WRITTEN ASSIGNMENT 2

Symmetries in CSPs. Several kinds of symmetries are exploited in constraint satisfaction problems to reduce the size of the search space. This happens in two steps. First we recognize that two or more states represent the same structure in the problem, and hence will behave similarly (i.e., will either both have a solution or both won't have a solution). As an example, consider a 7-queens representation of the problem where X_i represents the location of queen in the ith column. The board configuration $B=(X_1,...,X_7)$ is symmetric to configuration $B_v=(X_7,...,X_1)$ because B_v is a mirror image of B_1 along the vertical axis.

One way to break this symmetry we need to add constraint(s) so that only one of the symmetric configurations will be explored by the algorithm. For example, in the previous example we can add an additional "global" constraint $X_1 \le X_7$. Verify that this constraint allows it to break configurations symmetric along the vertical axis. In this question consider a similar symmetry along the horizontal axis of the board.

    (a) Give an algebraic representation of the configuration (B_h) that is symmetric along horizontal axis if B is ($X_1,...,X_7$).

    (b) Add constraint(s) so that only one of B and B_h are explored by the CSP algorithm.

    (c) Are any other symmetries present in 7-queens?

    (d) Symmetries can also be dynamic. After a value assignment new symmetries may become valid. Suppose the first queen we place is $X_1=1$. Are there new symmetries present in the problem? How about if the first queen is $X_4=1$?