Image Processing

Reading

Course Reader:

Jain et. Al. ***Machine Vision***
Chapter 4 and 5

Definitions

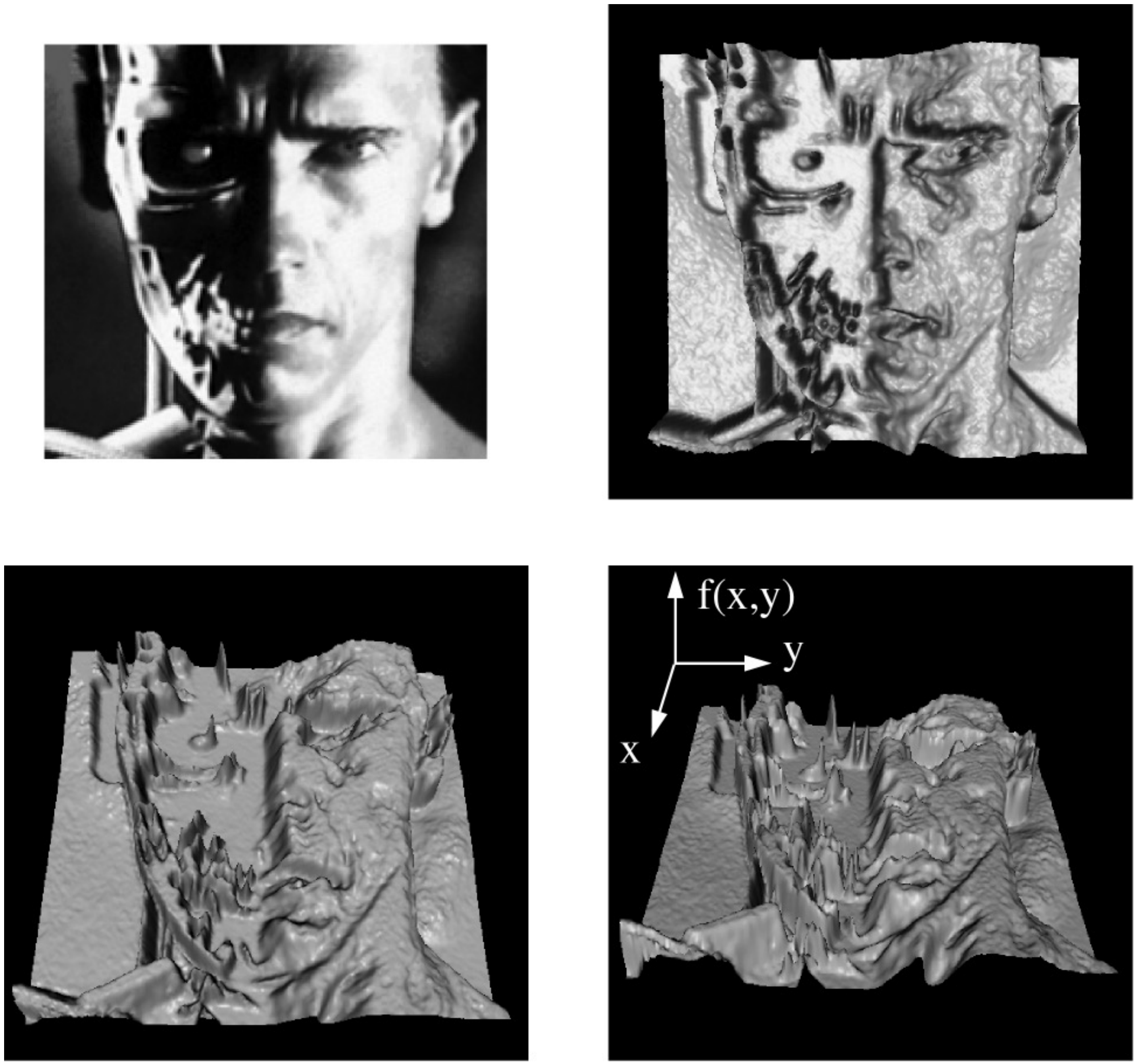
- Many graphics techniques that operate only on images
- **Image processing**: operations that take images as input, produce images as output
- In its most general form, an **image** is a function f from R^2 to R
  - $f(x, y)$ gives the intensity of a channel at position (x, y) defined over a rectangle, with a finite range:

$$f: [a,b] \times [c,d] \rightarrow [0,1]$$

  - A color image is just three functions pasted together:

$$\boldsymbol{f}(x, y) = (f_r(x, y), f_g(x, y), f_b(x, y))$$

Images as Functions



What is a digital image?

- In computer graphics, we usually operate on **digital** (**discrete**) images:
  - **Sample** the space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- If our samples are d apart, we can write this as:

$$f'[i,j] = Quantize(f(i \cdot d, j \cdot d))$$

Sampled digital image

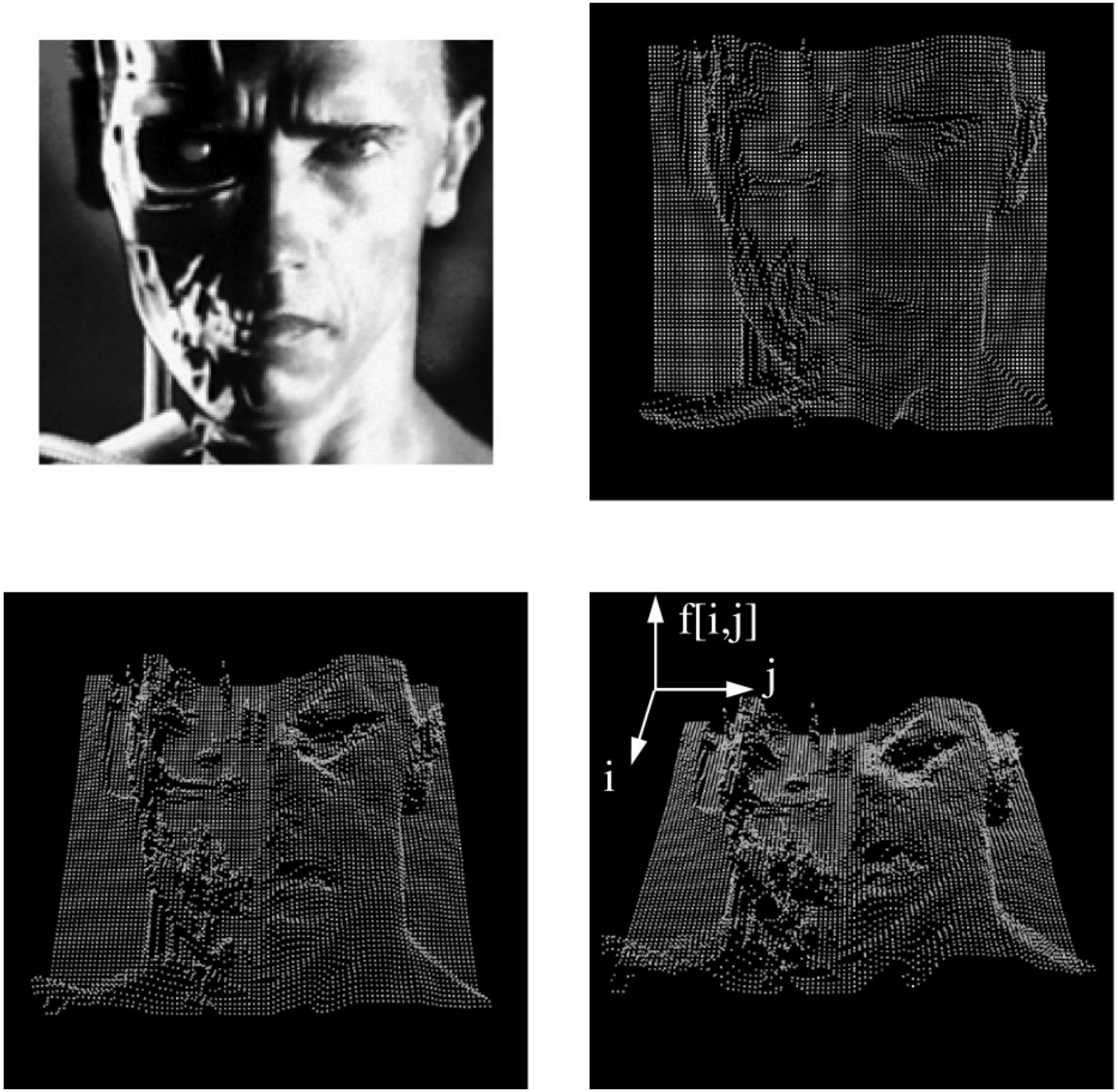


Image processing

- An **image processing** operation typically defines a new image *g* in terms of an existing image *f*.
- The simplest operations are those that transform each pixel in isolation. These pixel-to-pixel operations can be written:

$$g(x, y) = t(f(x, y))$$

- Example: threshold, RGB $\rightarrow$ grayscale

Pixel Movement

- Some operations preserve intensities, but move pixels around in the image

$$g(x, y) = f(u(x, y), v(x, y))$$

- Examples: many amusing warps of images

Multiple input images

- Some operations define a new image g in terms of n existing images (f_1, f_2, ... , f_n), where n is greater than 1

- Example: cross-dissolve between 2 input images

$$g(x, y) = \sum_i w_i f_i(x, y)$$

Noise

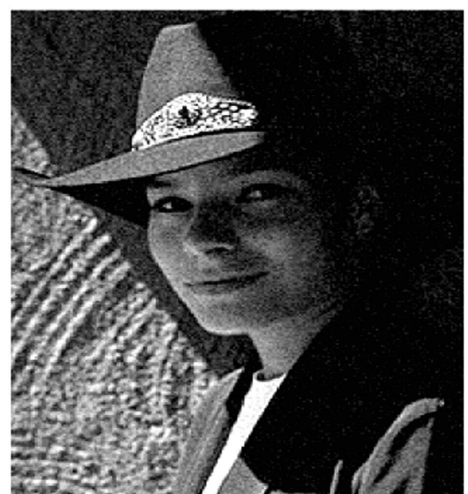
- Common types of noise:
  - **Salt and pepper noise**: contains random occurrences of black and white pixels
  - **Impulse noise:** contains random occurrences of white pixels
  - **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Noise Examples

Original

Salt and pepper noise

Impulse noise



Gaussian noise

Ideal noise reduction

Ideal noise reduction

Practical noise reduction

- How can we "smooth" away noise in a single image?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	100	130	110	120	110	0	0
0	0	0	110	90	100	90	100	0	0
0	0	0	130	100	90	130	110	0	0
0	0	0	120	100	130	110	120	0	0
0	0	0	90	110	80	120	100	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Cross-correlation filtering

- Let’s write this down as an equation. Assume the averaging window is (2k+1)x(2k+1):

$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]$$

- We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

- This is called a **cross-correlation** operation and written:

$$G = H \otimes F$$

- H is called the “filter,” “kernel,” or “mask.”
- The above allows negative filter indices. When you implement need to use: H[u+k,v+k] instead of H[u,v]

Mean kernel

- What's the kernel for a 3x3 mean filter?

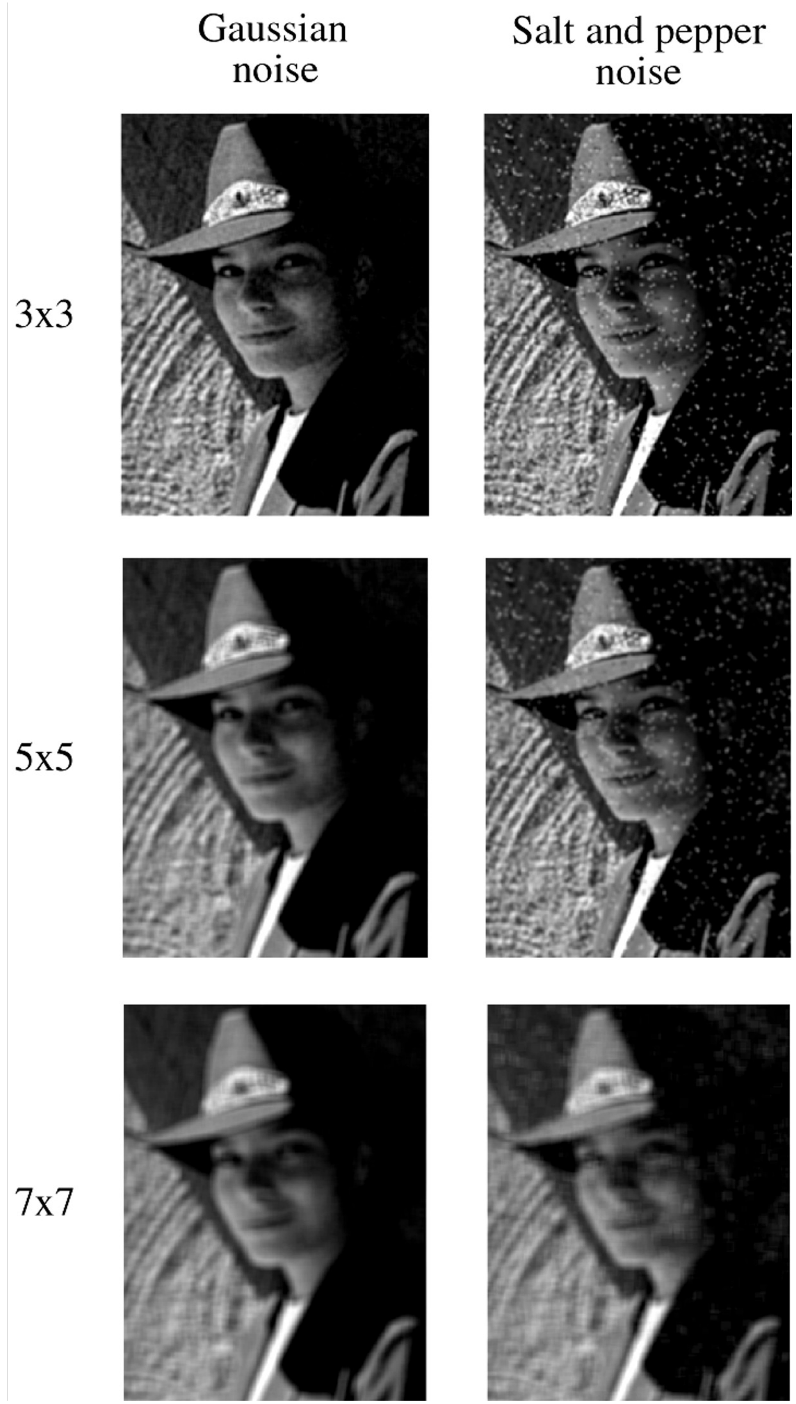
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$H[u, v]$

Mean Filters



Gaussian Filtering

- A Gaussian kernel gives less weight to pixels further from the center of the window

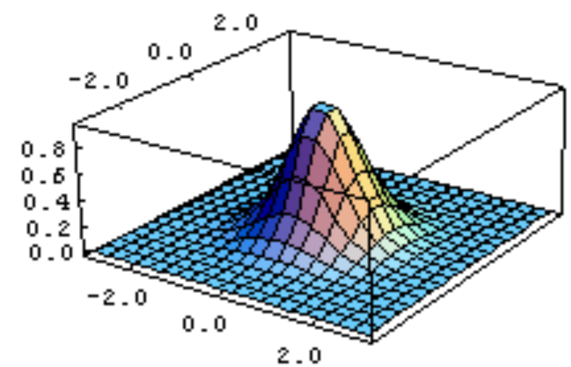
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H[u, v]$



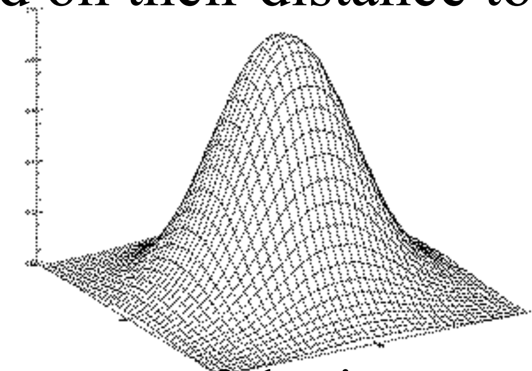
Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

Gaussian Filters

- Gaussian filters weigh pixels based on their distance to the location of convolution.

$$h[i,j] = e^{-\left(i^2+j^2\right)/2\sigma^2}$$



- Blurring noise while preserving features of the image
- Smoothing the same in all directions
- More significance to neighboring pixels
- Width parameterized by s
- Gaussian functions are separable
- Convolving with multiple Gaussian filters results in a single Gausian filter

Convolution

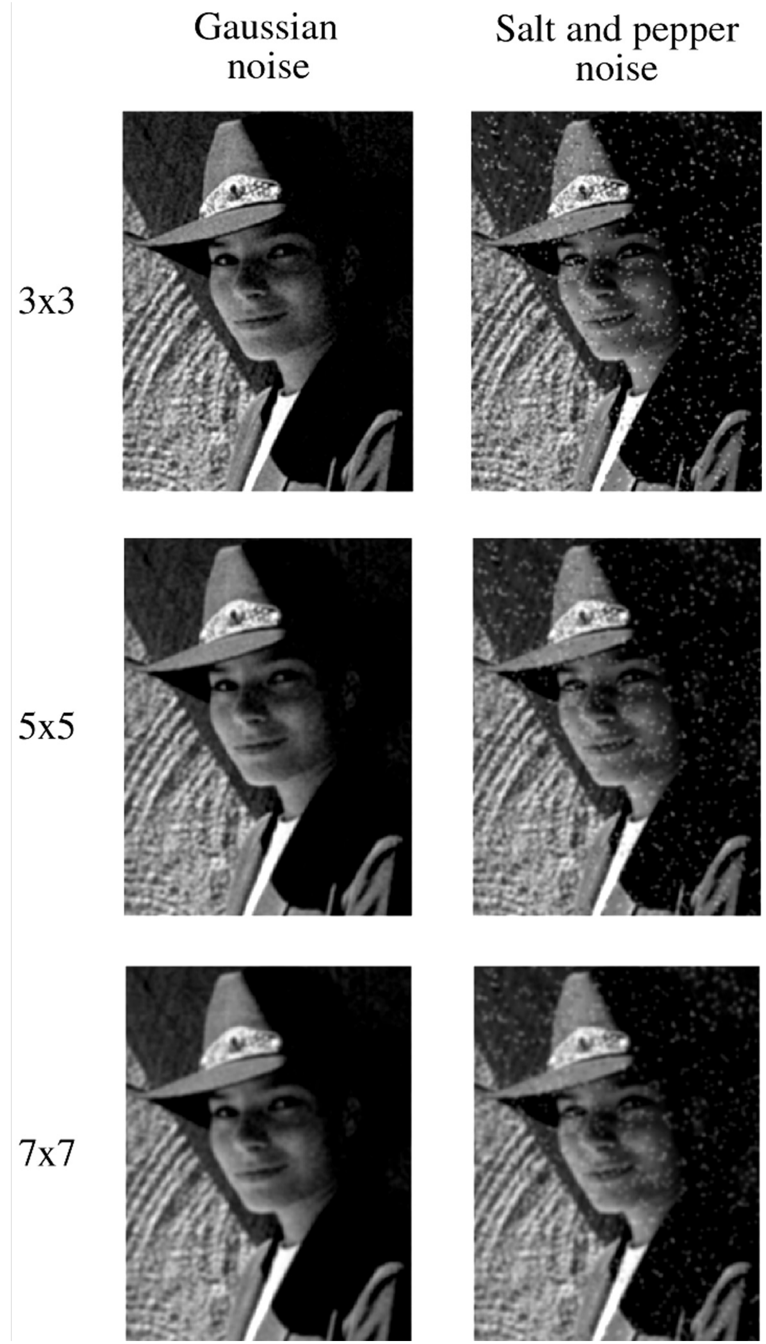
- A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

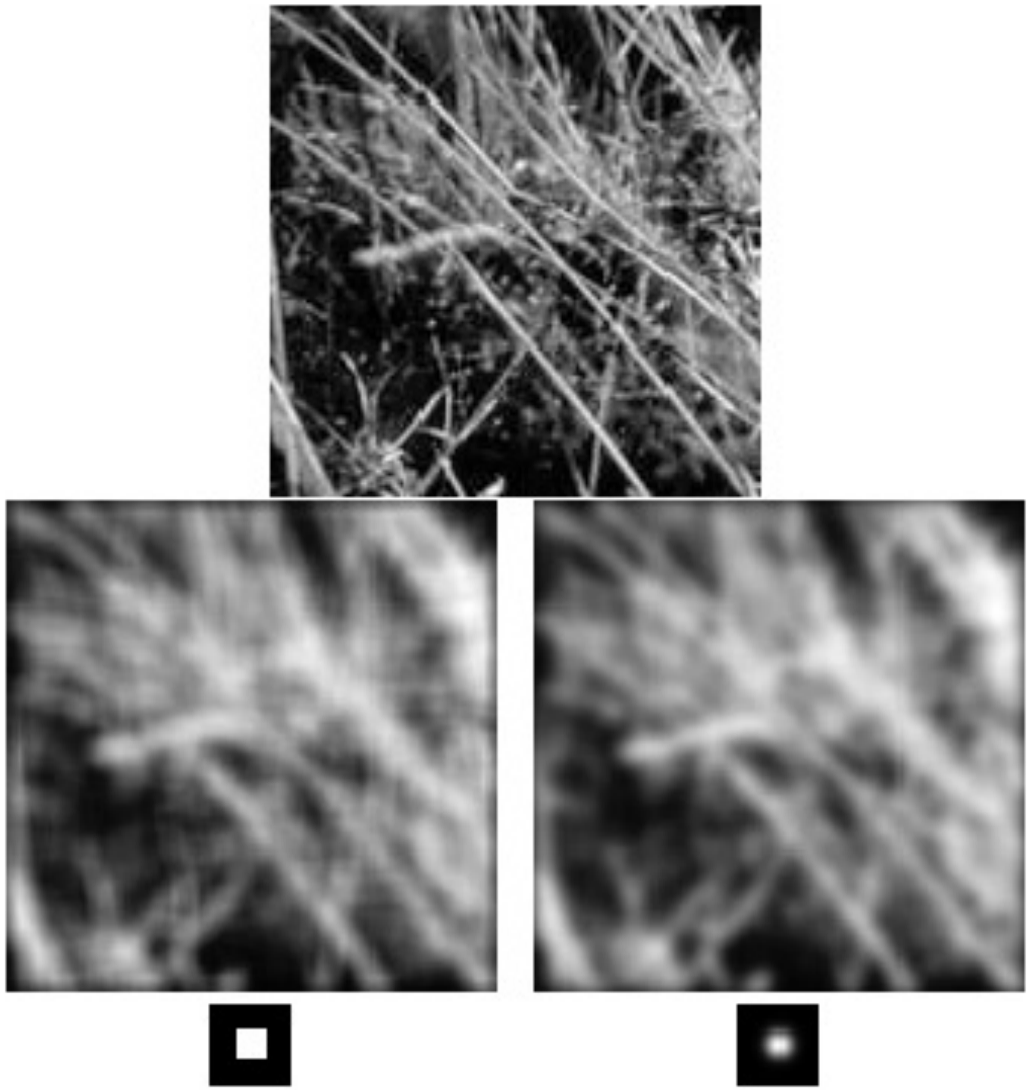
- It is written: $G = H \star F$

- Suppose H is a Gaussian or mean kernel. How does convolution differ from cross-correlation?

Gaussian Filters



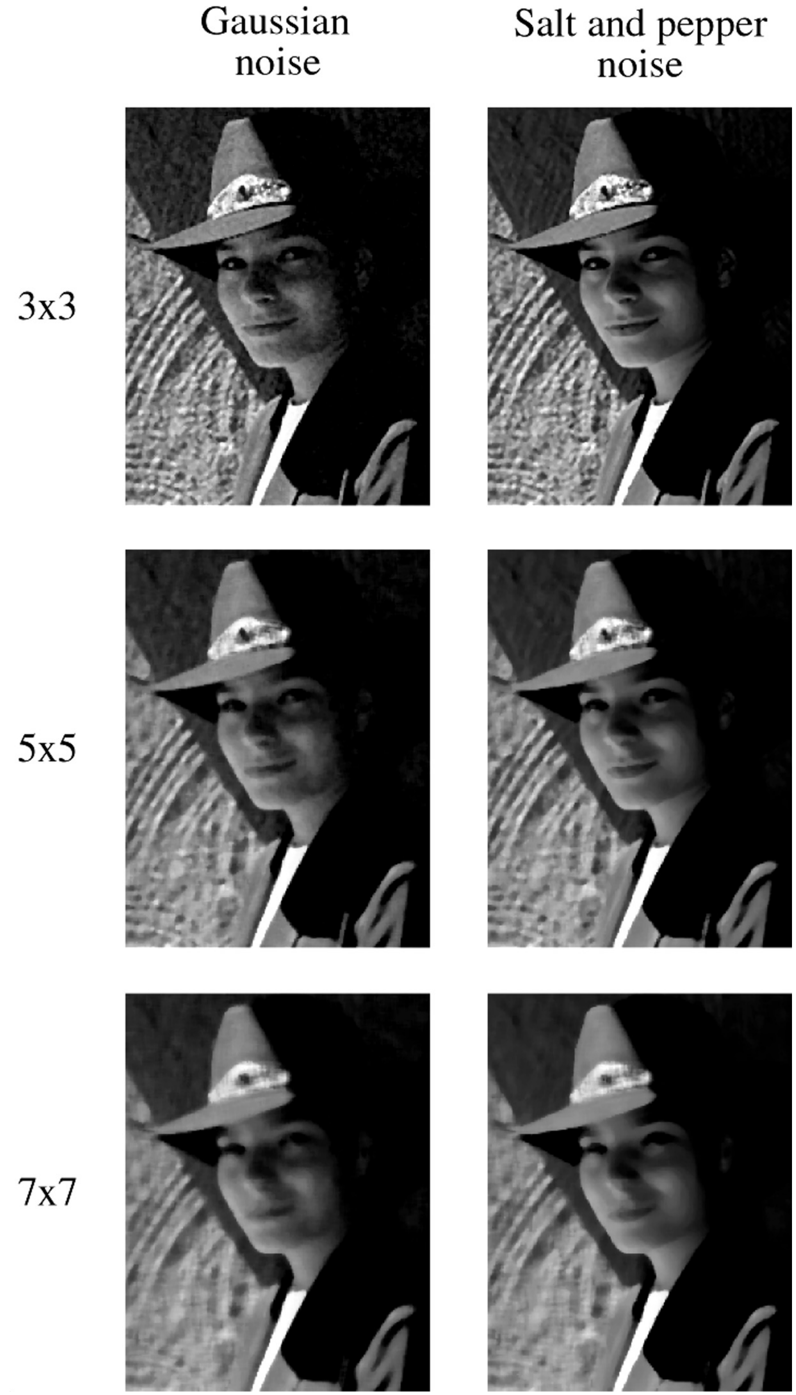
Mean vs. Gaussian filtering

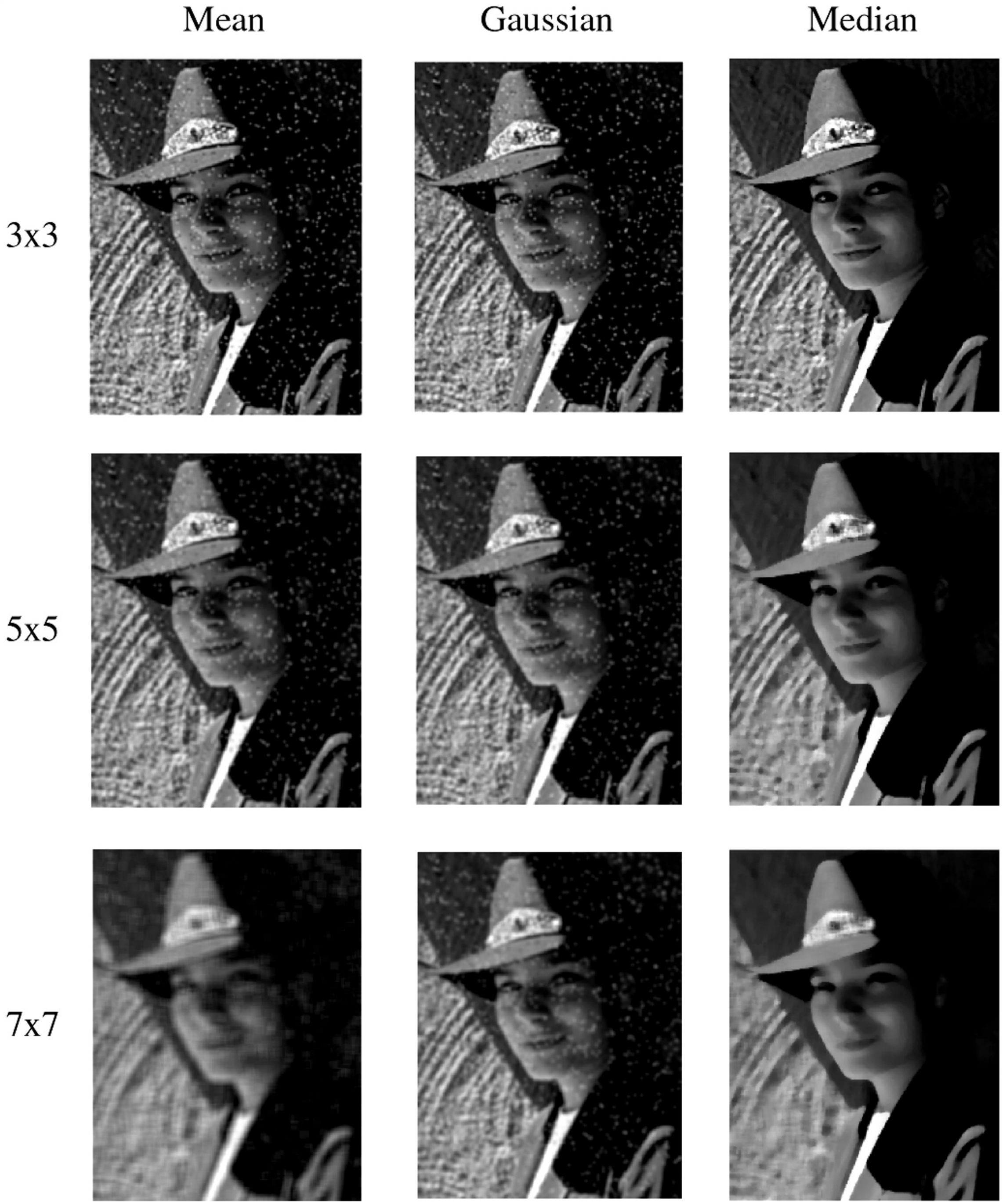


Median Filters

- A **Median Filter** operates over a k £ k region by selecting the median intensity in the region.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

Median Filters




Mean
Gaussian
Median
3x3
5x5
7x7

Sampling theorem

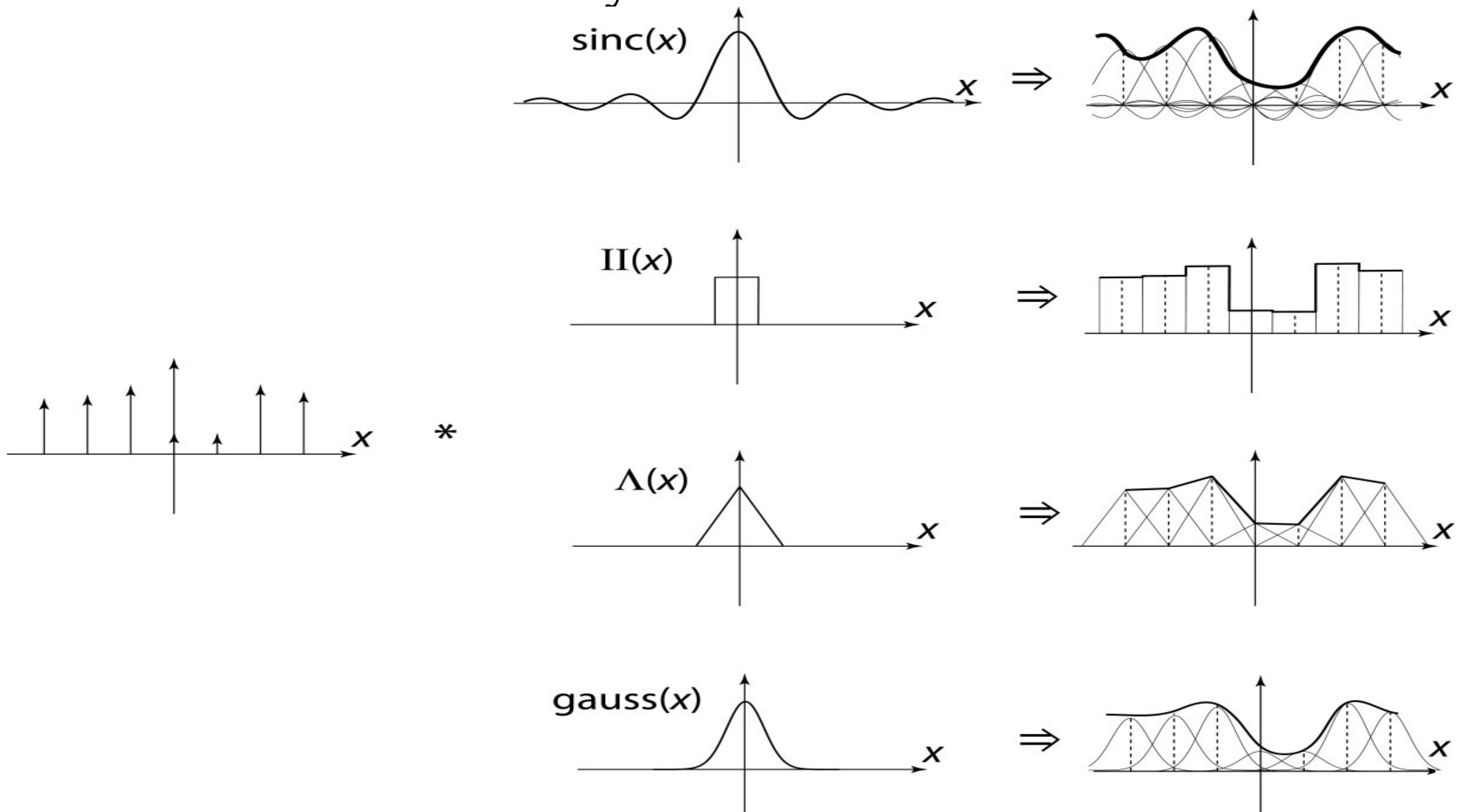
- This result is known as the **Sampling Theorem** and is due to Claude Shannon who first discovered it in 1949:

  A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above ½ the sampling frequency.

- For a given **bandlimited** function, the minimum rate at which it must be sampled is the **Nyquist frequency**.

Reconstruction filters

- The sinc filter, while “ideal”, has two drawbacks:
  - It has large support (slow to compute)
  - It introduces ringing in practice
- We can choose from many other filters…

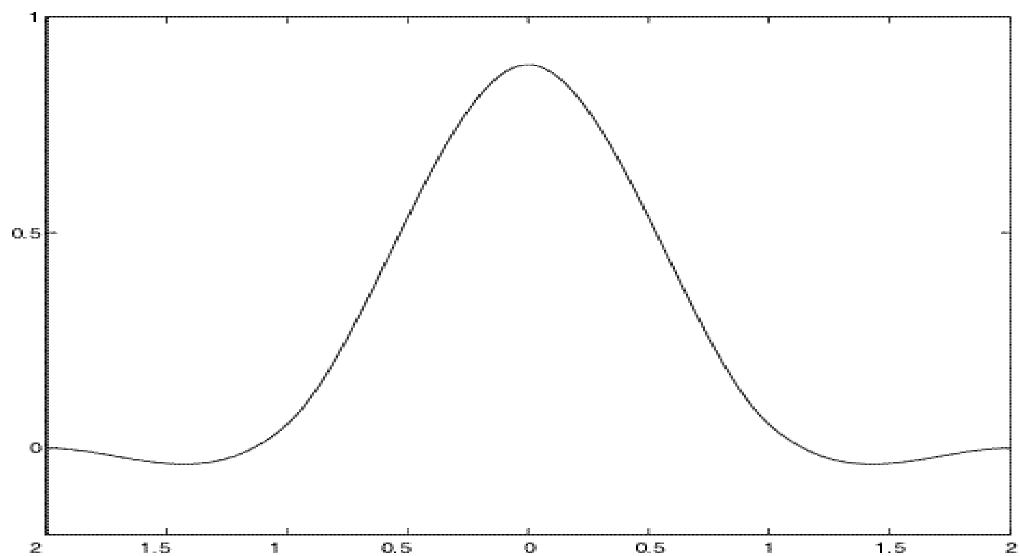


Cubic filters

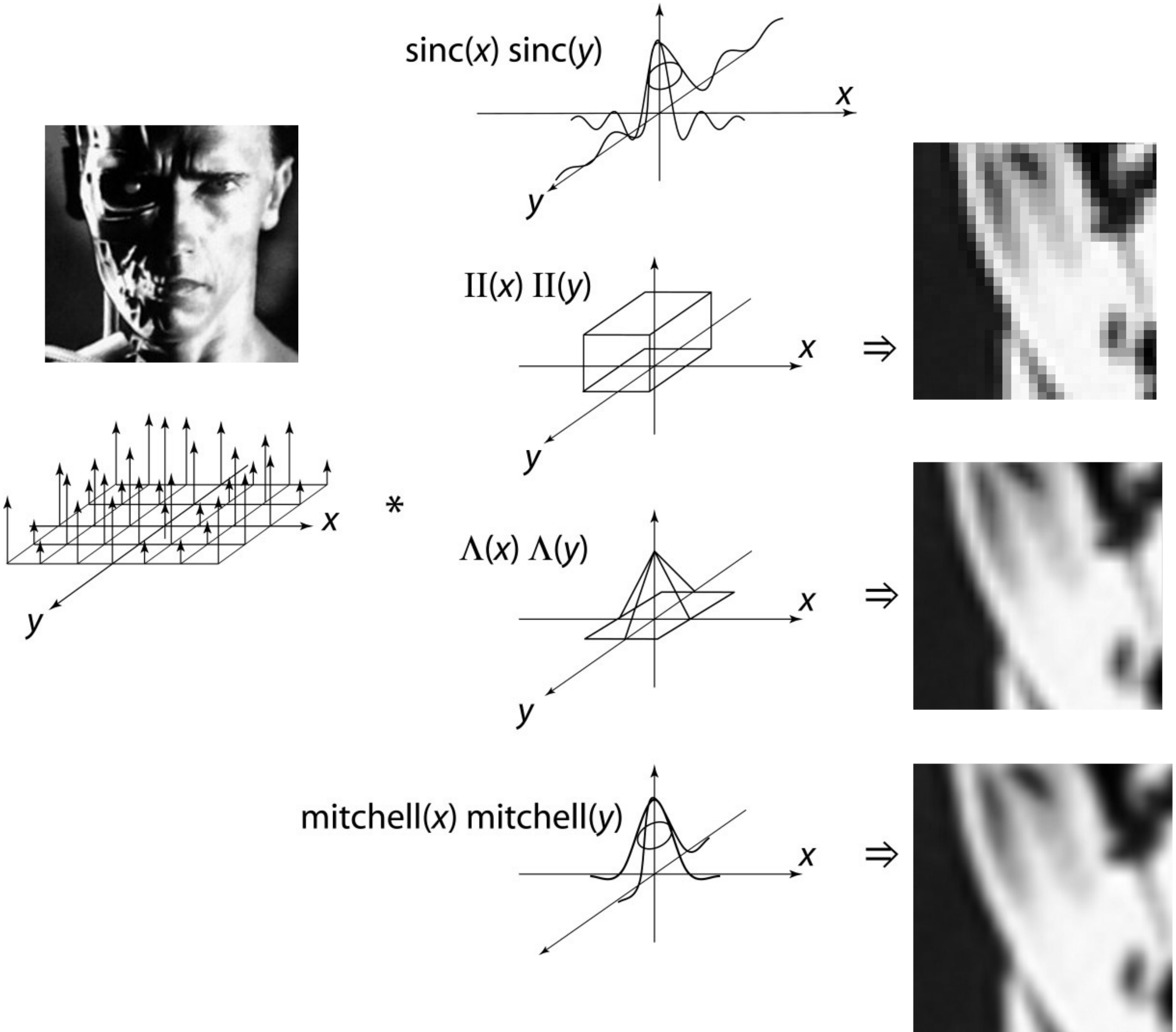
•Mitchell and Netravali (1988) experimented with cubic filters, reducing them all to the following form:

$$r(x) = \frac{1}{6}\begin{cases} (12-9B-6C)|x|^3 + (-18+12B+6C)|x|^2 + (6-2B) & |x| < 1 \\ ((-B-6C)|x|^3 + (6B+30C)|x|^2 + (-12B-48C)|x| + (8B+24C) & 1 \le |x| < 2 \\ 0 & otherwise \end{cases}$$

•The choice of B or C trades off between being too blurry or having too much ringing. B=C=1/3 was their "visually best" choice: "Mitchell filter."



Reconstruction filters in 2D

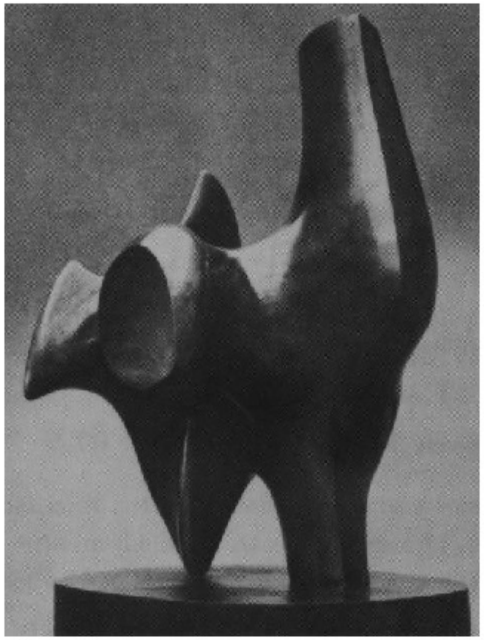


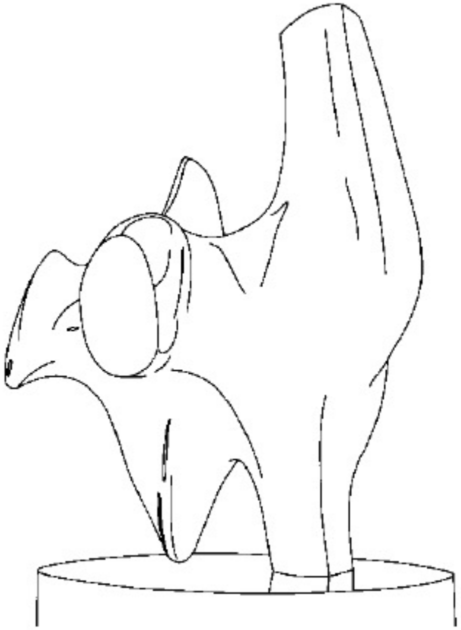
Edge detection

•One of the most important uses of image processing is **edge detection:**

- Really easy for humans
- Really difficult for computers

- Fundamental in computer vision
- Important in many graphics applications



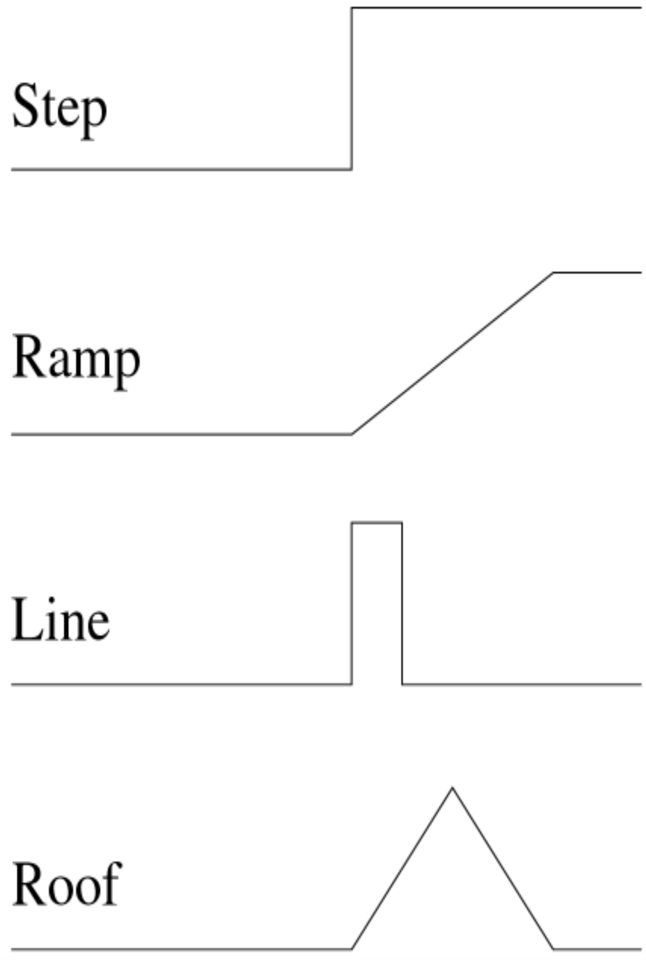


•How to tell if a pixel is on an edge?

Edge Detection

- One of the most important uses of image processing is **edge detection**
  - Really easy for humans
  - Really difficult for computers

  - Fundamental in computer vision
  - Important in many graphics applications
- What defines an edge?



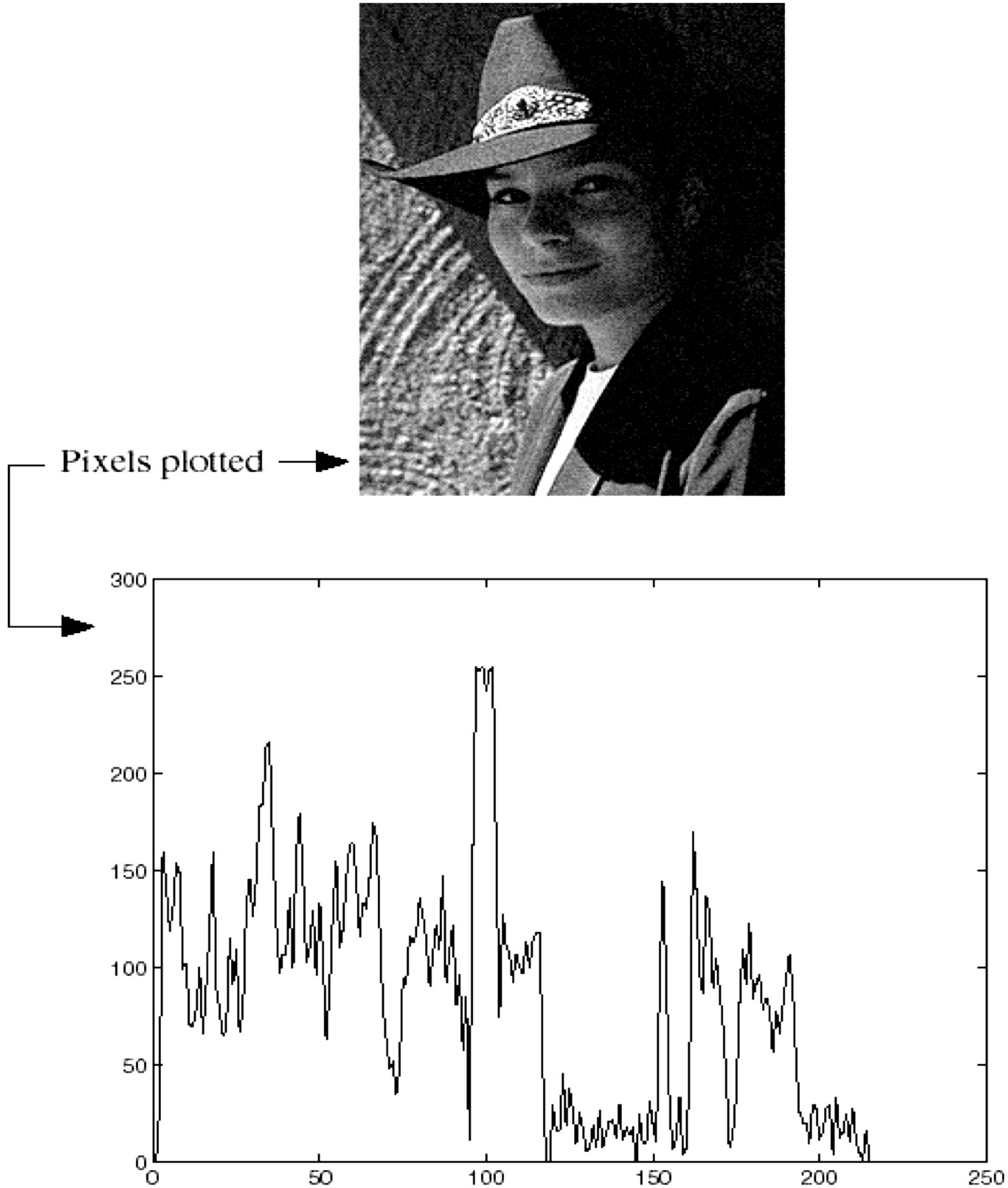
Gradient

- The **gradient** is the 2D equivalent of the derivative:

$$\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

- Properties of the gradient
  - It's a vector
  - Points in the direction of maximum increase of f
  - Magnitude is rate of increase
- How can we approximate the gradient in a discrete image?

Less than ideal edges



Edge Detection Algorithms

- Edge detection algorithms typically proceed in three or four steps:
  - Filtering: cut down on noise
  - Enhancement: amplify the difference between edges and non-edges
  - Detection: use a threshold operation
  - Localization (optional): estimate geometry of edges beyond pixels

Edge Enhancement

- A popular gradient magnitude computation is the **Sobel operator**:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- We can then compute the magnitude of the vector (s_x, s_y)

Sobel Operator


Original


Smoothed


Sx + 128

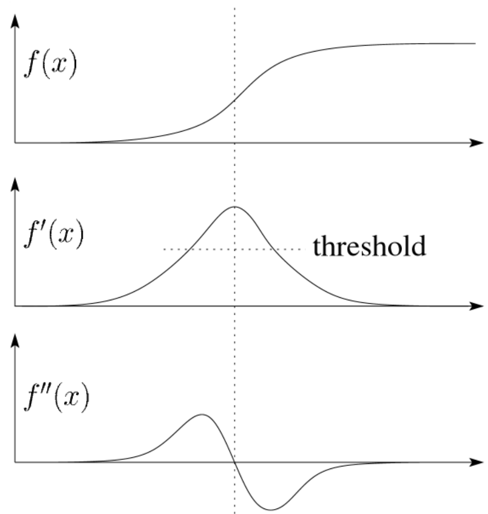

Sy + 128


Magnitude


Threshold = 64


Threshold = 128

Second derivative operators



- The Sobel operator can produce thick edges. Ideally, we're looking for infinitely thin boundaries.
- An alternative approach is to look for local extrema in the first derivative: places where the change in the gradient is highest.
- **Q**: A peak in the first derivative corresponds to what in the second derivative?

Localization with the Laplacian

- An equivalent measure of the second derivative in 2D is the **Laplacian**:

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Using the same arguments we used to compute the gradient filters, we can derive a Laplacian filter to be:

$$\Delta^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Zero crossings of this filter correspond to positions of maximum gradient. These zero crossings can be used to localize edges.

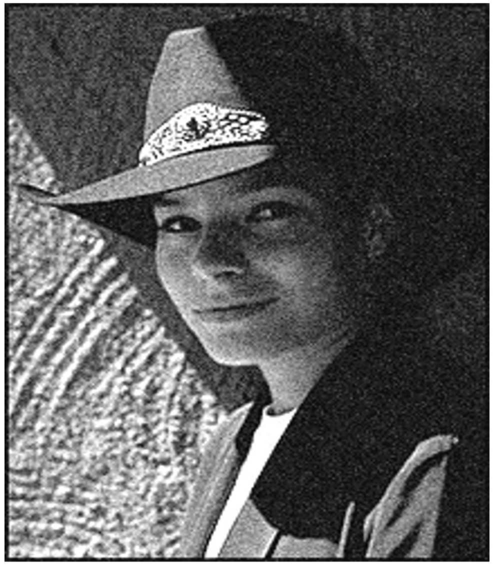
Laplacian alternatives

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

−1	2	−1
2	−4	2
−1	2	−1

Localization with the Laplacian



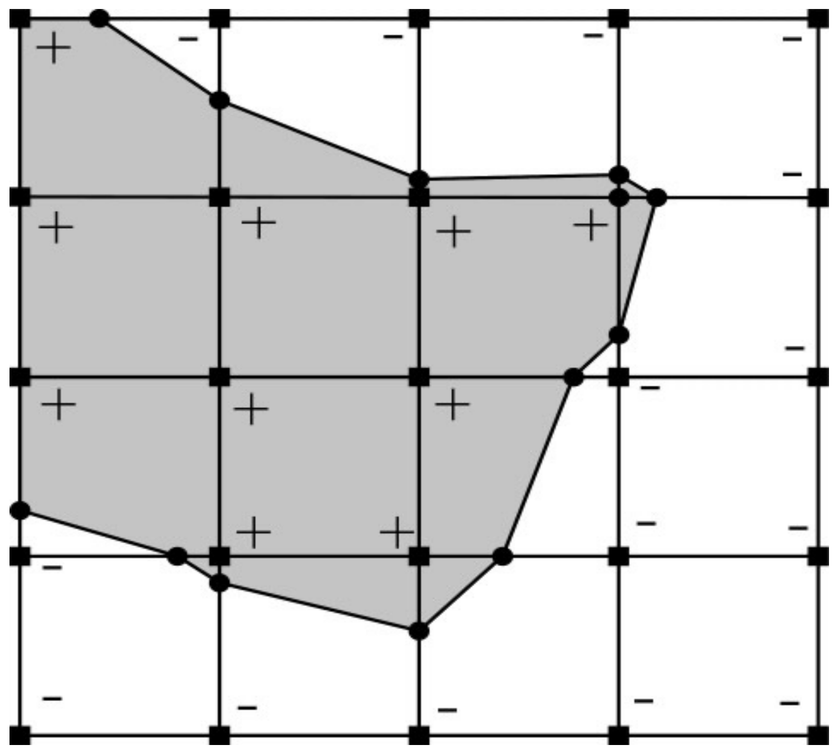
Original

Smoothed

Laplacian (+128)

Marching squares

- We can convert these signed values into edge contours using a "marching squares" technique:



Sharpening with the Laplacian

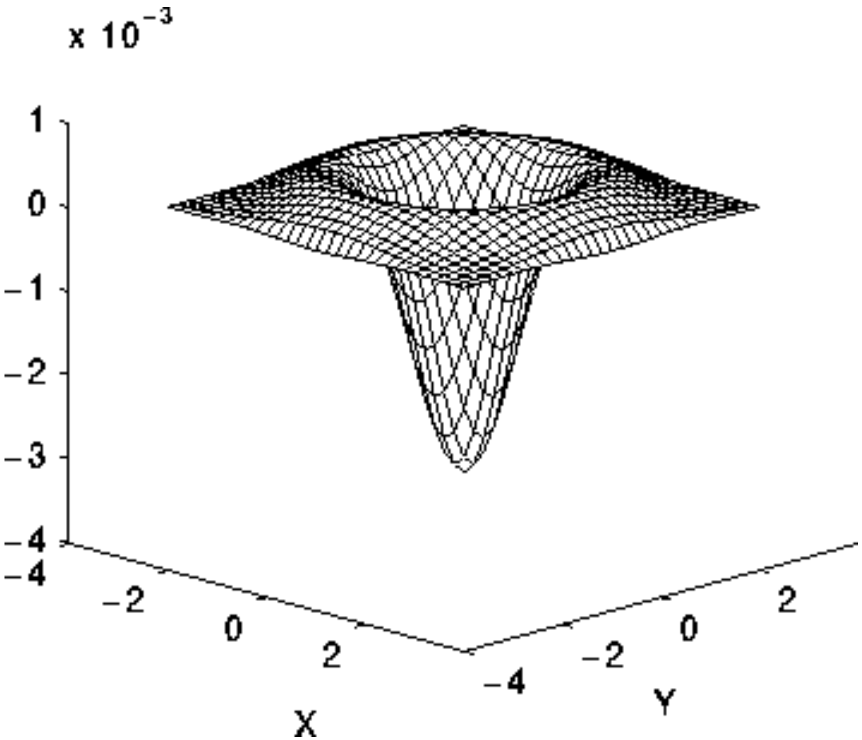
Original

Laplacian (+128)

Original + Laplacian

Original - Laplacian

Laplacian of Gaussian



0	0	3	2	2	2	3	0	0
0	2	3	5	5	5	3	2	0
3	3	5	3	0	3	5	3	3
2	5	3	-12	-23	-12	3	5	2
2	5	0	-23	-40	-23	0	5	2
2	5	3	-12	-23	-12	3	5	2
3	3	5	3	0	3	5	3	3
0	2	3	5	5	5	3	2	0
0	0	3	2	2	2	3	0	0

Summary

- Formal definitions of image and image processing
- Kinds of image processing: pixel-to-pixel, pixel movement, convolution, others
- Types of noise and strategies for noise reduction
- Definition of convolution and how discrete convolution works
- The effects of mean, median and Gaussian filtering
- How edge detection is done
- Gradients and discrete approximations