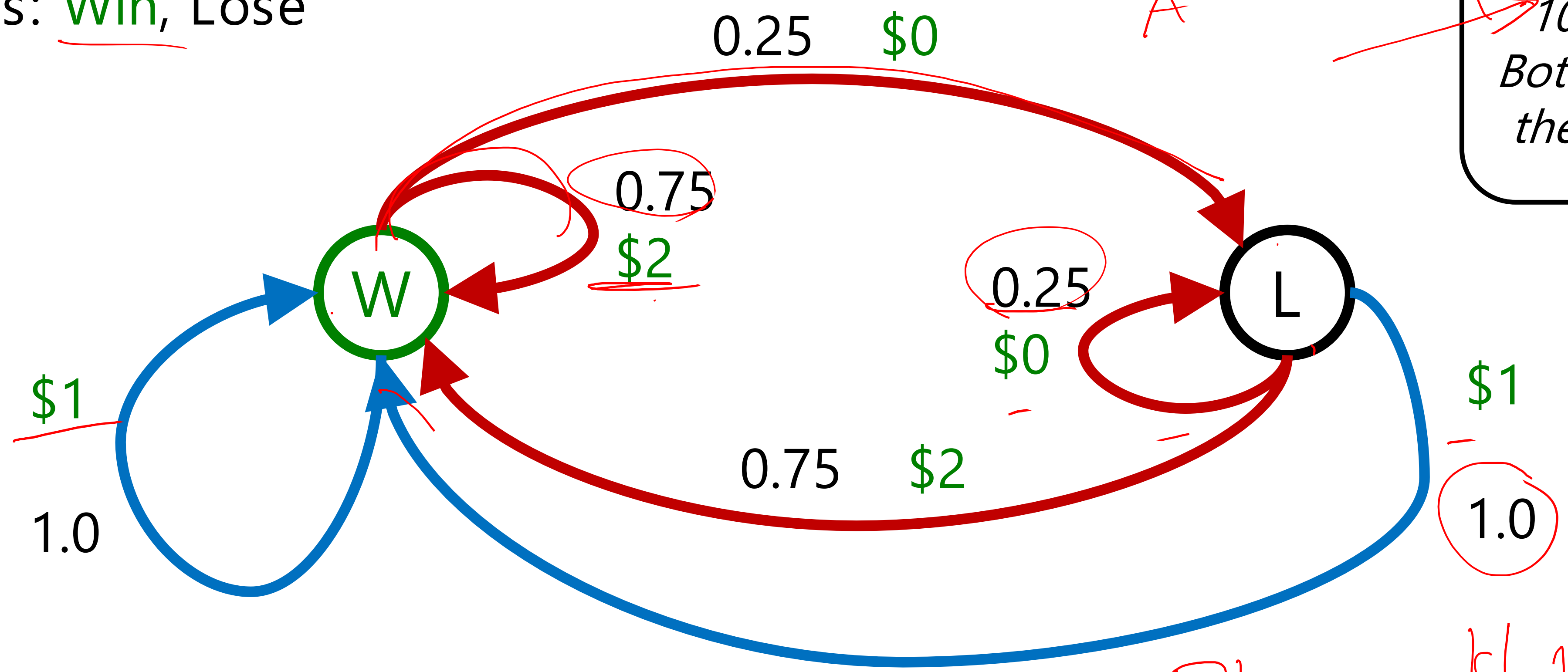
Double Bandits



Double-Bandit MDP

S
T
R
A

- Actions: *Blue*, *Red*
- States: Win, Lose



No discount
10 time steps
Both states have the same value

Blue : $10
Red : $15

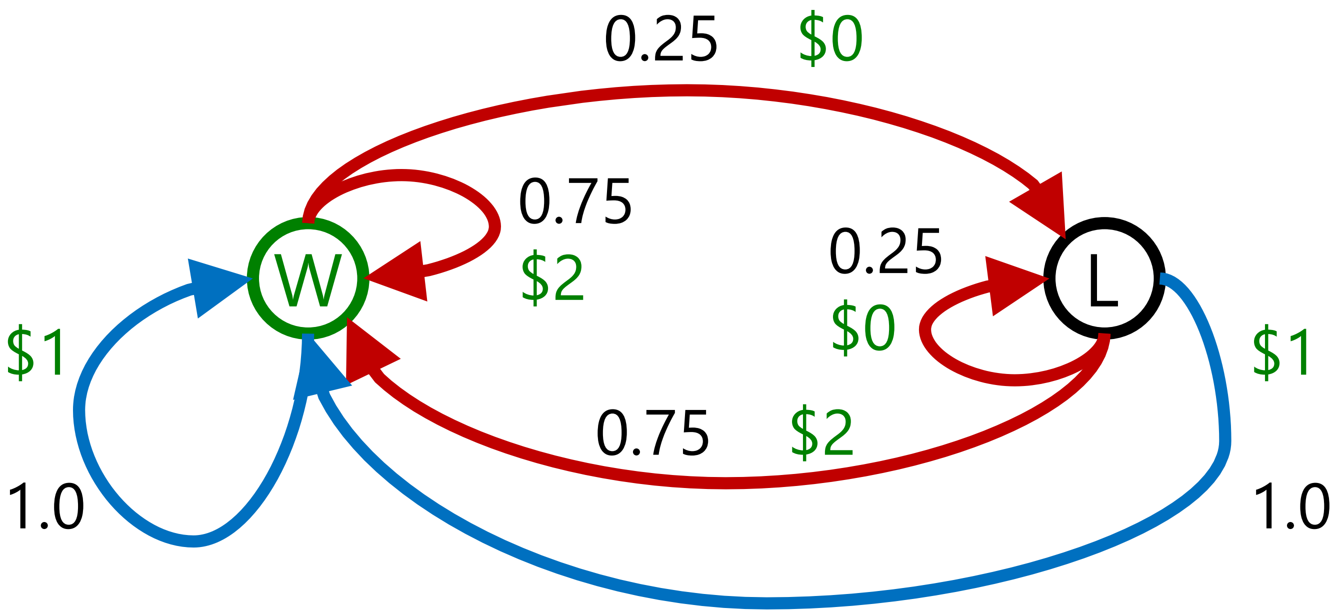
Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount

10 time steps

	Value
Play Red	15
Play Blue	10



Let's Play!

$15

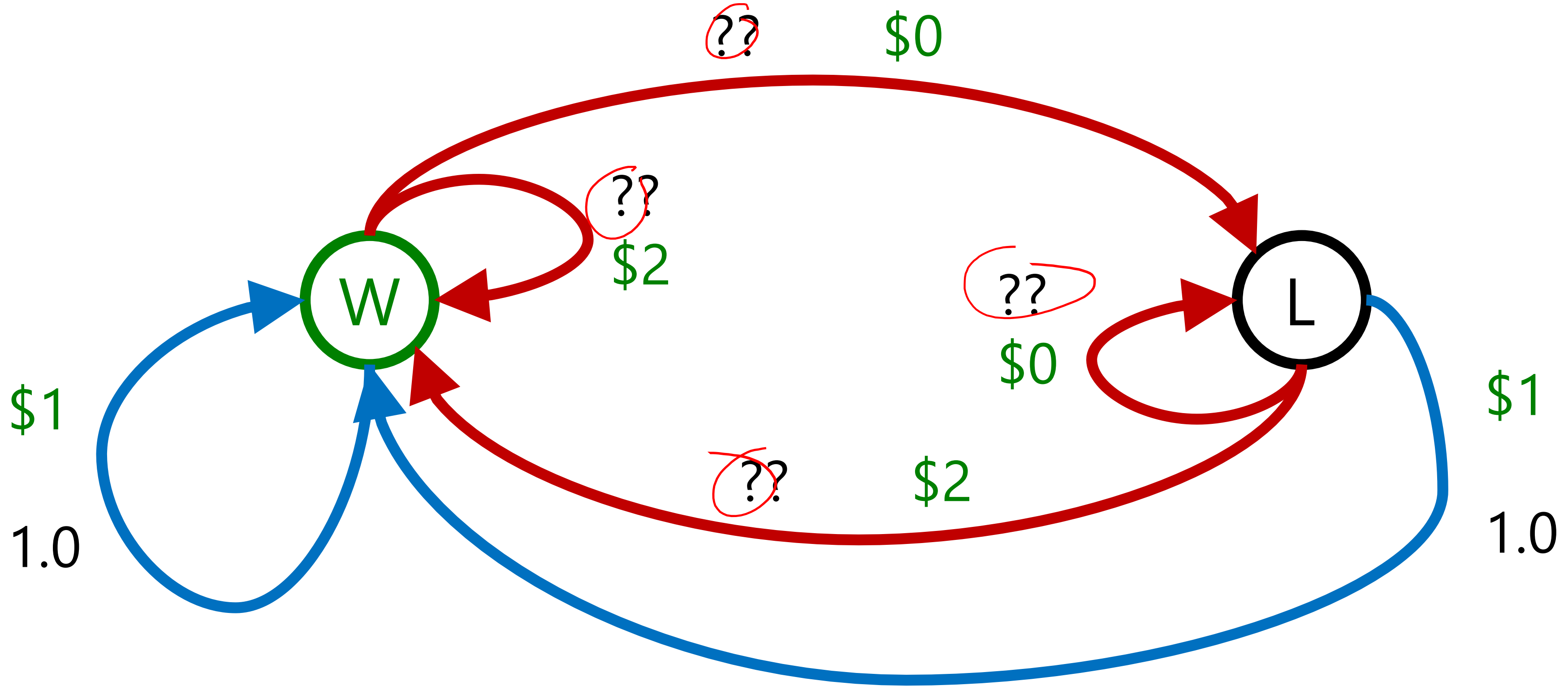
$2 $2 $0 $2 $2

$2 $2 $0 $0 $0

$12

Online Planning

- Rules changed! Red's win chance is different.



Let's Play!

\$0 \$0 \$2 \$0

\$0 \$2 \$2 \$0 \$0

\$0

What Just Happened?

- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out

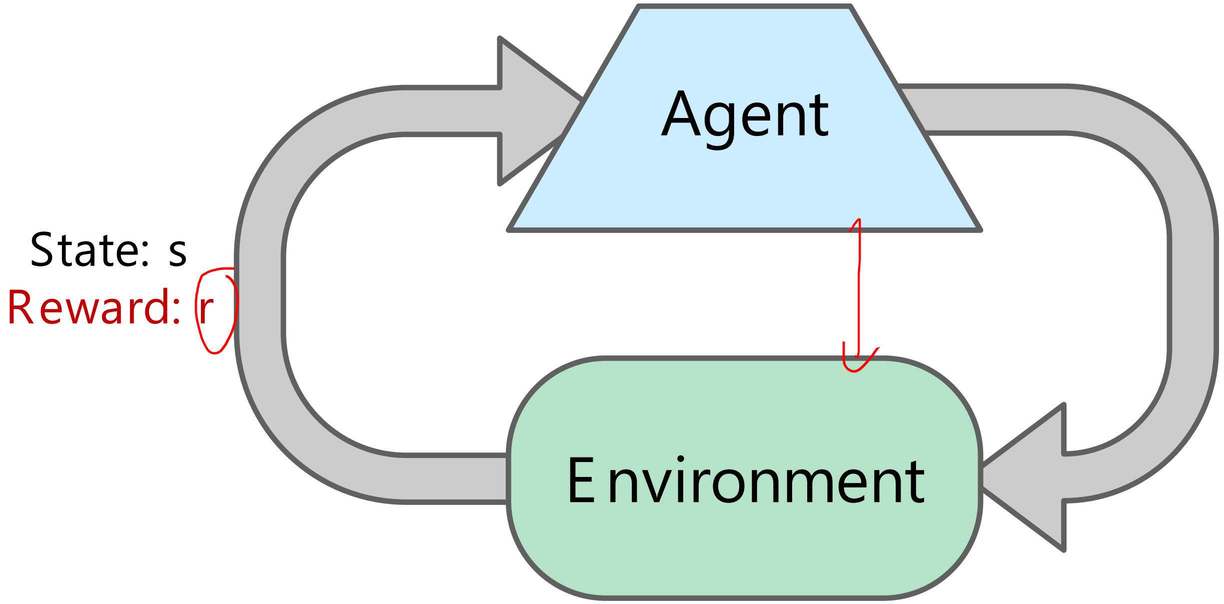
- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model $T(s,a,s')$
  - A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn

Reinforcement Learning



R

+2, 0, -

$R(s, a, s')$

- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

Example: Learning to Walk

Initial

A Learning Trial

After Learning [1K Trials]

[Kohl and Stone, ICRA 2004]

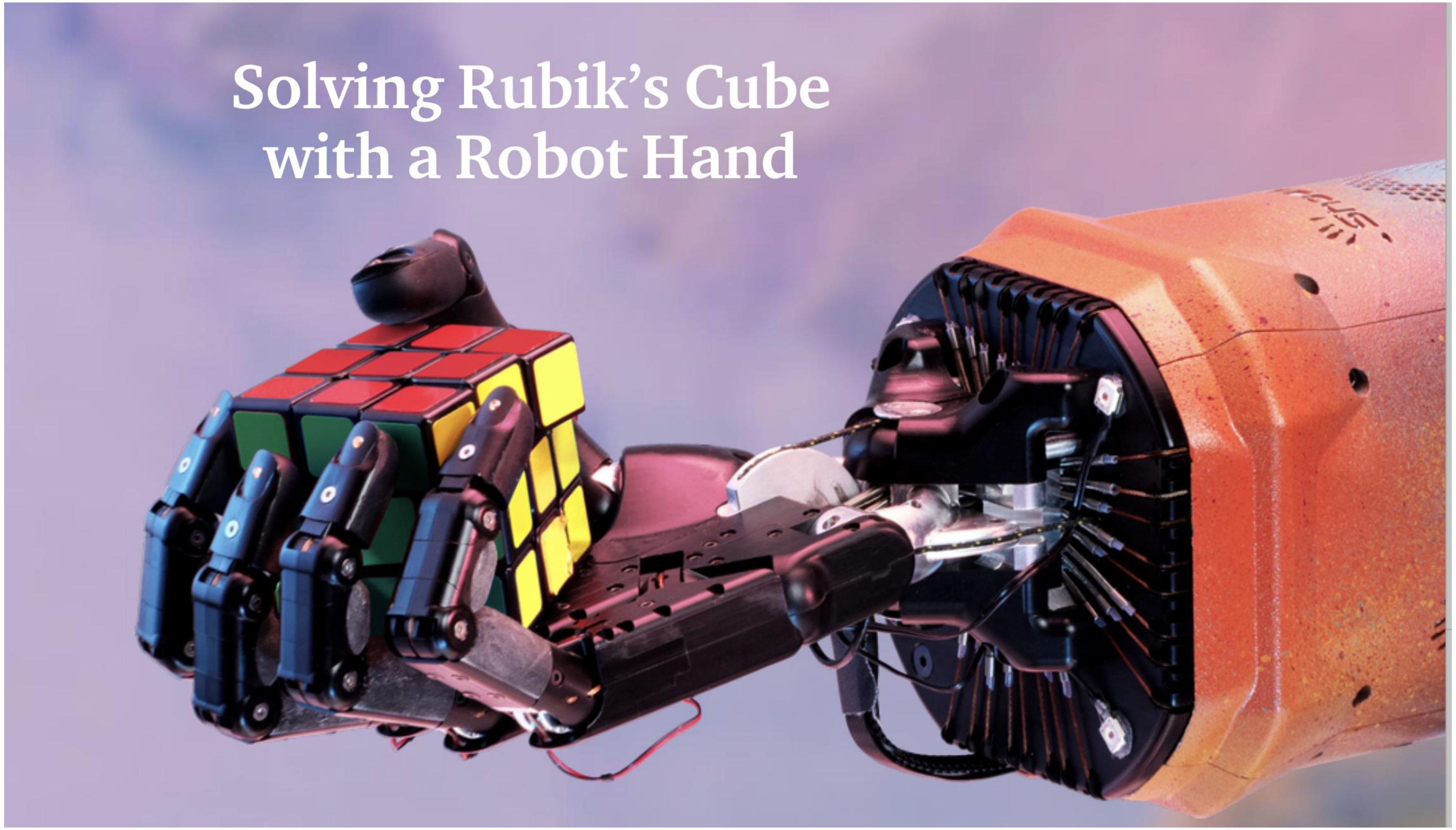
Example: Toddler Robot

[Tedrake, Zhang and Seung, 2005]

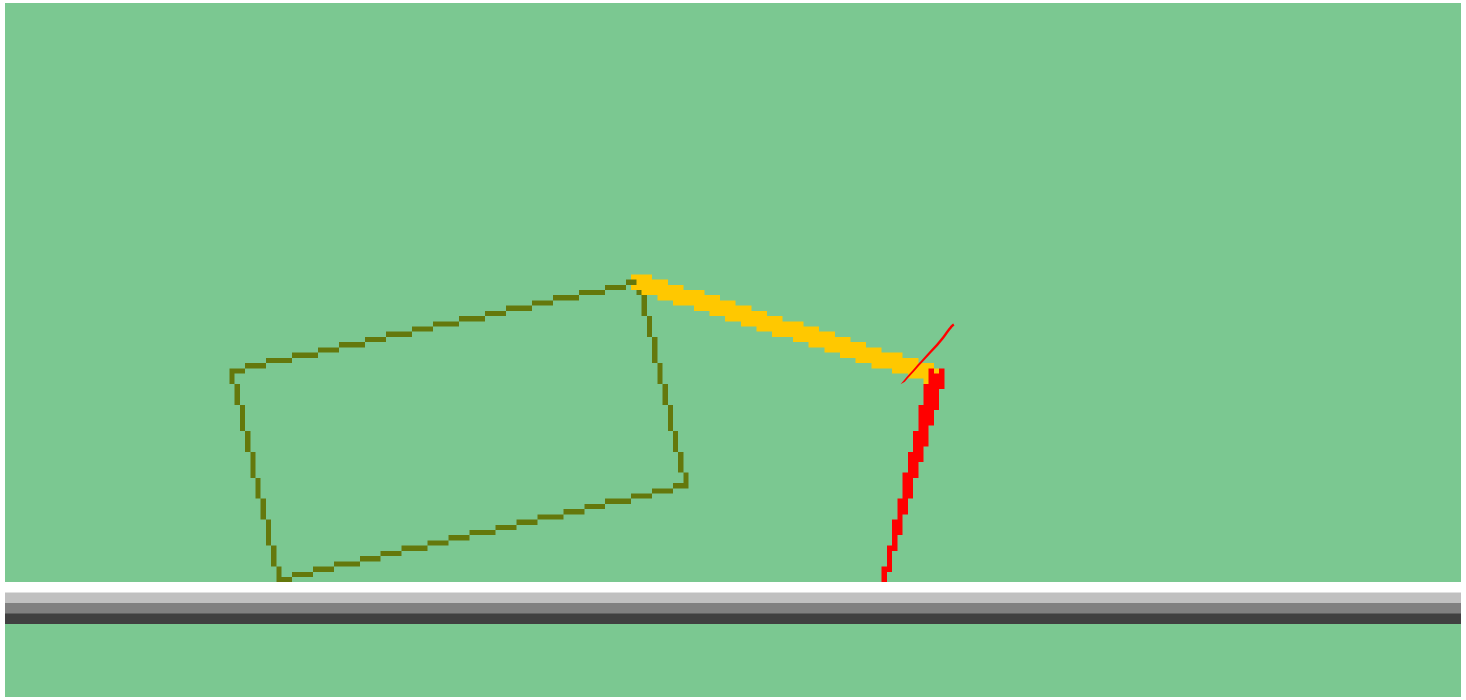
[Video: TODDLER – 40s]

Robotics Rubik Cube

- https://www.youtube.com/watch?v=x4O8pojMF0w

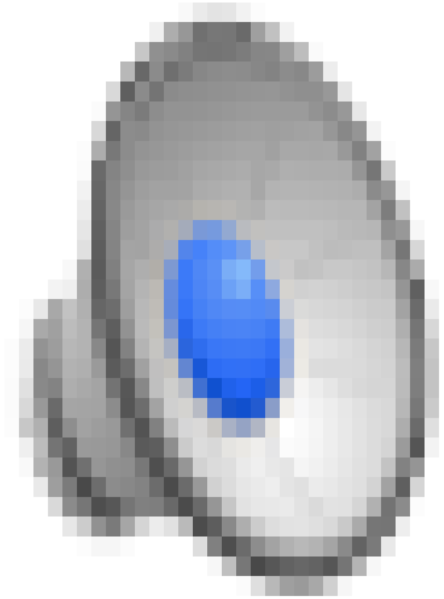


The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project 3]

Video of Demo Crawler Bot



Reinforcement Learning

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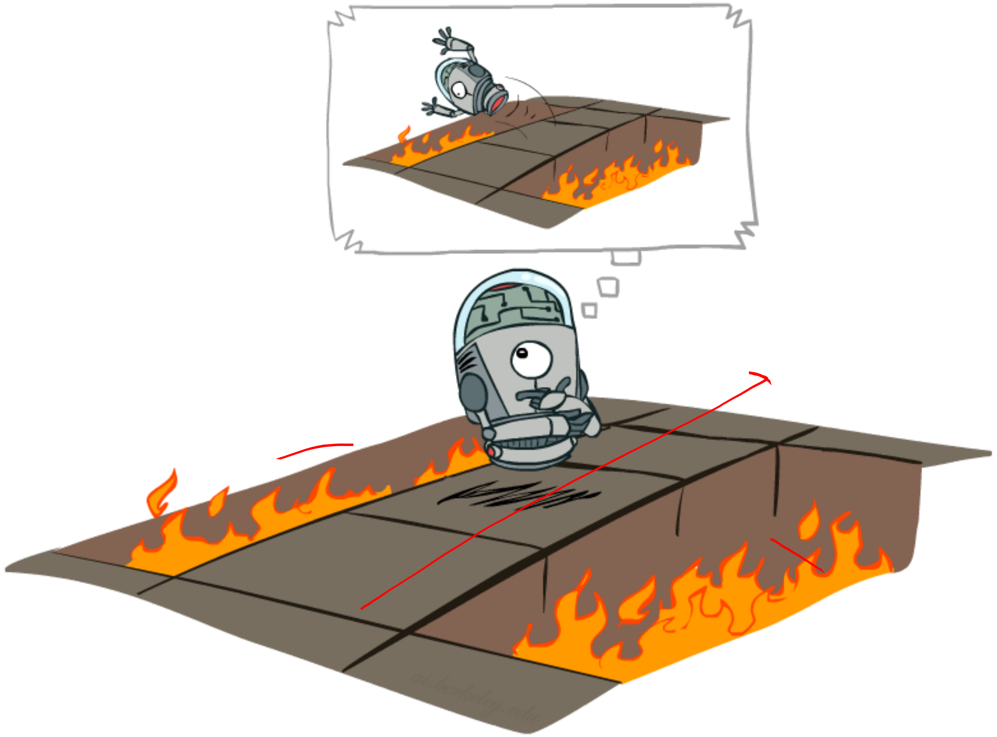
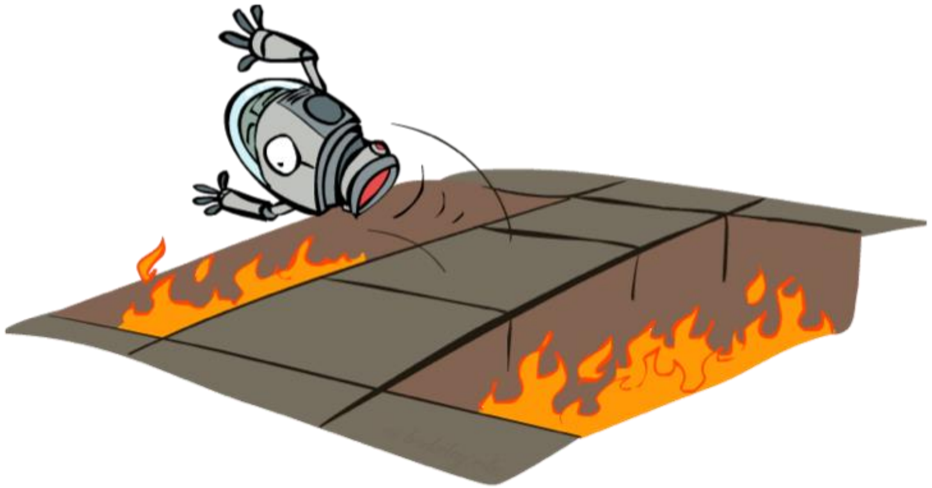
- New twist: don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn

Offline (MDPs) vs. Online (RL)

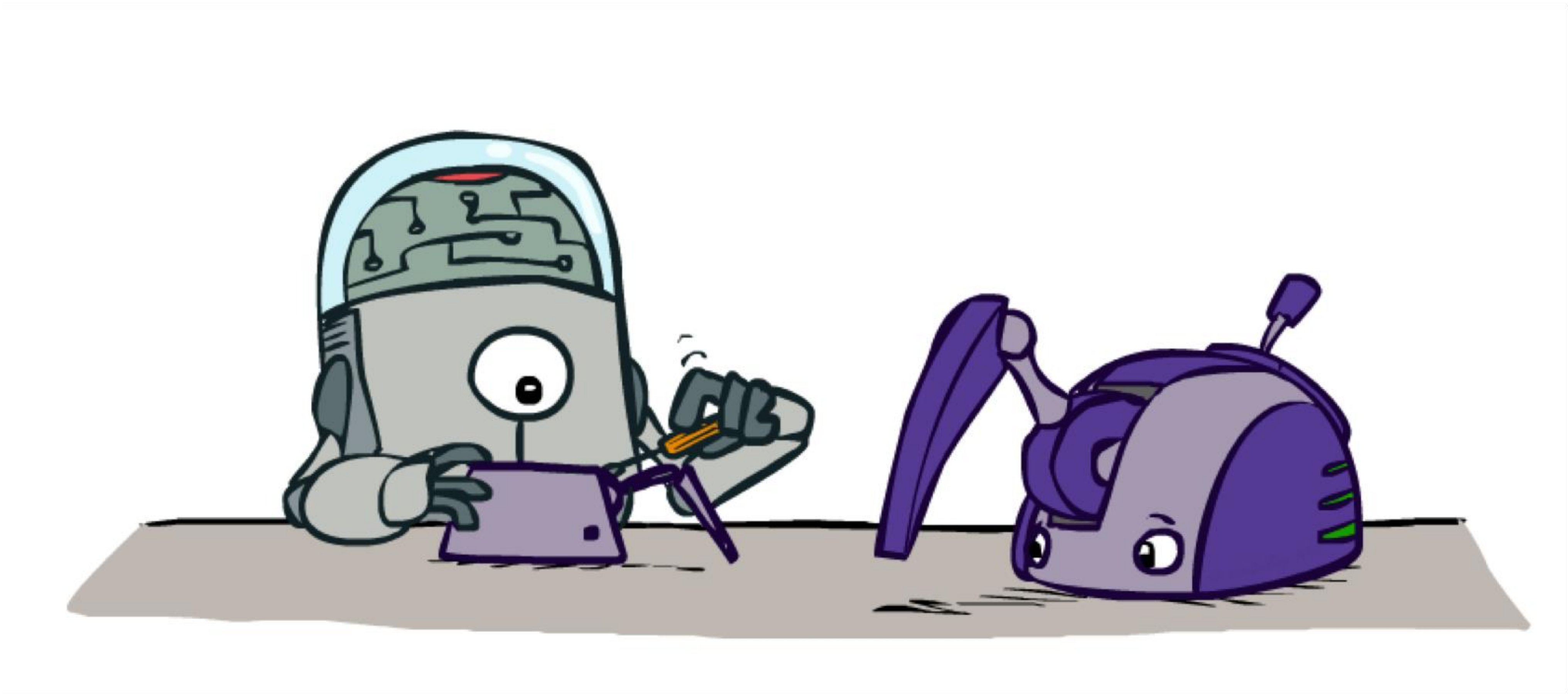


Offline Solution



Online Learning

Model-Based Learning



Model-Based Learning

T R

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct

- Step 1: Learn empirical MDP model
  - Count outcomes s′ for each s, a
  - Normalize to give an estimate $\hat{T}(s, a, s')$
  - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s′)

- Step 2: Solve the learned MDP
  - For example, use value iteration, as before

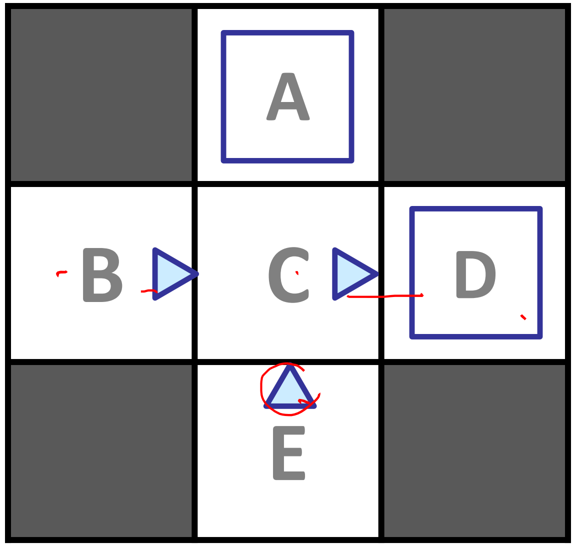
Example: Model-Based Learning

T(B, east, C) = 1.00

T(C, east, D) = 3/4

T(C, east, A) = 1/4

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

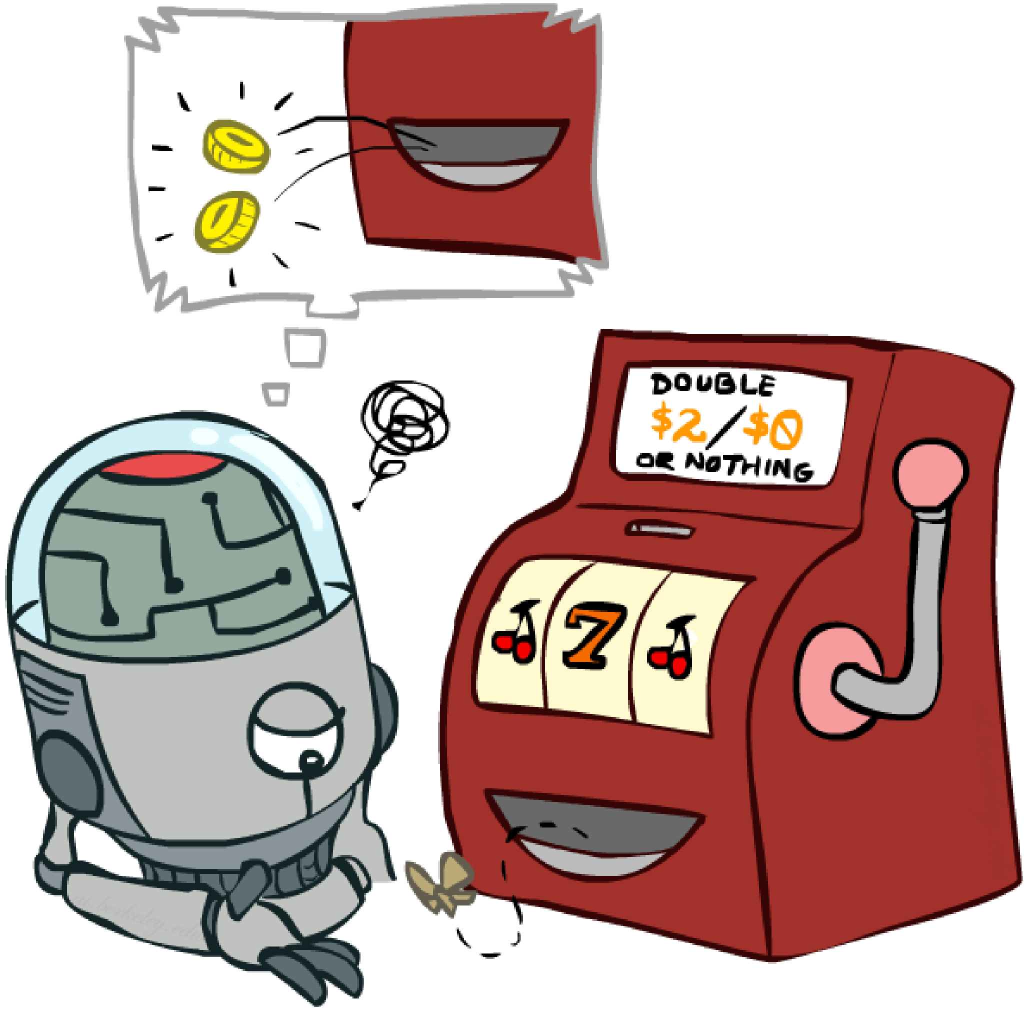
$\hat{T}(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$\hat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Model-Free Learning



Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
  - Act according to π
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples
- This is called direct evaluation

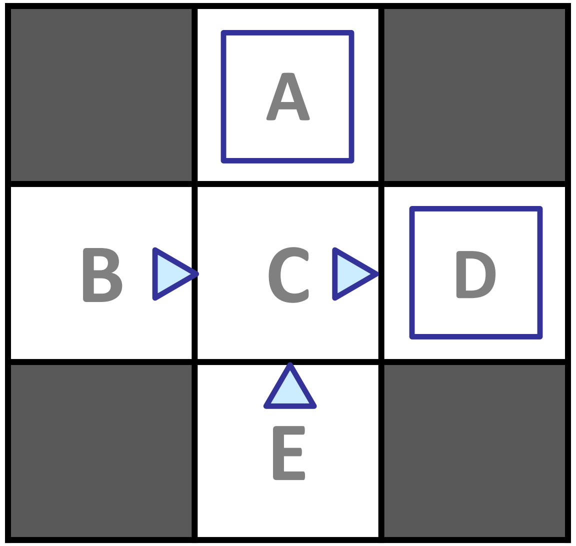
Example: Direct Evaluation

T

R

Input Policy π

V(B)



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

8

B, east, C, -1
C, east, D, -1
D, exit, x, +10

9

Episode 2

8

B, east, C, -1
C, east, D, -1
D, exit, x, +10

9

Episode 3

8

E, north, C, -1
C, east, D, -1
D, exit, x, +10

9

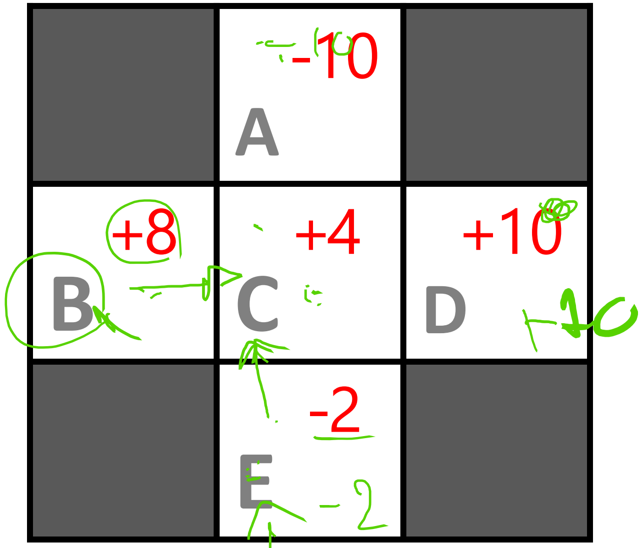
Episode 4

-12

E, north, C, -1
C, east, A, -1
A, exit, x, -10

-11

Output Values



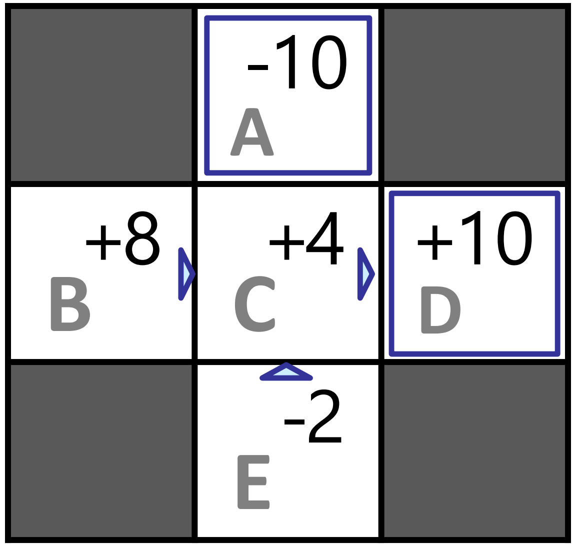
If B and E both go to C under this policy, how can their values be different?

Problems with Direct Evaluation

- What's good about direct evaluation?
  - It's easy to understand
  - It doesn't require any knowledge of T, R
  - It eventually computes the correct average values, using just sample transitions

- What bad about it?
  - It wastes information about state connections
  - Each state must be learned separately
  - So, it takes a long time to learn

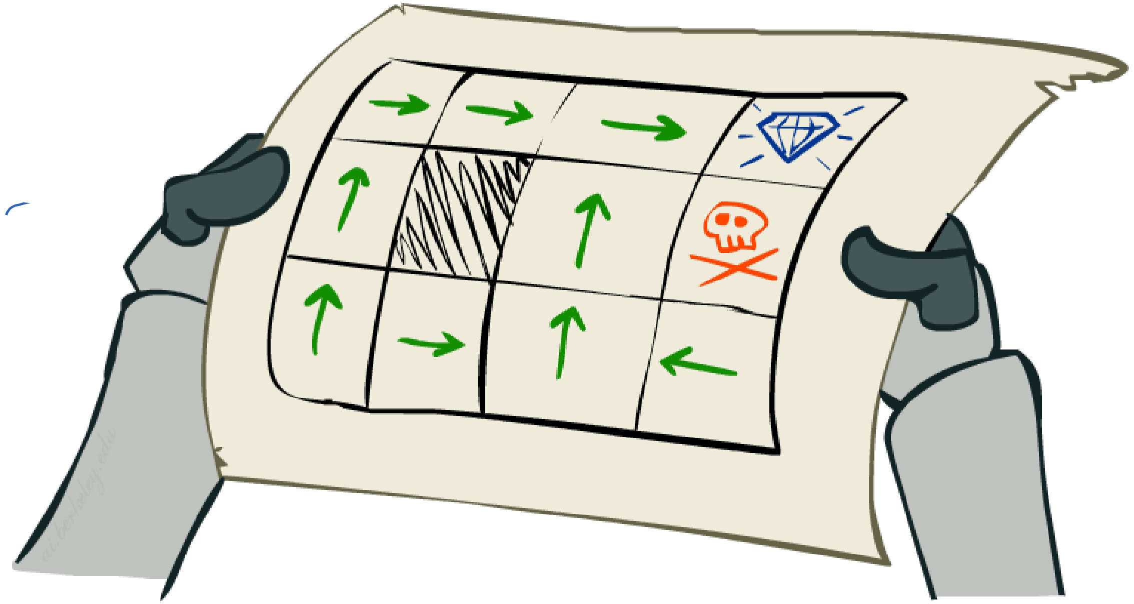
Output Values



If B and E both go to C under this policy, how can their values be different?

Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know the transitions $T(s,a,s')$
  - You don't know the rewards $R(s,a,s')$
  - Goal: learn the state values



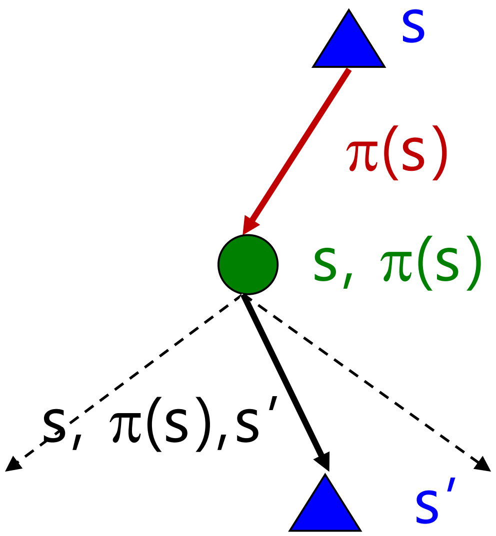
- In this case:
  - Learner is "along for the ride"
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - This is NOT offline planning! You actually take actions in the world.

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:
  - Each round, replace V with a one-step-look-ahead layer over V

$$V_0^{\pi}(s) = 0$$

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$



  - This approach fully exploited the connections between the states
  - Unfortunately, we need T and R to do it!

- Key question: how can we do this update to V without knowing T and R?
  - In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} \boxed{T(s,\pi(s),s')}[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

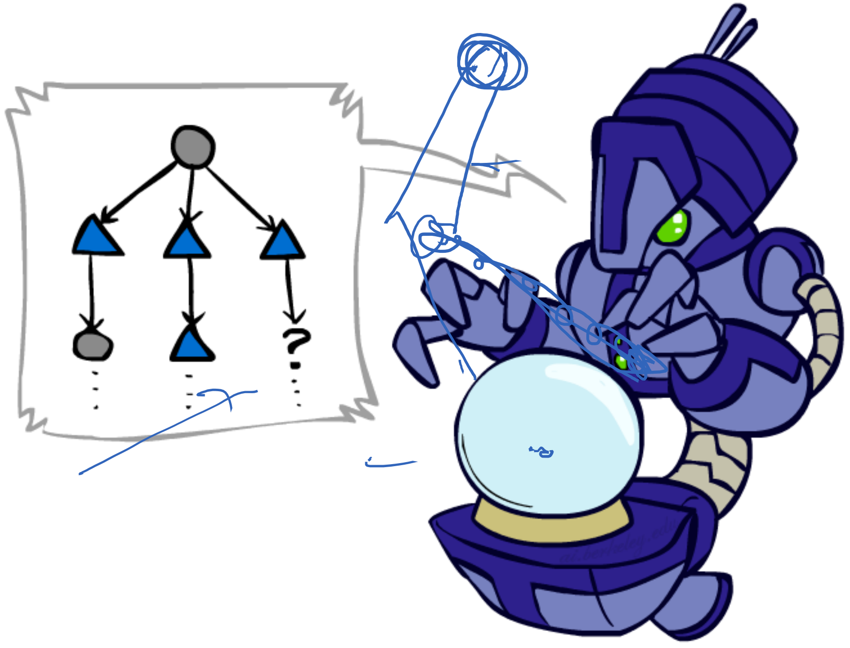
1 $$sample_1 = R(s,\pi(s),s_1') + \gamma V_k^{\pi}(s_1')$$

2 $$sample_2 = R(s,\pi(s),s_2') + \gamma V_k^{\pi}(s_2')$$

$$\ldots$$

$$sample_n = R(s,\pi(s),s_n') + \gamma V_k^{\pi}(s_n')$$

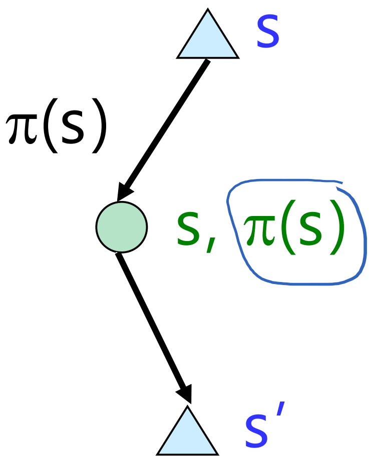
$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n}\sum_i sample_i$$



from state s.

Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s′, r)
  - Likely outcomes s′ will contribute updates more often



- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average

$(1-\alpha)V(s) + \alpha \text{ sample}$

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

α

Exponential Moving Average

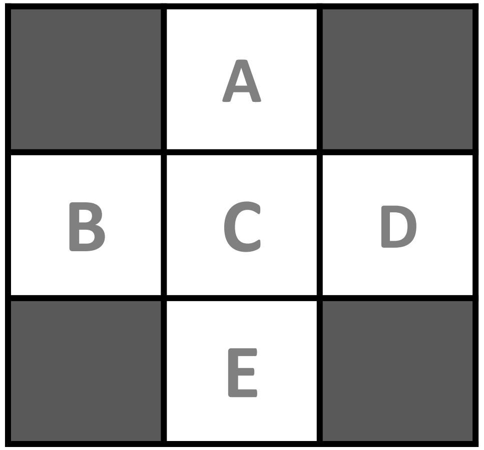
- Exponential moving average
  - The running interpolation update: $\bar{x}_n = (1-\alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$

    $[(1-\alpha)\tilde{x}_{n-2} + \alpha x_{n-1}] + \alpha \cdot \boxed{x_n}$
  - Makes recent samples more important
  - Forgets about the past (distant past values were wrong anyway)

    $+\alpha$
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States



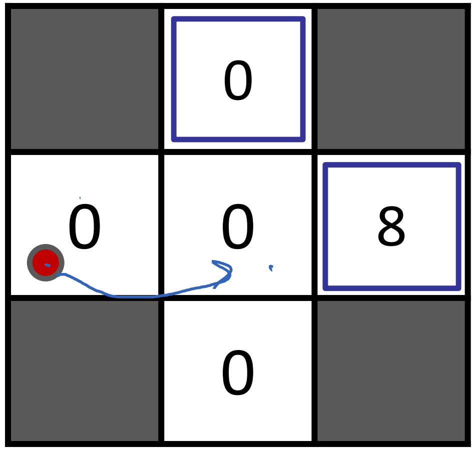
Assume: $\gamma = 1$, $\alpha = 1/2$

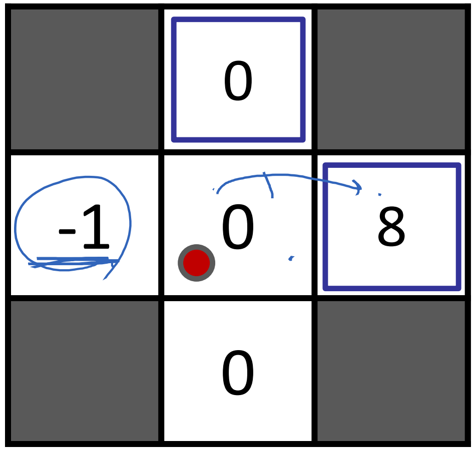
Observed Transitions

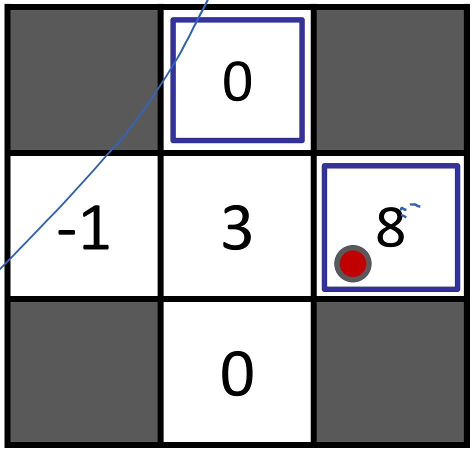
B, east, C, -2

C, east, D, -2

R







$$V^{\pi}(s) \leftarrow \underbrace{(1-\alpha)V^{\pi}(s)}_{\emptyset} + \alpha\left[R(s,\pi(s),s') + \gamma \boxed{V^{\pi}}(s')\right]$$

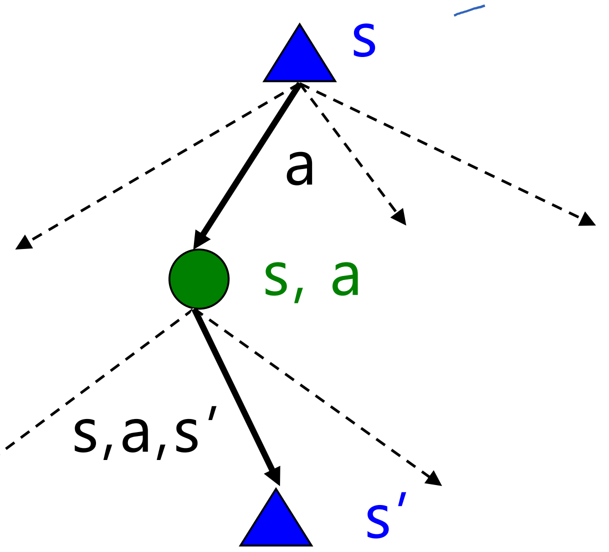
$+ 1/2 \; [\, -2 \; + \; 8 \,]$

Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V(s')\right]$$



- Idea: learn Q-values, not values
- Makes action selection model-free too!

Announcements

- Project Proposal: Feb 11th
- Paper report: Feb 18th
- PS3: Feb 22nd

- Google Cloud credit is available for you to use.

Recap: Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn
- Big Idea: Compute all averages over T using sample outcomes

The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	Value Learning

Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
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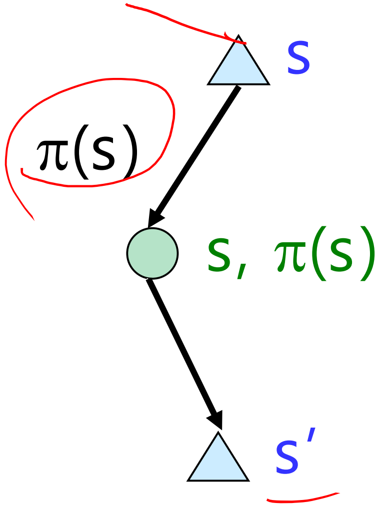

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

learning rate

Update to V(s): $V^{\pi}(s) \leftarrow (1 - \alpha) V^{\pi}(s) + (\alpha) sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

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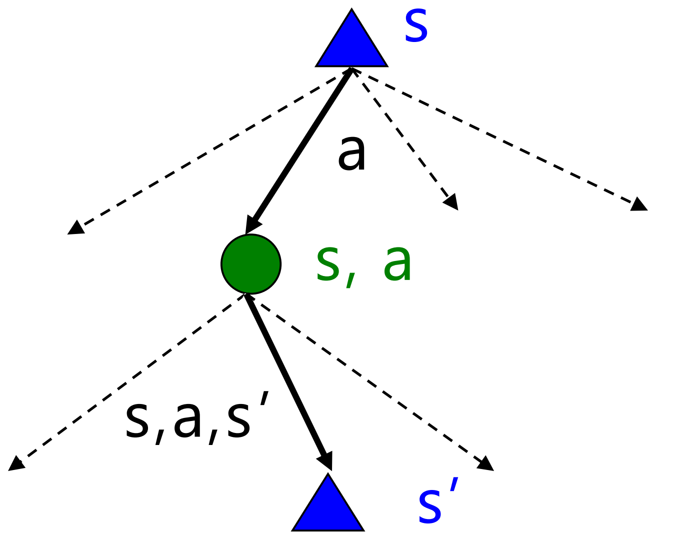


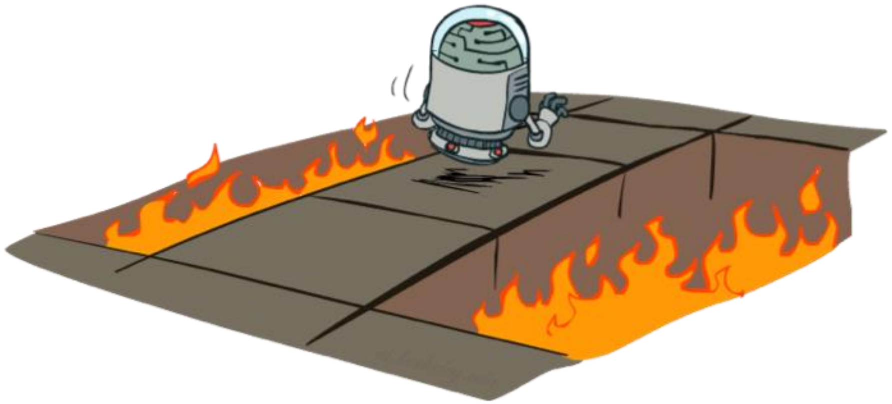
- Idea: learn Q-values, not values
- Makes action selection model-free too!

Discussion: Model-Based vs Model-Free RL

- Model-Based vs. Model Free

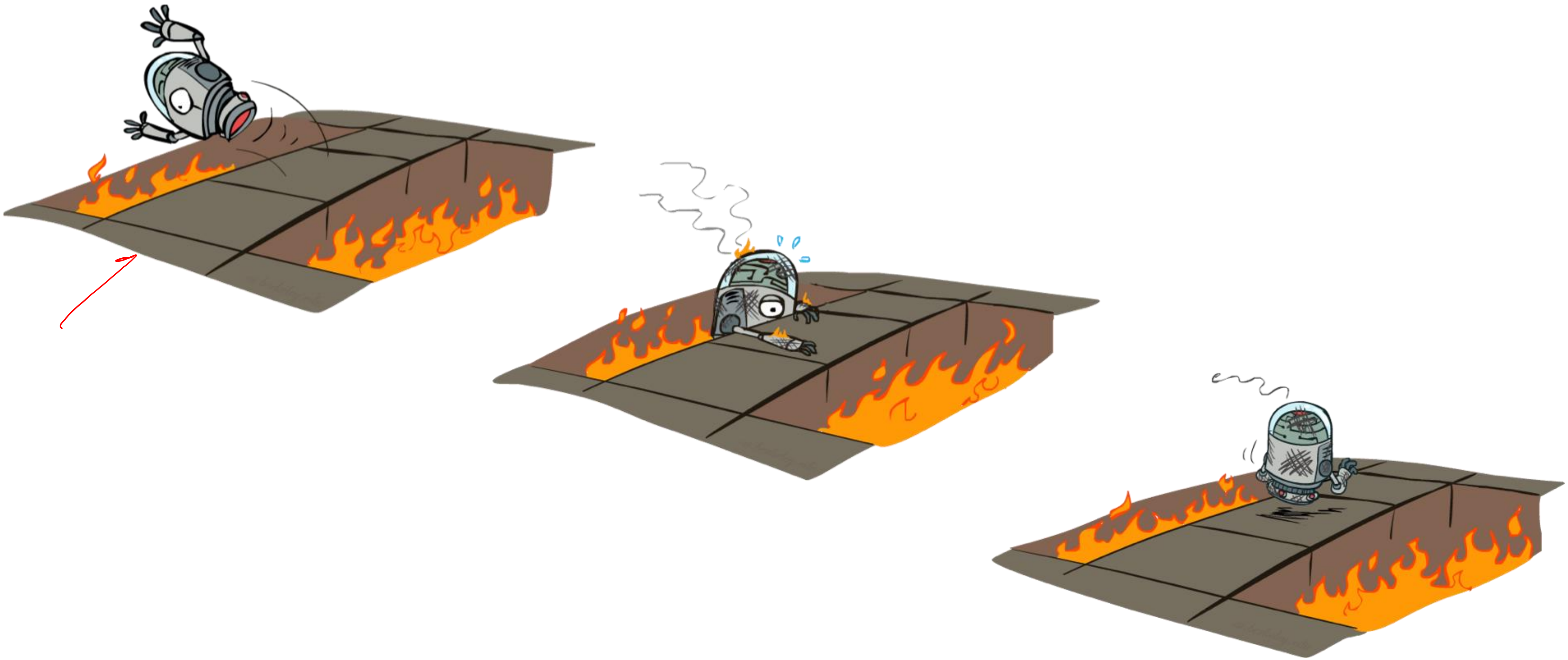
TD value

- Active vs. Passive



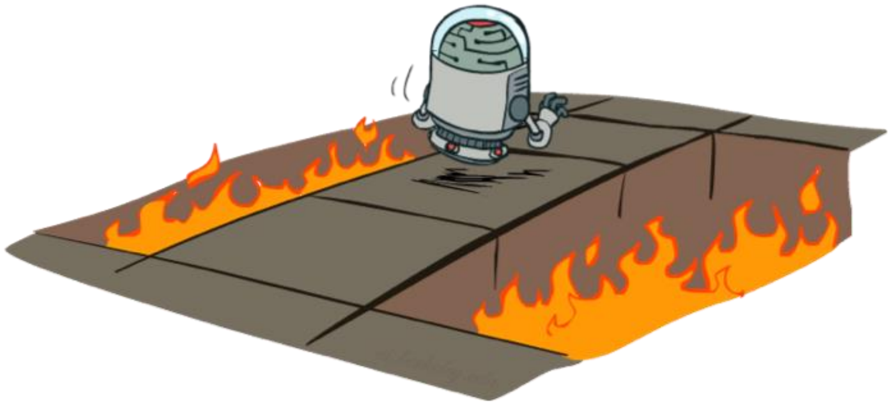
- Active Reinforcement Learning:
  - act according to current optimal (based on Q-Values)
  - but also explore…

Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You choose the actions now
  - Goal: learn the optimal policy / values

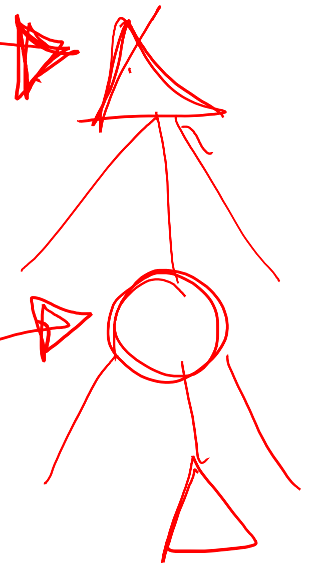


- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$



- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Q_k, calculate the depth k+1 q-values for all q-states:

$V(s')$

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

Q_K

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

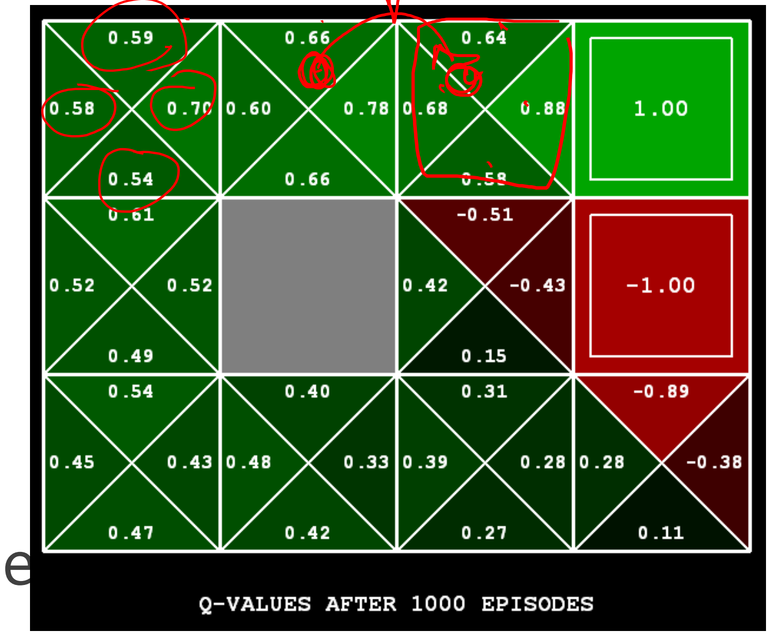
- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

no longer policy evaluation!

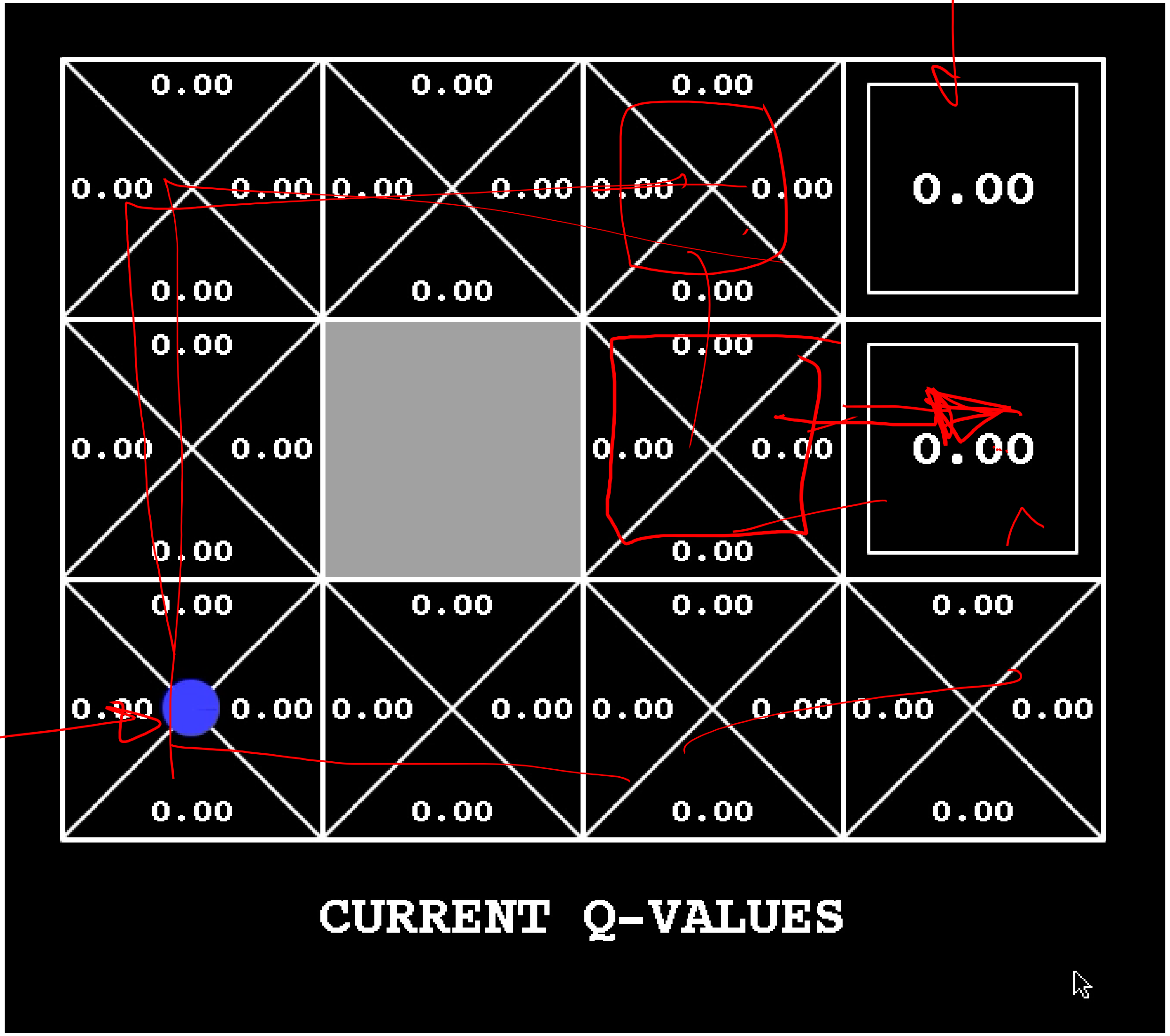
  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha) Q(s,a) + (\alpha) [sample]$$

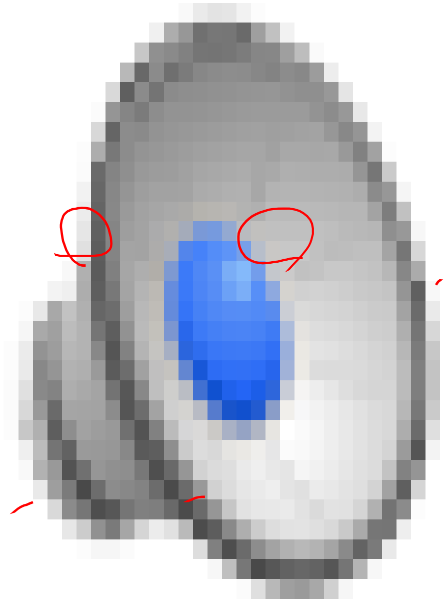


[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

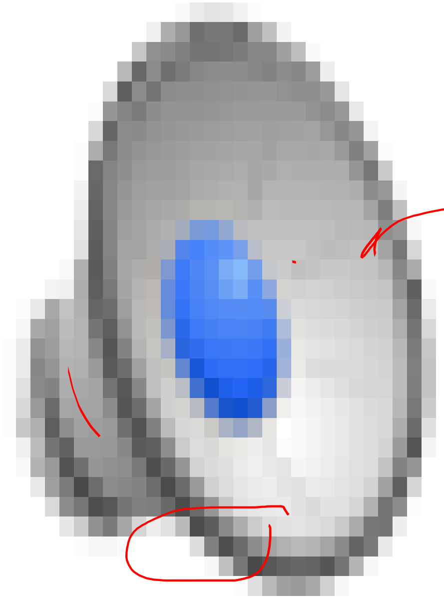
Q-Learning Demo



Video of Demo Q-Learning -- Gridworld



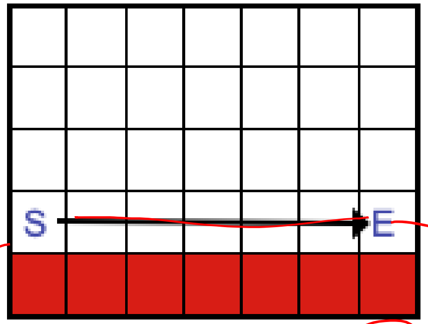
Video of Demo Q-Learning -- Crawler

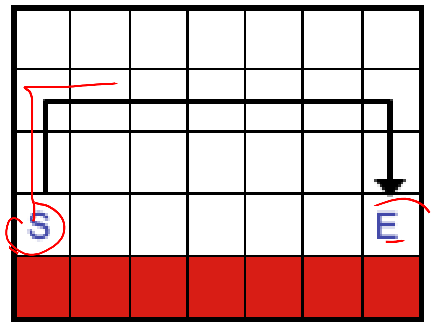


Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!

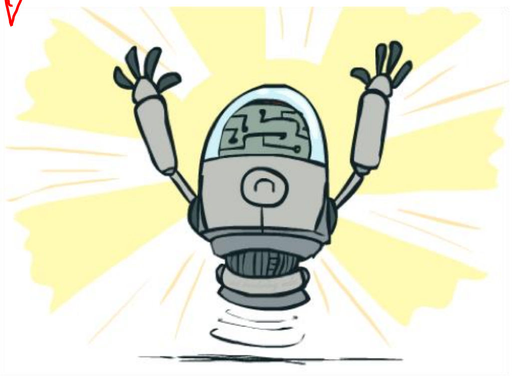
- This is called off-policy learning



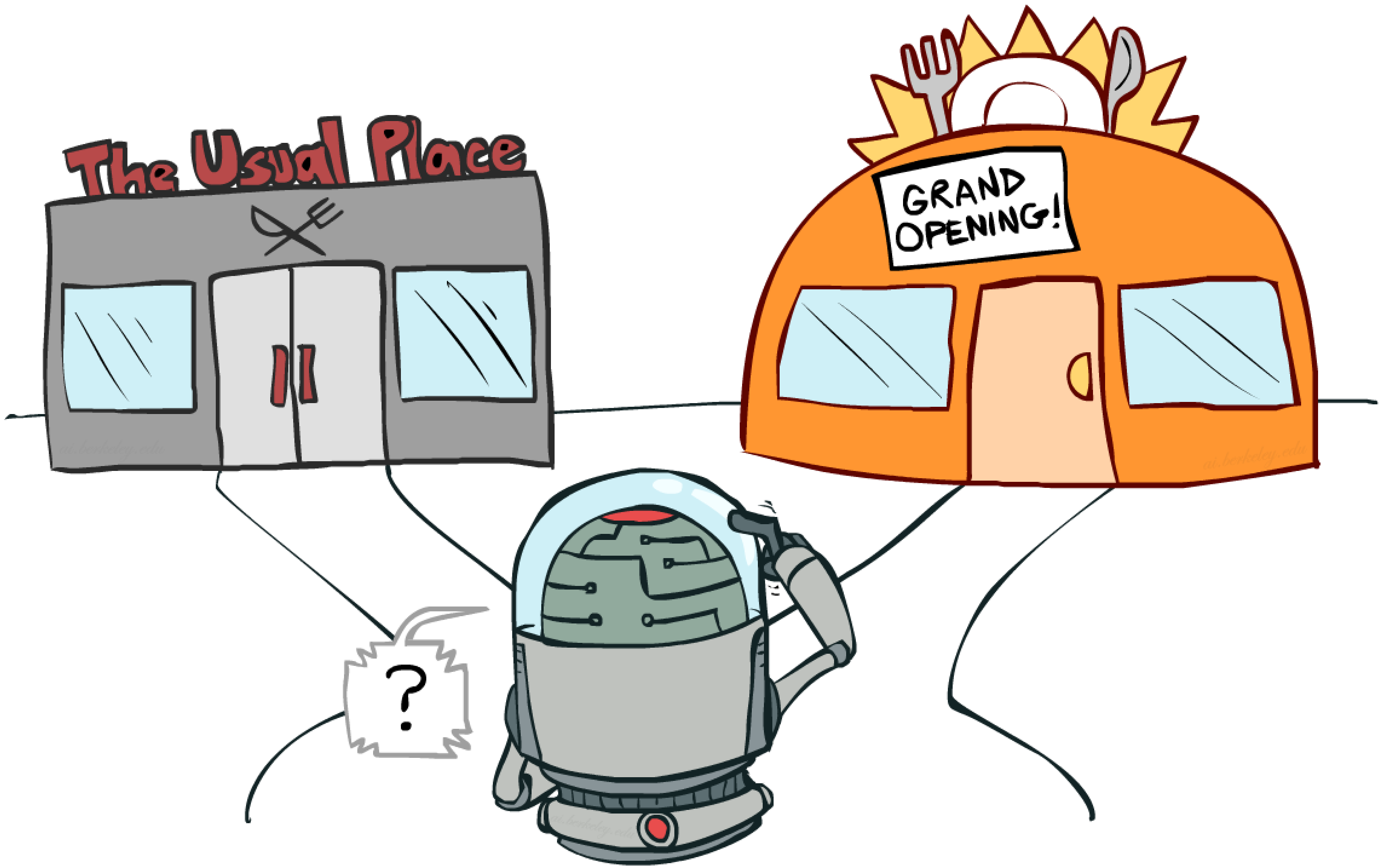


$Q \leftarrow Q + \alpha \, [\quad]$

- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions



Exploration vs. Exploitation



How to Explore?

$\alpha \downarrow$, $\epsilon \downarrow$

- Several schemes for forcing exploration
  - Simplest: random actions (ε-greedy)
    - Every time step, flip a coin
    - With (small) probability ε, act randomly
    - With (large) probability 1-ε, act on current policy

  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

α

Exploration Functions

- When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring
- Exploration function
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g. $f(u,n) = u + k/n$

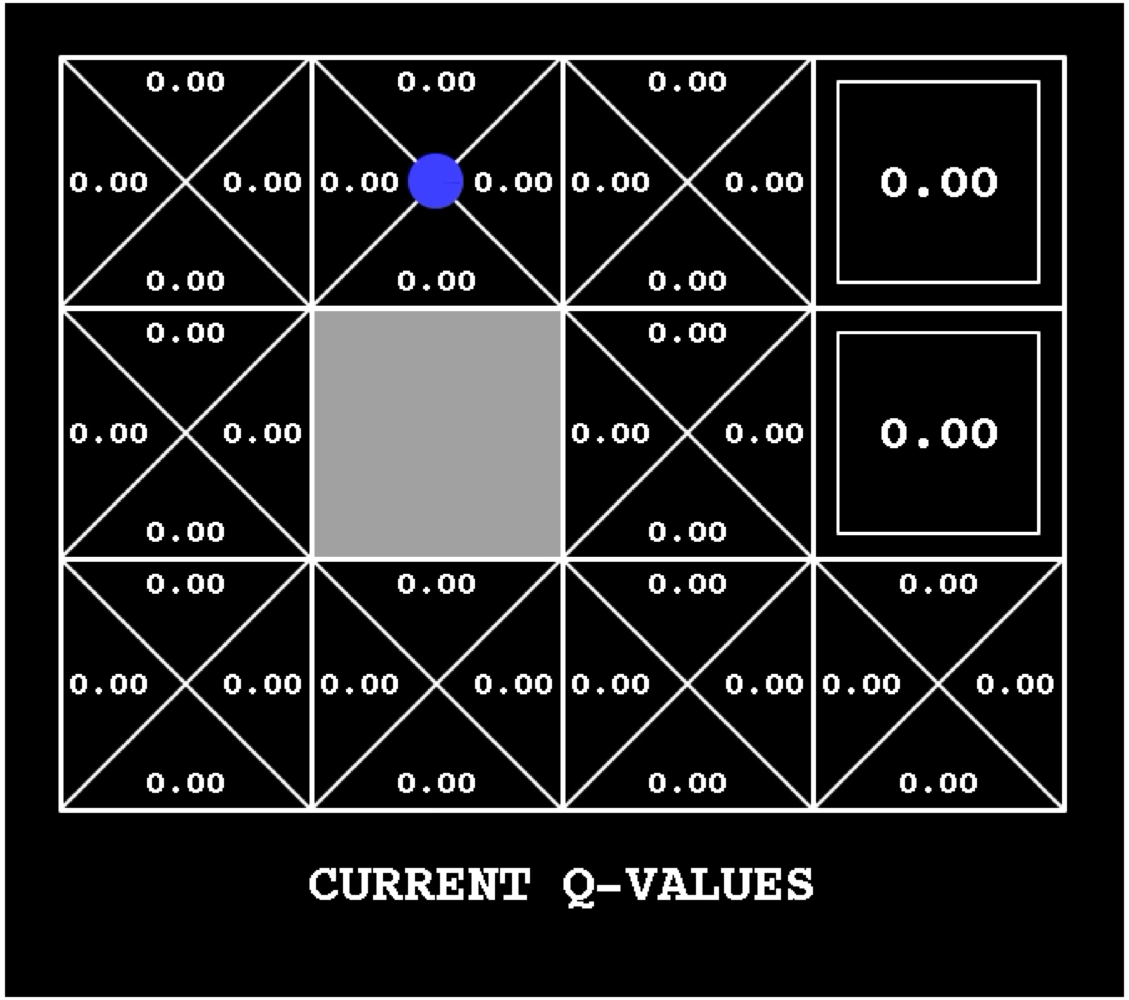
    Regular Q-Update: $Q(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} Q(s',a')$

    Modified Q-Update: $Q(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} f(Q(s',a'), N(s',a'))$

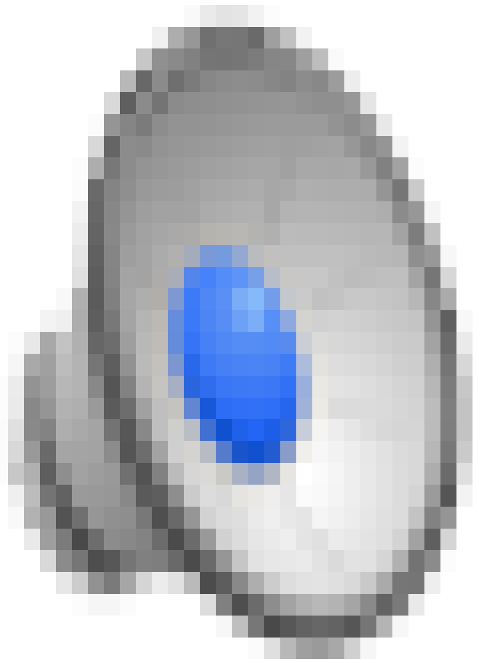
  - Note: this propagates the "bonus" back to states that lead to unknown states as well!

[Demo: exploration – Q-learning – crawler – exploration function (L11D4)]

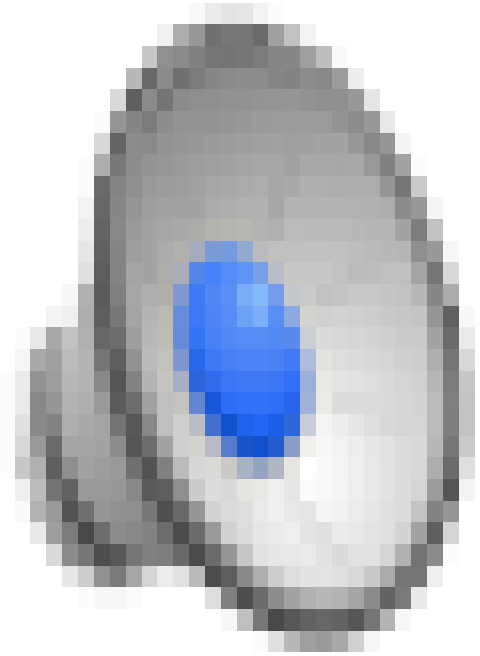
Q-Learn Epsilon Greedy



Video of Demo Q-learning – Epsilon-Greedy – Crawler

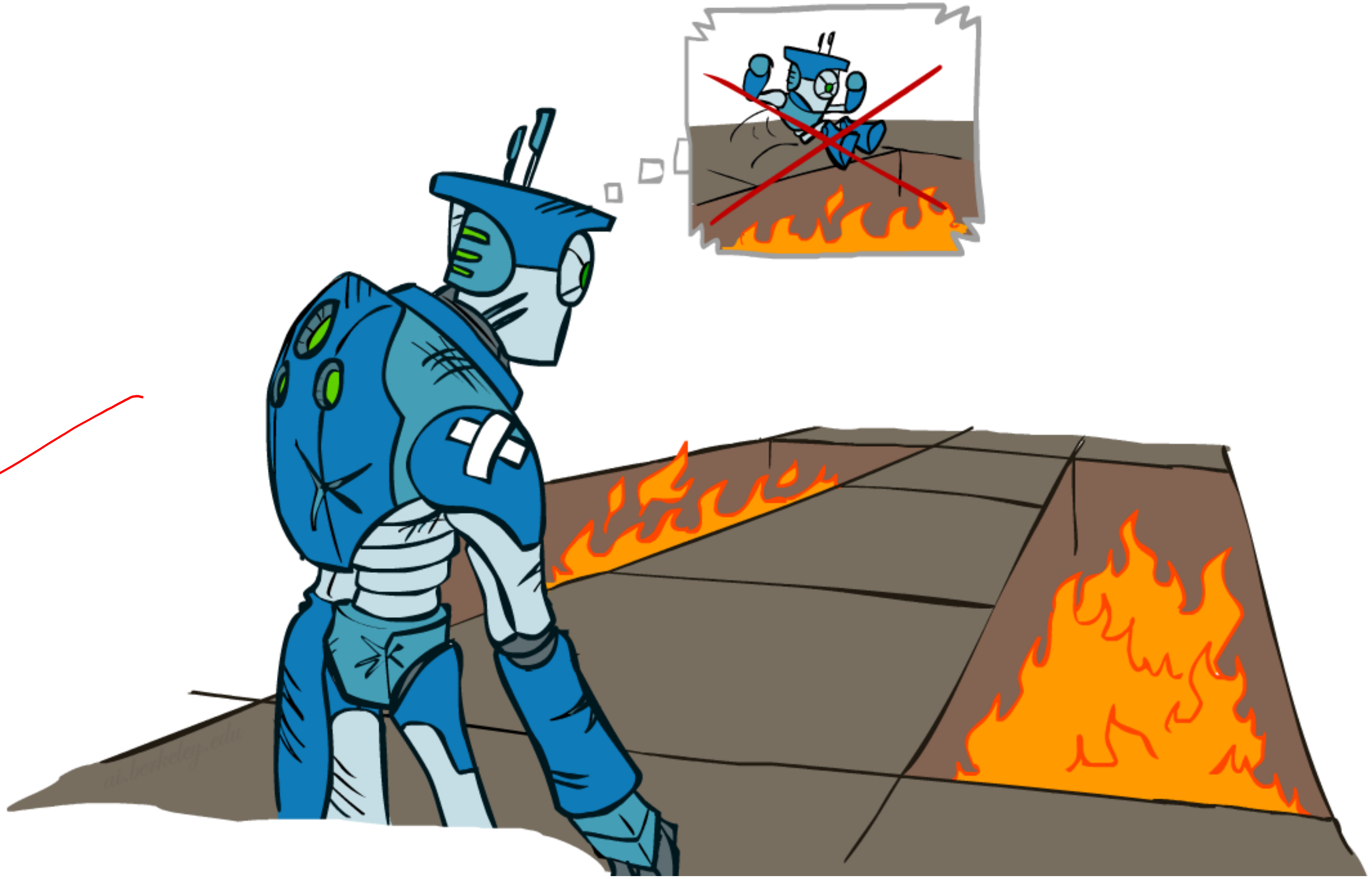


Video of Demo Q-learning – Exploration Function – Crawler

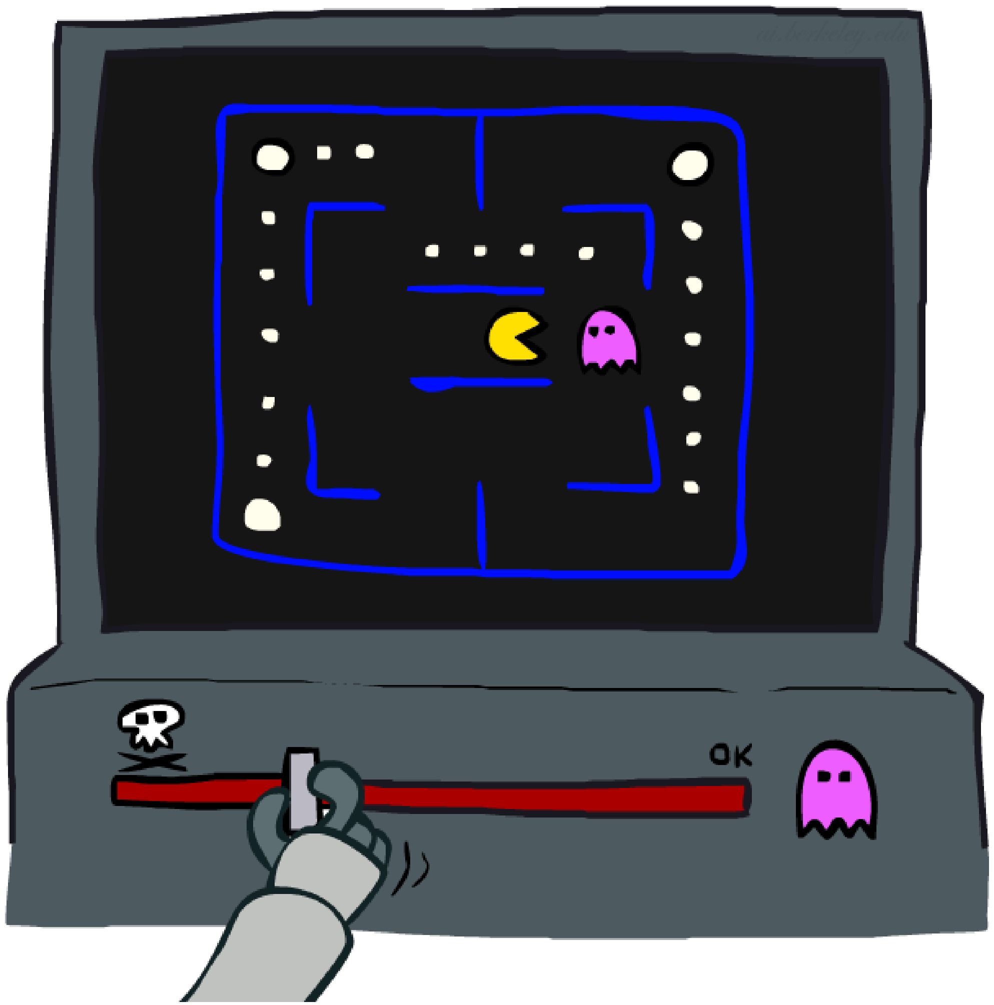


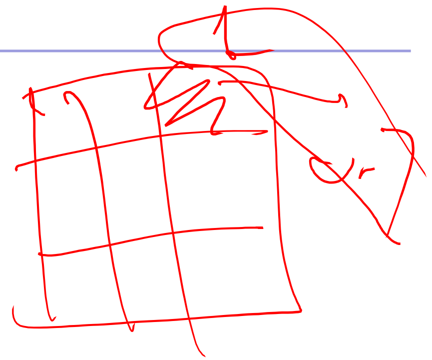
Regret

- Even if you learn the optimal policy you still make mistakes along the way!
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret



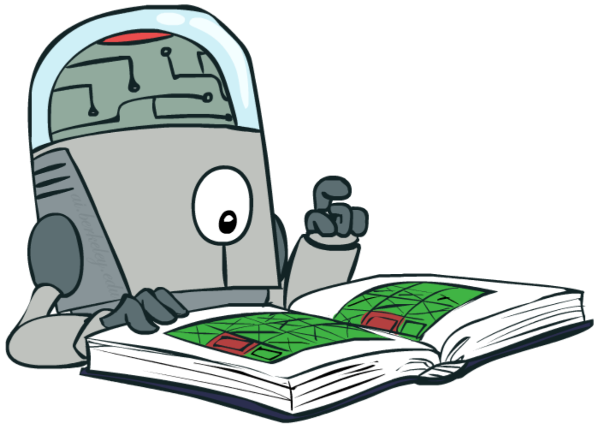
Approximate Q-Learning

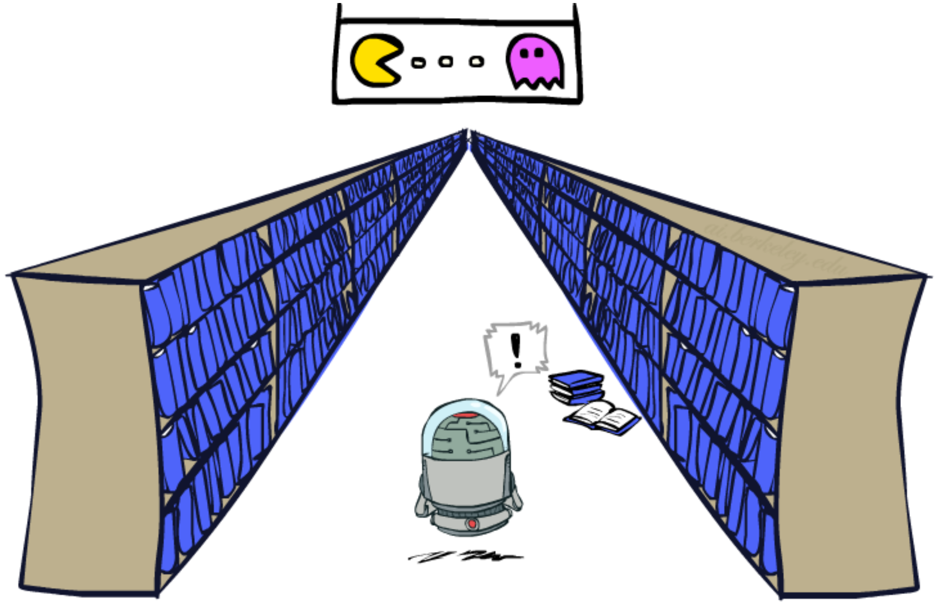




Generalizing Across States

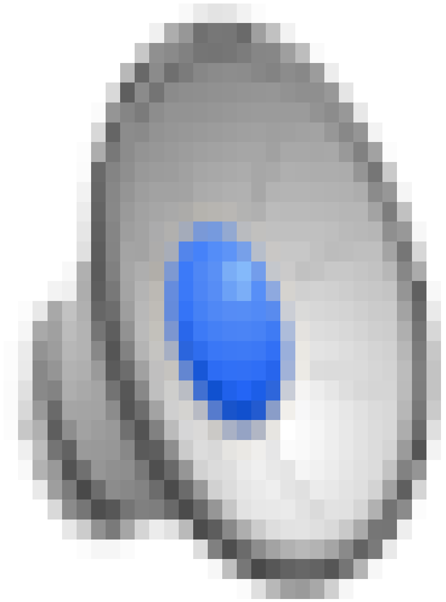
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again





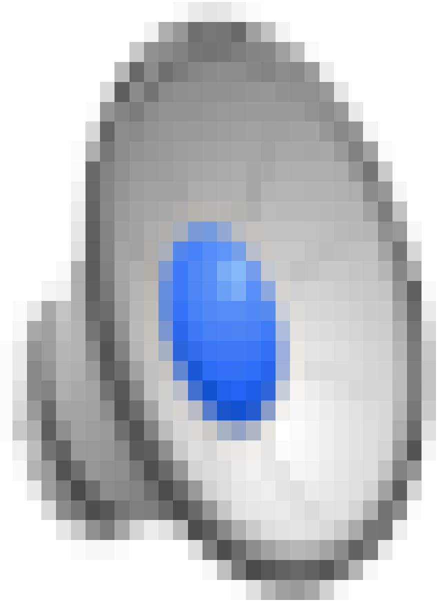
[demo – RL pacman]

Video of Demo Q-Learning Pacman – Tiny – Watch All



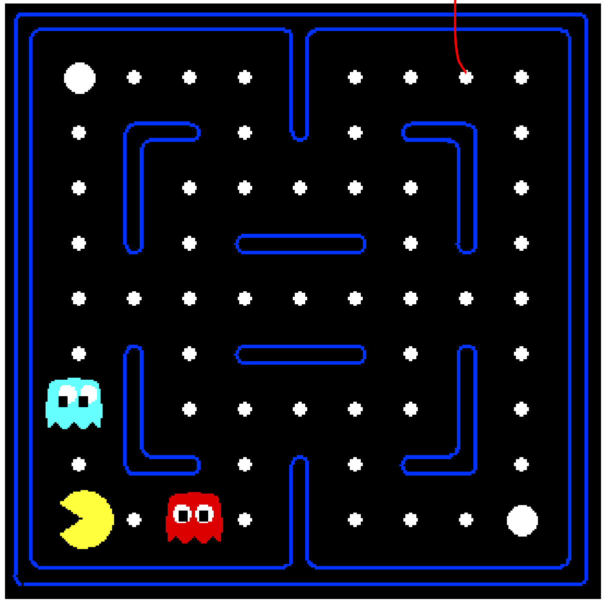
Video of Demo Q-Learning Pacman – Tiny – Silent Train

Video of Demo Q-Learning Pacman – Tricky – Watch All

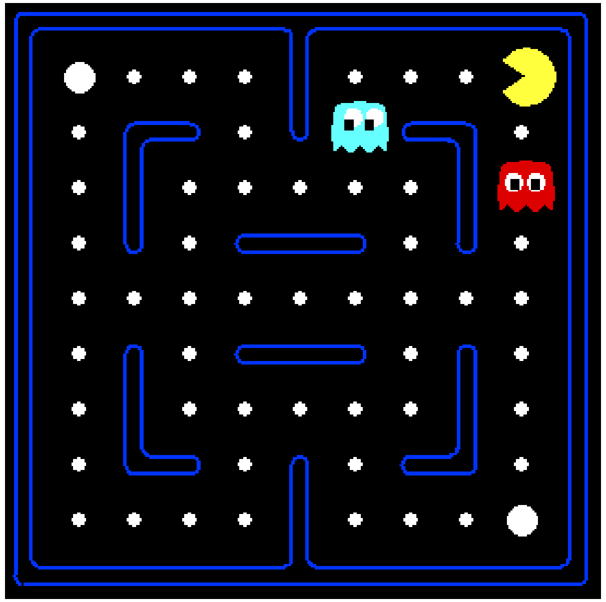


Example: Pacman

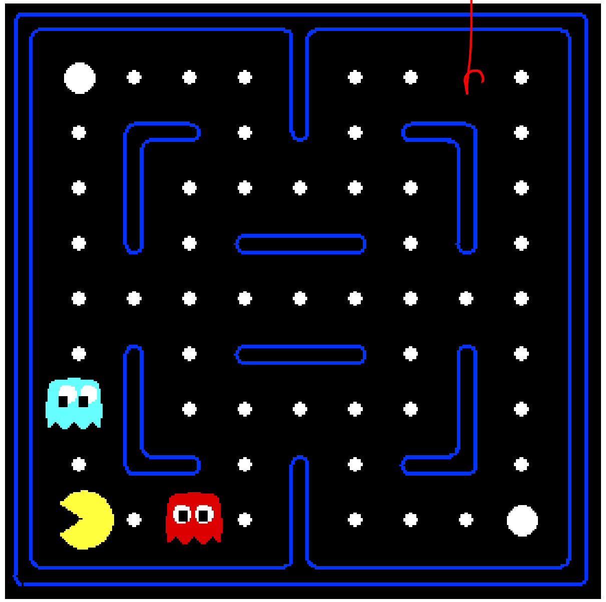
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:

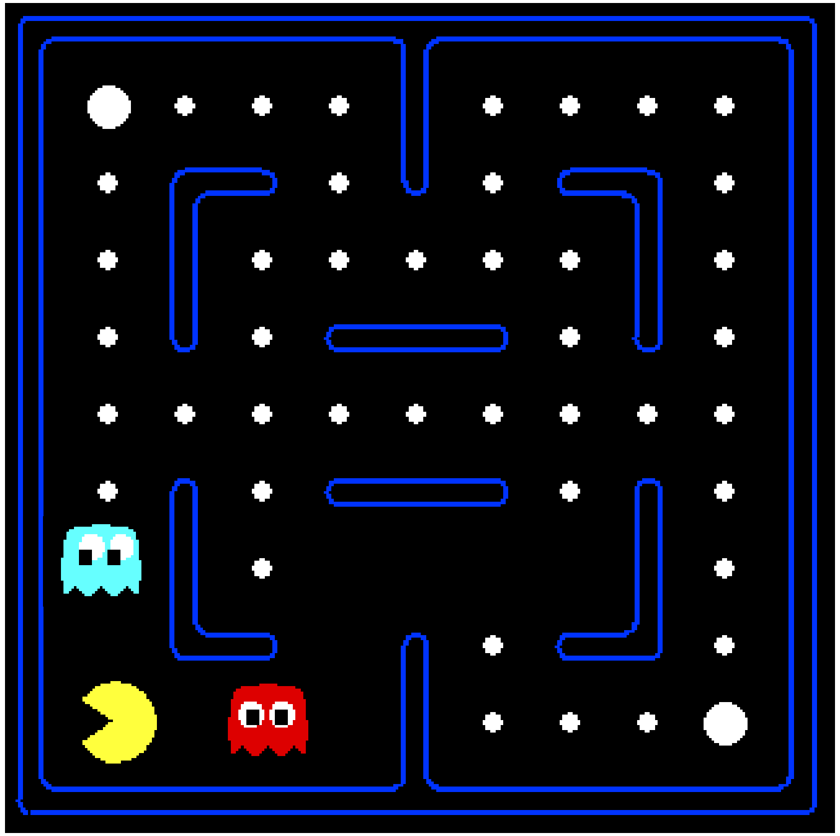


Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n \boxed{f_n(s,a)}$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear Q-functions:

transition $= (s, a, r, s')$

difference $= \left[r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)$

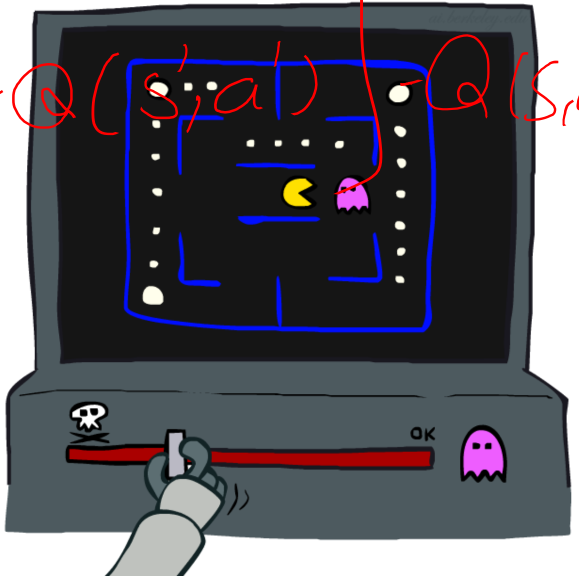
$$Q(s,a) \leftarrow Q(s,a) + \alpha\, [\text{difference}]$$

$$w_i \leftarrow w_i + \alpha\, [\text{difference}]\, \boxed{f_i(s,a)}$$

$\left[r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)$

Exact Q's

Approximate Q's

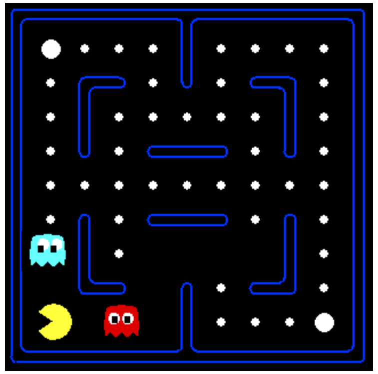


- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s

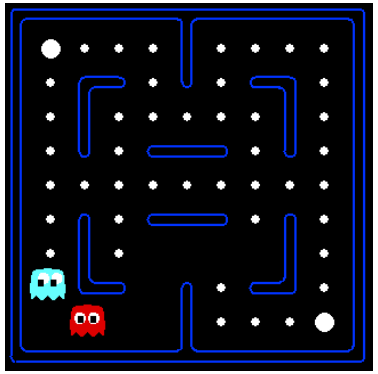


$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$a = \text{NORTH}$
$r = -500$

s'



$Q(s, \text{NORTH}) = +1$

$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

$Q(s', \cdot) = 0$

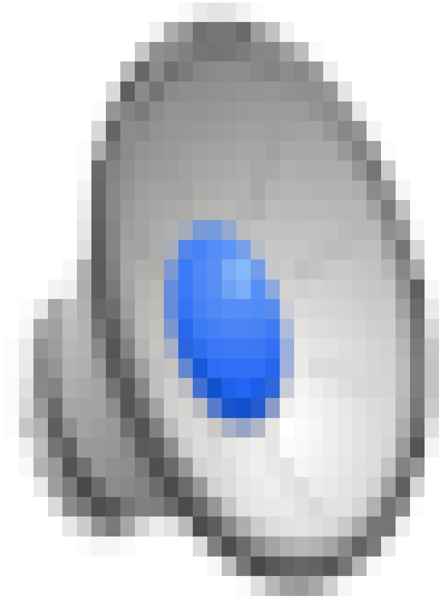
difference $= -501$

$w_{DOT} \leftarrow 4.0 + \alpha\, [-501]\, 0.5$

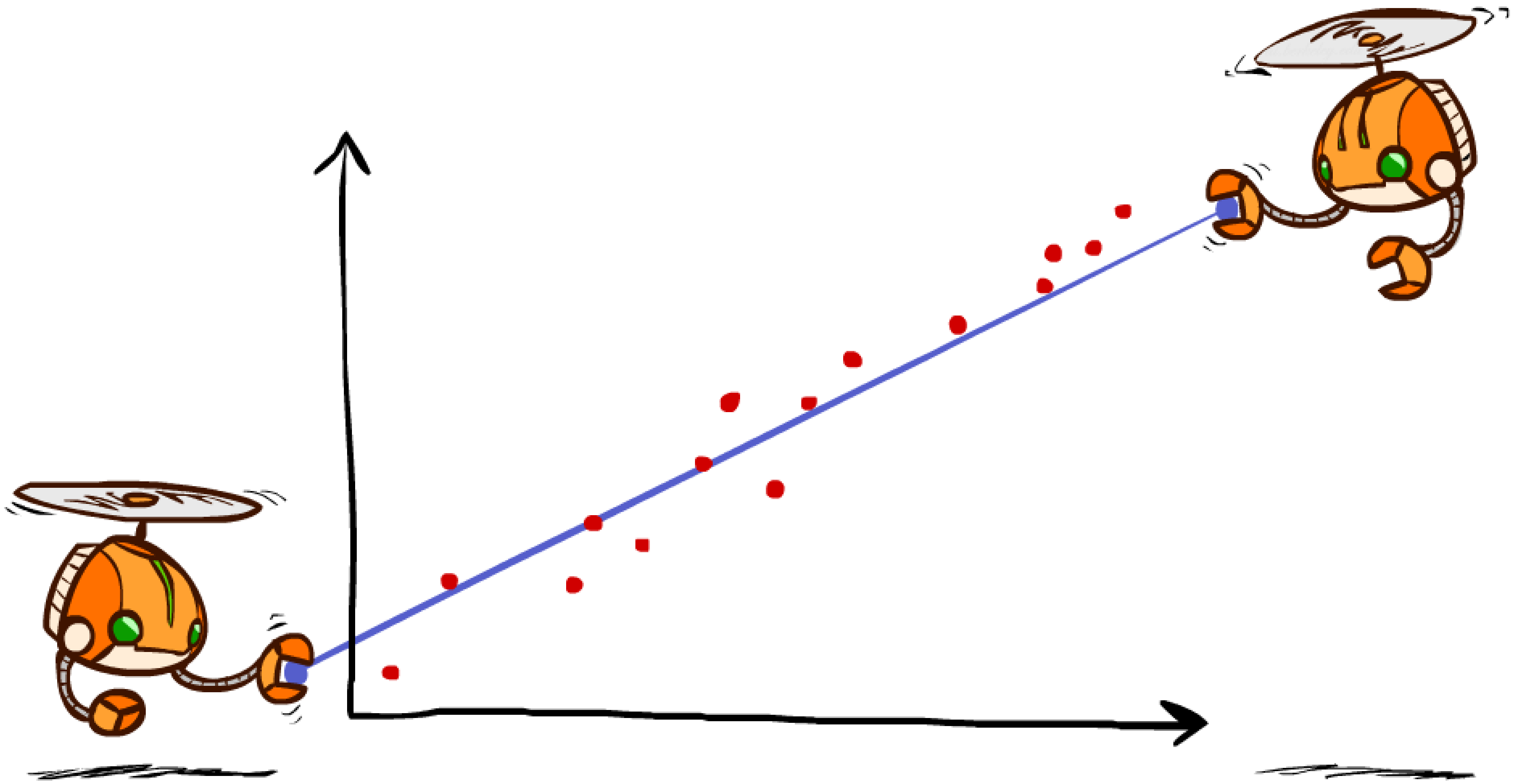
$w_{GST} \leftarrow -1.0 + \alpha\, [-501]\, 1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

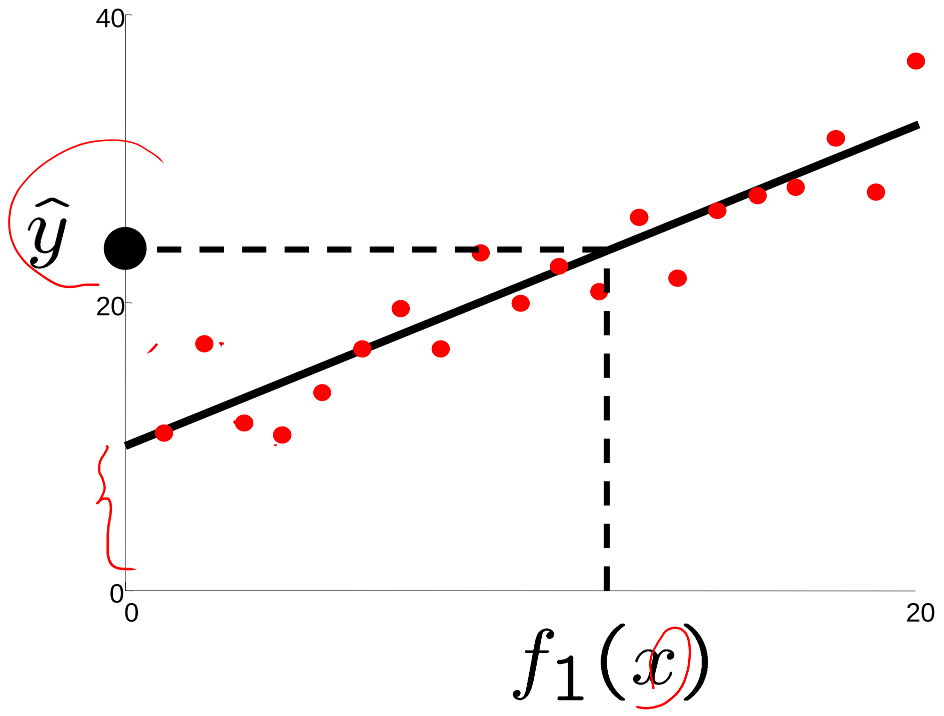
Video of Demo Approximate Q-Learning -- Pacman

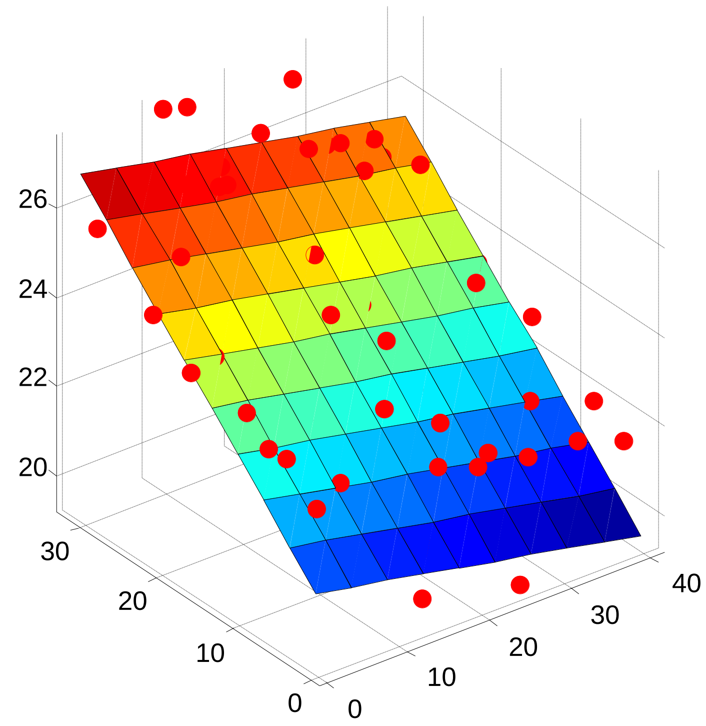


Q-Learning and Least Squares



Linear Approximation: Regression





Prediction:

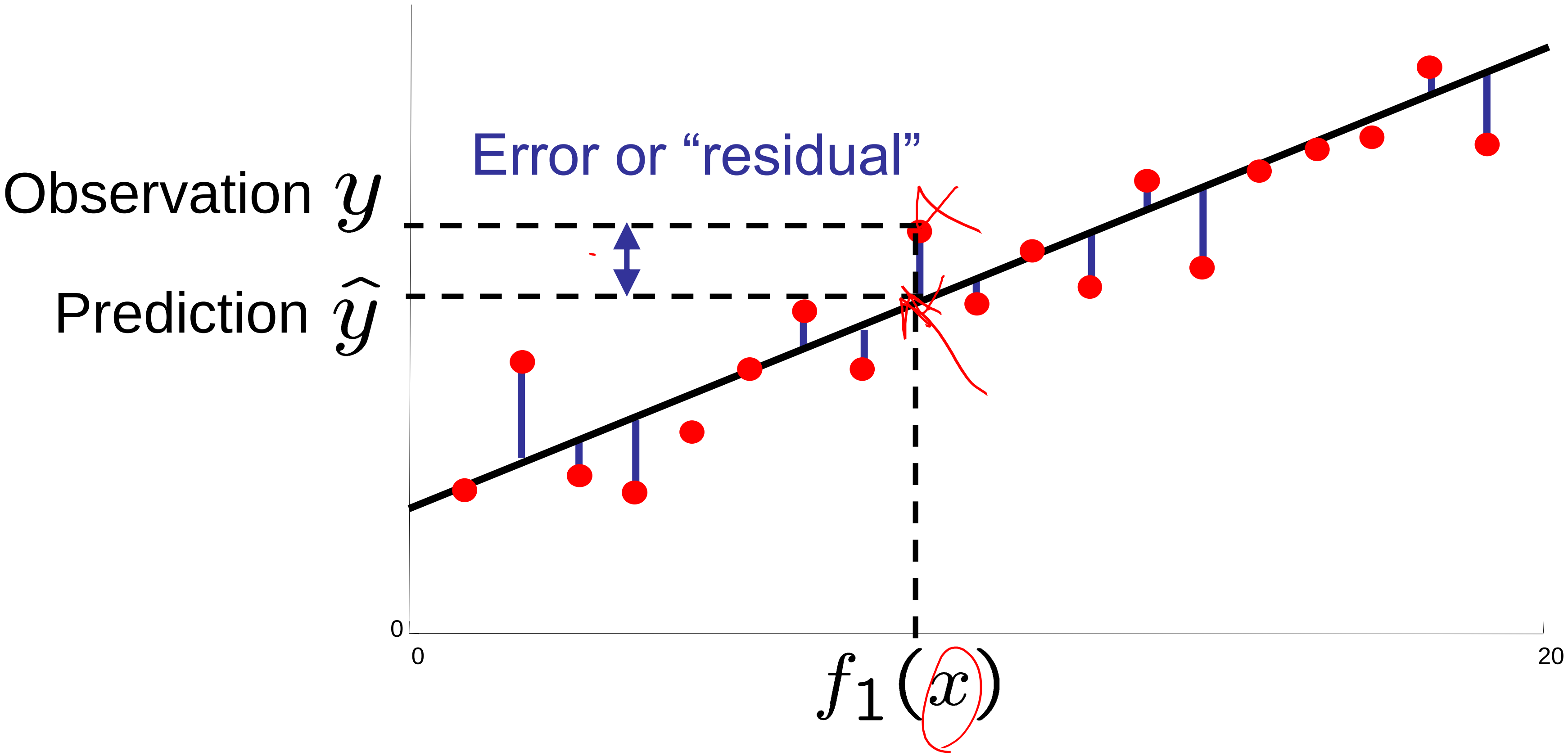
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares

$$\text{total error} = \underbrace{\sum_i (y_i - \hat{y}_i)^2} = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



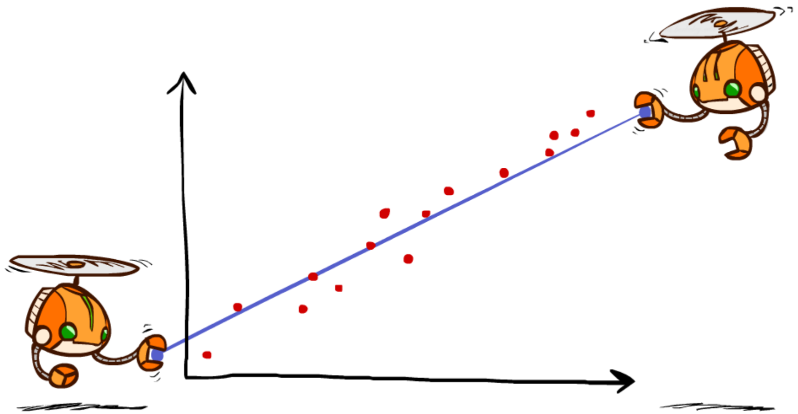
Minimizing Error

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\, \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

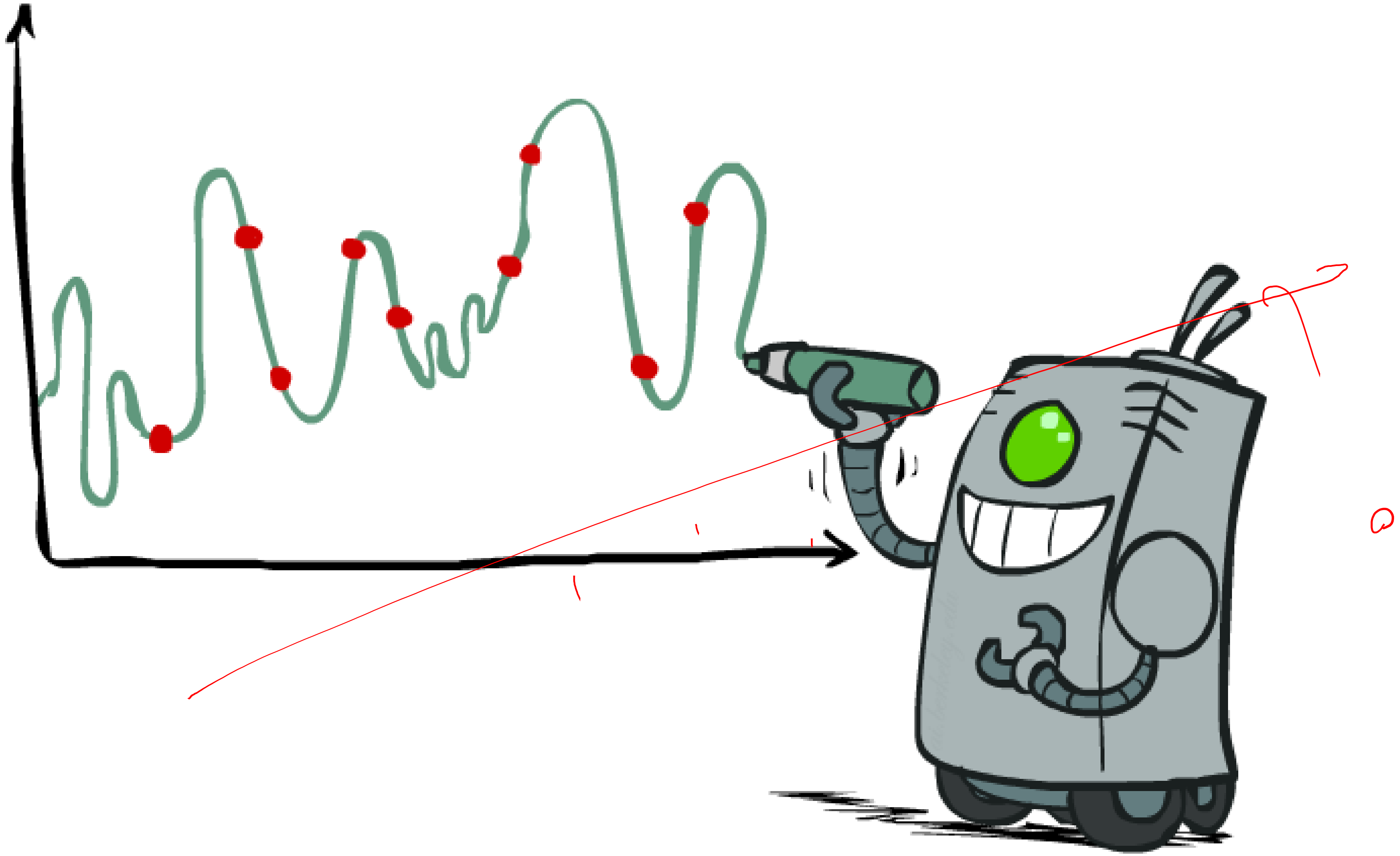
$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$



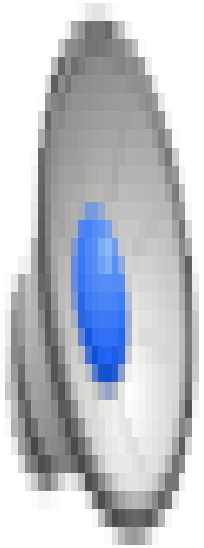
Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[\underbrace{r + \gamma \max_a Q(s', a')}_{\text{“target”}} - \underbrace{Q(s,a)}_{\text{“prediction”}}\right] f_m(s,a)$$

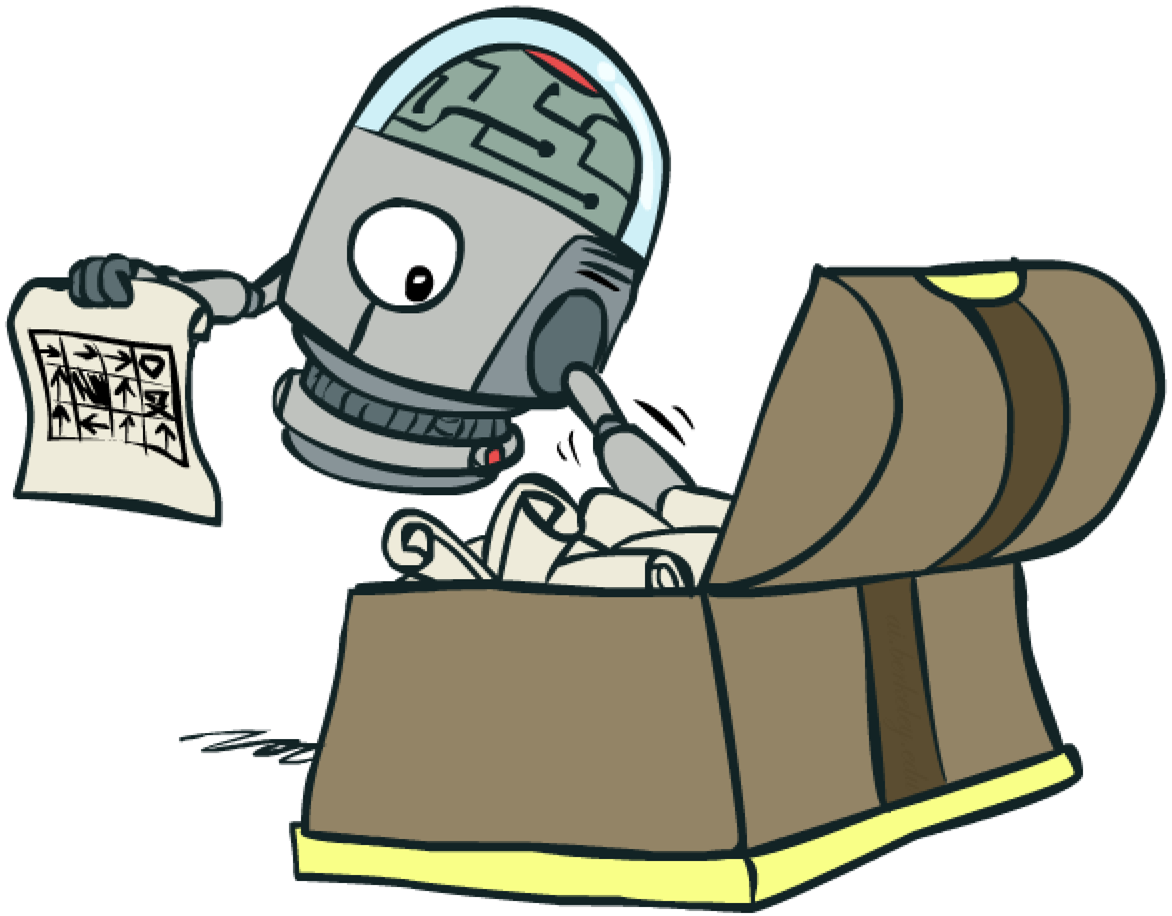
Overfitting: Why Limiting Capacity Can Help



New in Model-Free RL
Playing Atari Games



Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)
  - We'll see this distinction between modeling and prediction again later in the course

- Solution: learn policies that maximize rewards, not the values that predict them

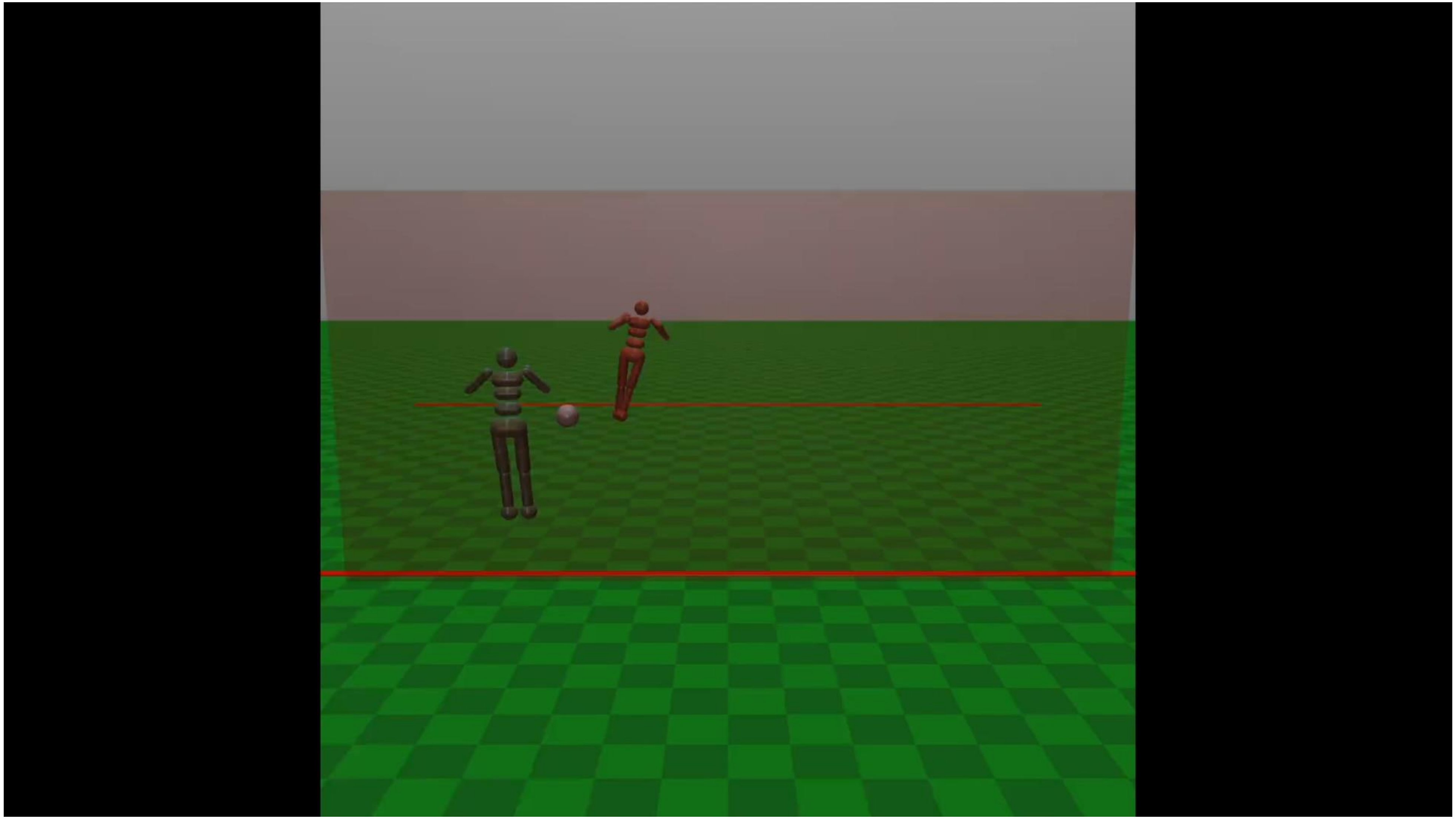
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before

- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

- Better methods exploit lookahead structure, sample wisely, change multiple parameters…



Summary: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

**use features to generalize*

Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

**use features to generalize*

Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	Value Learning

Conclusion

- We've seen how AI methods can solve problems in:
  - Search
  - Games
  - Markov Decision Problems
  - Reinforcement Learning

- Next up: Uncertainty and Learning!