ALIASING

Version 2.4: 2003/08/21 09:42:24.188 GMT-5

Don Johnson
Justin Romberg

Abstract

This module introduces the idea of aliasing and gives examples of it in sampling
and reconstruction problems.

Aliasing

1 Introduction

When considering the reconstruction[1] of a signal, you should already be familiar with the
idea of the Nyquist rate[2]. This concept allows us to find the sampling rate that will provide
for perfect reconstruction of our signal. If we sample at too low of a rate (below the
Nyquist rate), then problems will arise that will make perfect reconstruction impossible -
this problem is known as **aliasing**. Aliasing occurs when there is an overlap in the shifted,
perioidic copies of our original signal's FT, i.e. spectrum.

In the frequency domain, one will notice that part of the signal will overlap with the
periodic signals next to it. In this overlap the values of the frequency will be added together
and the shape of the signals spectrum will be unwantingly altered. This overlapping, or
aliasing, makes it impossible to correctly determine the correct strength of that frequency.
The figure belows provides a visual example of this phenomenon:

2 Aliasing and Sampling

If we sample too slowly, i.e.,

$$\forall, \left(T > \frac{\pi}{\Omega_B} \right) : \Omega_s < 2\Omega_B$$

We cannot recover the signal from its samples due to aliasing.

Example 1:

Let $f_1(t)$ have CTFT.

Let $f_2(t)$ have CTFT.

Try to sketch and answer the following questions on your own:
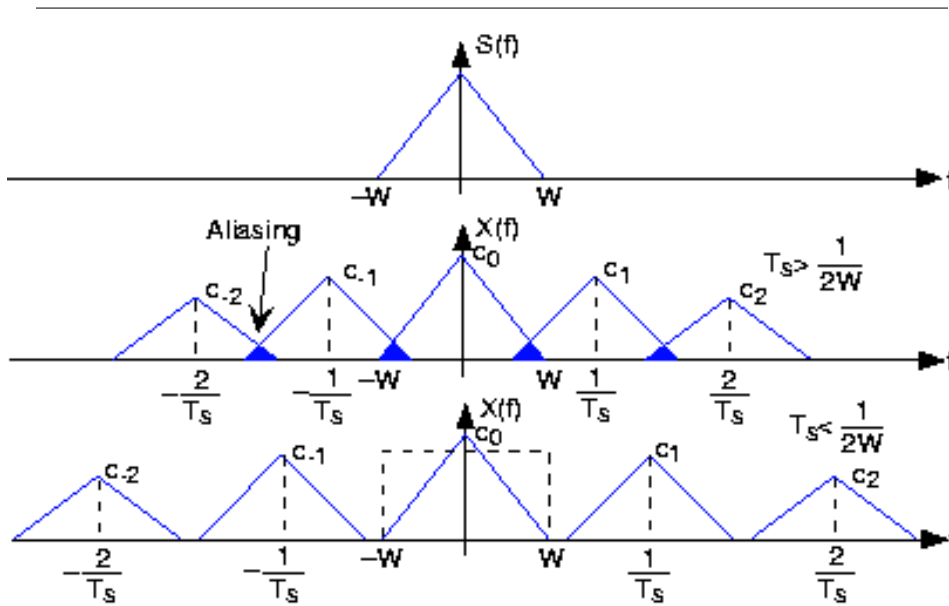

Figure 1: The spectrum of some bandlimited (to W Hz) signal is shown in the top plot. If the sampling interval T_s is chosen too large relative to the bandwidth W, aliasing will occur. In the bottom plot, the sampling interval is chosen sufficiently small to avoid aliasing. Note that if the signal were not bandlimited, the component spectra would always overlap.
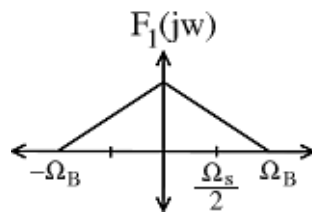

Figure 2: In the above figure, note the following equation: $\Omega_B - \frac{\Omega_s}{2} = a$
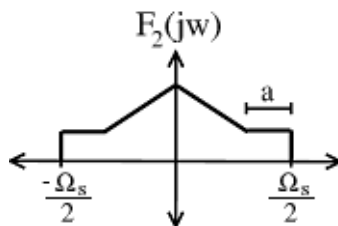

Figure 3: The horizontal portions of the signal result from overlap with shifted replicas - showing visual proof of aliasing.
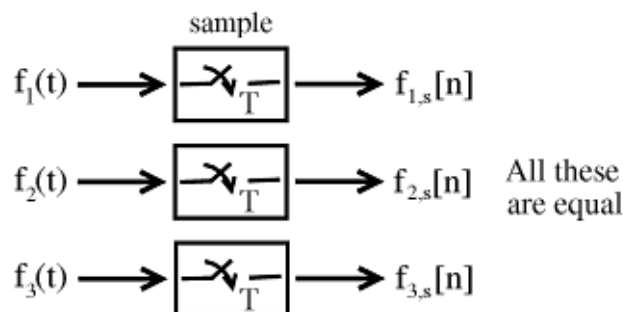
Figure 4: These are all equal!

- •What does the DTFT of $f_{1,s}[n] = f_1(nT)$ look like?
- •What does the DTFT of $f_{2,s}[n] = f_2(nT)$ look like?
- •Do any other signals have the same DTFT as $f_{1,s}[n]$ and $f_{2,s}[n]$?

CONCLUSION: If we sample below the Nyquist frequency, there are many signals that could have produced that given sample sequence.

Why the term "aliasing"? Because the same sample sequence can represent different CT signals (as opposed to when we sample above the Nyquist frequency, then the sample sequence represents a unique CT signal).

Example 2:

$f(t) = \cos(2\pi t)$

Case 1: Sample $\Omega_s = 8\pi$ rad/sec $\Rightarrow T = \frac{1}{4}$ sec.

   NOTE: $\Omega_s > 2\Omega_B$

Case 2: Sample $\frac{w}{\Omega_s} = \frac{8}{3}\pi$ rad/sec $\Rightarrow T = \frac{3}{4}$ sec

   NOTE: $\Omega_s < 2\Omega_B$

When we run the DTFT from Case 2 through the reconstruction steps, we realize that we end up with the following cosine:

$$\tilde{f}(t) = \cos\left(\frac{\pi}{2}t\right)$$

This is a "stretched" out version of our original. Clearly, our sampling rate was not high enough to ensure correct reconstruction from the samples.

You may have seen some effects of aliasing such as a wagon wheel turning backwards in a western movie. Here are some applets[3] that illustrate such effects of undersampling and aliasing. Aliasing in images[4] can result in Moire Patterns. Here is an example of an image that has Moire artifacts[5] as a result of scanning at too low a frequency.

---

[3]http://flowers.ofthenight.org/wagonWheel
[4]http://ptolemy.eecs.berkeley.edu/eecs20/week13/moire.html
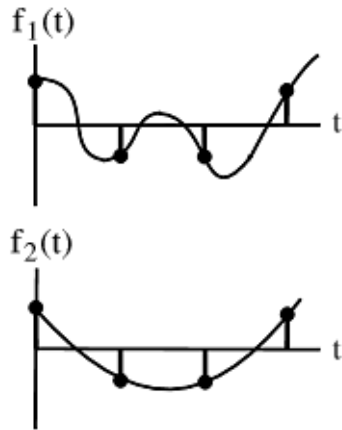[5]http://www.dvp.co.il/filter/moire.html

Figure 5:   These two signals contain the same four samples, yet are very different signals.
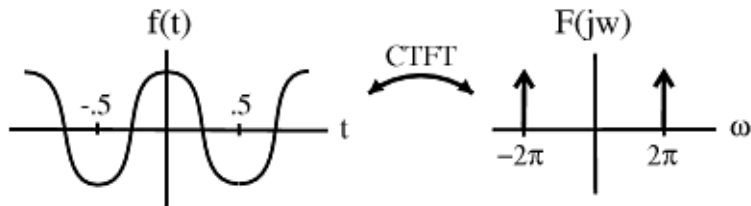

Figure 6:   The figure shows the cosine function, $f(t) = \cos(2\pi t)$, and its CTFT.