

Hierarchical Modeling

Reading

Optional:

- ◆ Angel, sections 8.1 – 8.6, 8.8

Further reading:

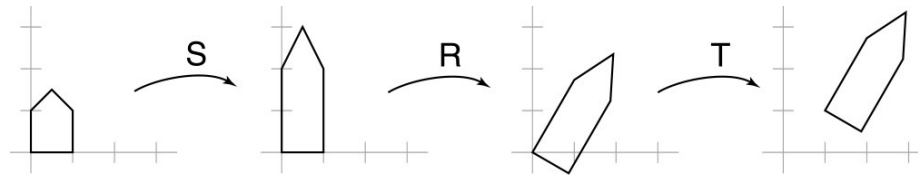
- ◆ *OpenGL Programming Guide*, chapter 3

Symbols and instances

Most graphics APIs support a few geometric **primitives**:

- ◆ spheres
- ◆ cubes
- ◆ cylinders

These symbols are **instanced** using an **instance transformation**.



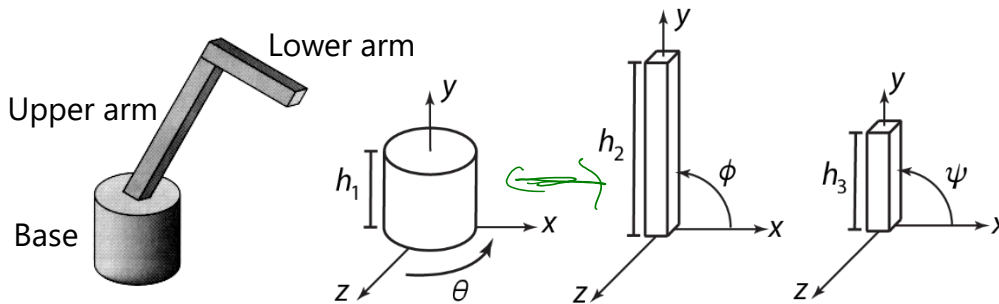
Q: What is the matrix for the instance transformation above?

$$TRS \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

3D Example: A robot arm

Let's build a robot arm out of a cylinder and two cuboids, with the following 3 degrees of freedom:

- ◆ Base rotates about its vertical axis by θ
- ◆ Upper arm rotates in its xy -plane by ϕ
- ◆ Lower arm rotates in its xy -plane by ψ



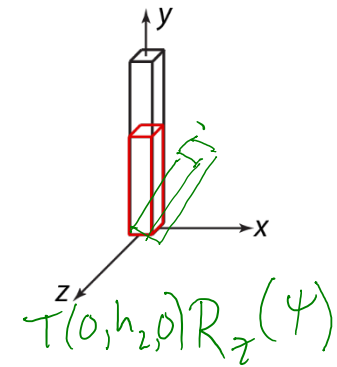
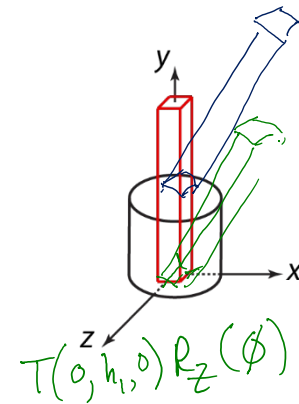
[Angel, 2011]

(Note that the angles are set to zero in the figures on the right; i.e., the parts are shown in their "default" positions.)

Suppose we have transformations $R_x(\square)$, $R_y(\square)$, $R_z(\square)$, $T(\square, \square, \square)$, $\square, \square, \square$.

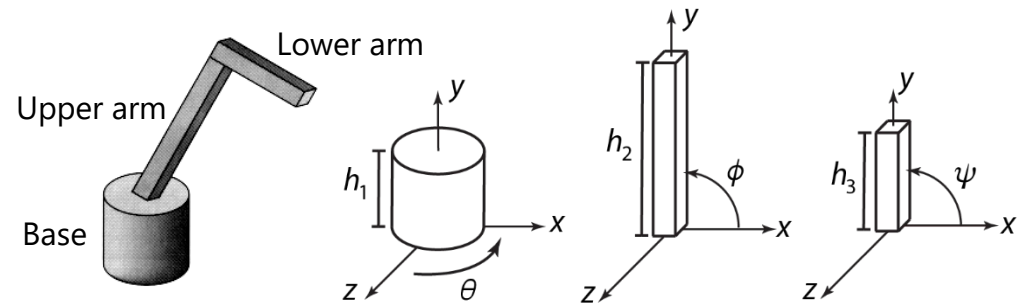
Q: What matrix do we use to transform the base?

Q: What matrix product for the upper arm?

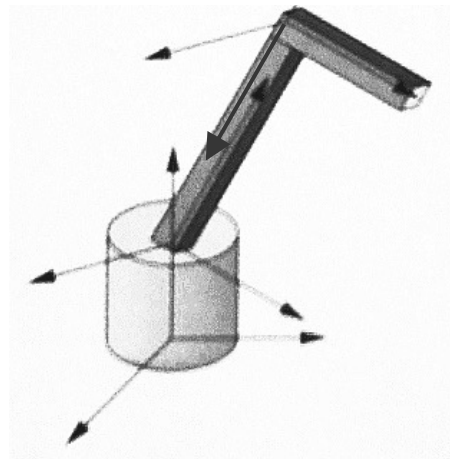


3D Example: A robot arm

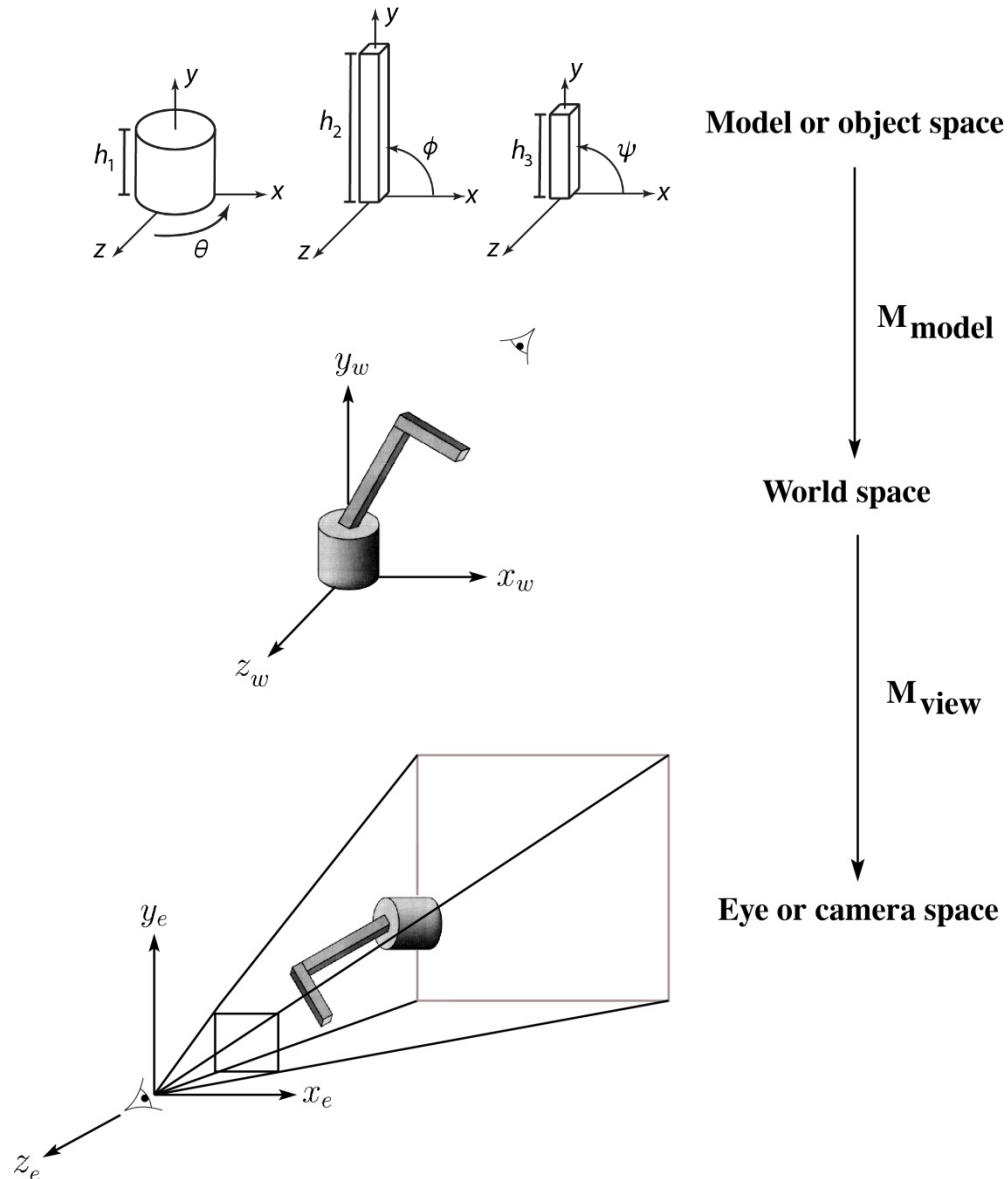
An alternative interpretation is that we are taking the original coordinate frames...



...and translating and rotating them into place:



From parts to model to viewer



Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M, M_model, M_view;
```

```
main()
```

```
{  
    . . .  
    M_view = compute_view_transform();  
    robot_arm();  
    . . .  
}
```

```
robot_arm()
```

```
{  
    M_model = R_y(theta);  
    M = M_view*M_model;  
    base();  
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);  
    M = M_view*M_model;  
    upper_arm();  
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)*T(0,h2,0)*R_z(psi);  
    M = M_view*M_model;  
    lower_arm();  
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place* by concatenating matrices on the right:

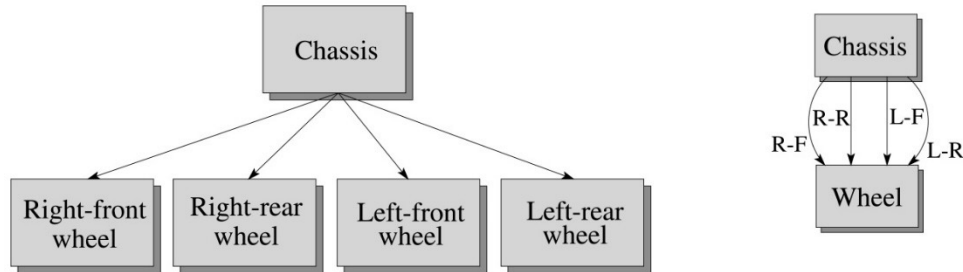
```
Matrix M_modelview;

main()
{
    . . .
    M_modelview = compute_view_transform();
    robot_arm();
    . . .
}

robot_arm()
{
    M_modelview *= R_y(theta);
    base();
    M_modelview *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_modelview *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```


Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:



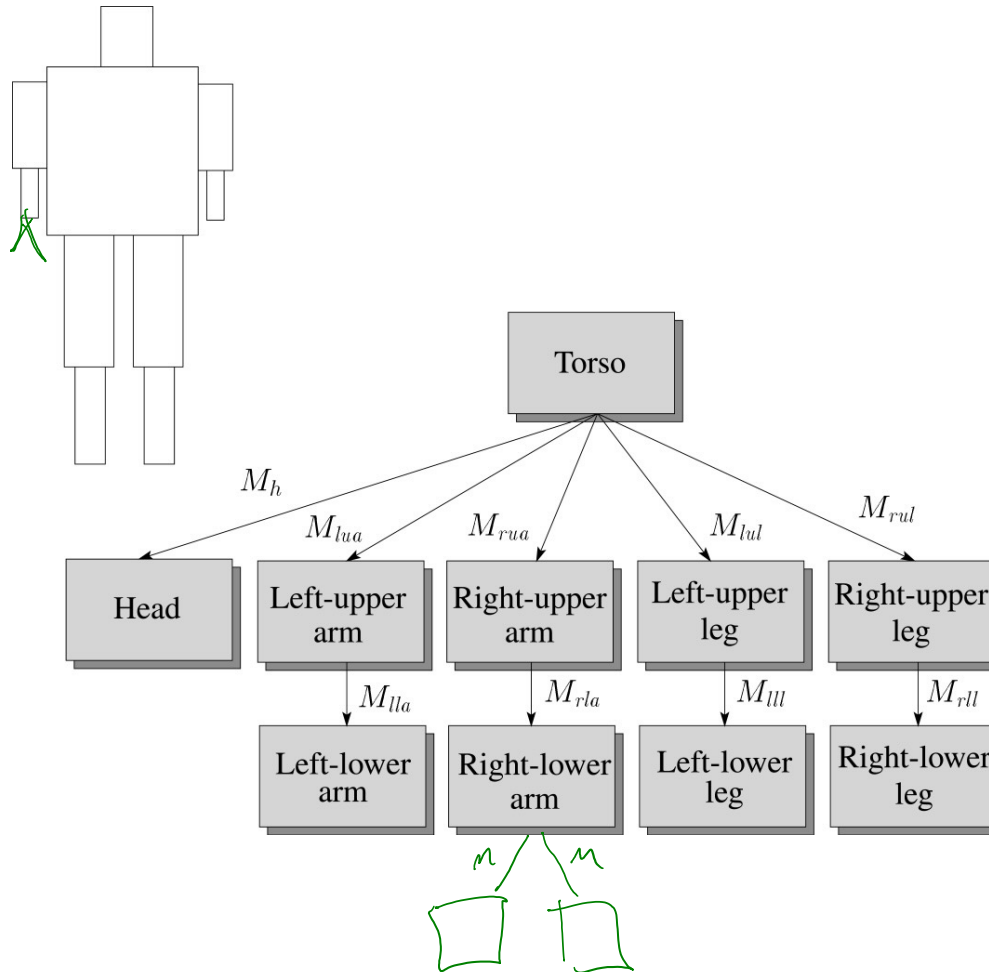
- ◆ edges contain geometric transformations
- ◆ nodes contain geometry (and possibly drawing attributes)

We will use trees for hierarchical models.

How might we draw the tree for the robot arm?



A complex example: human figure



Q: What's the most sensible way to traverse this tree?

depth first traversal

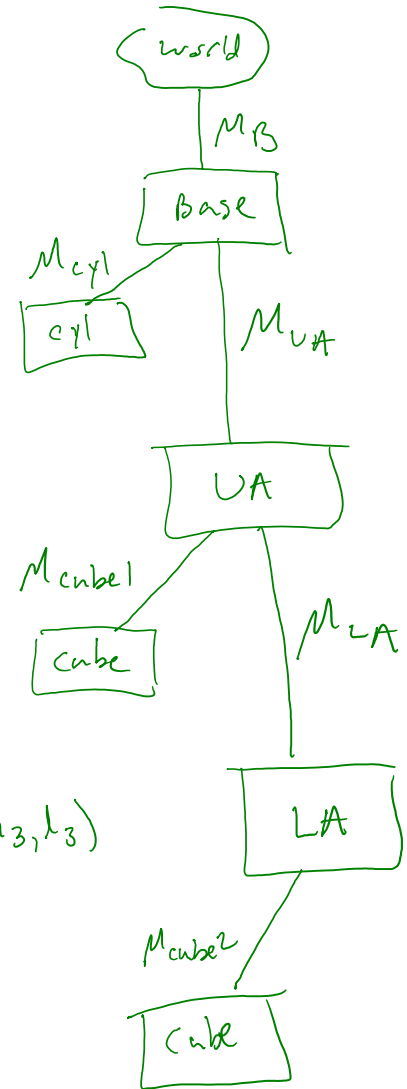
Stack

Using canonical primitives

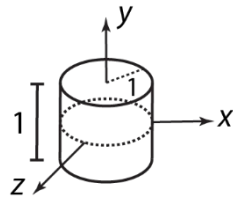
Consider building the robot arm again, but this time the building blocks are canonical primitives like a unit cylinder and a unit cube.

What additional transformations are needed?
 What does the hierarchy look like now?

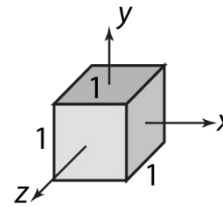
$$S(-, -, -)$$



Canonical primitives



Unit cylinder



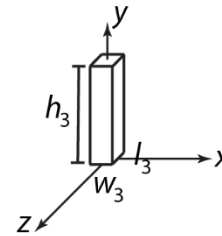
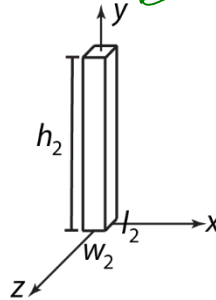
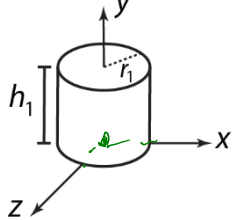
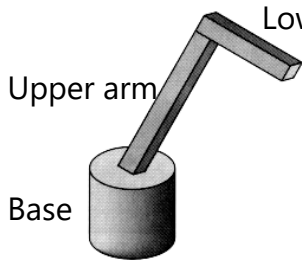
Unit cube

$$M_{cyl} = T(0, \frac{h}{2}, 0) S(r, h, r)$$

$$S(r_1, h_1, r_1)$$

$$M_{cube1} = T(0, \frac{h_2}{2}, 0) S(w_2, h_2, l_2)$$

$$M_{cube2} = T(0, \frac{h_3}{2}, 0) S(w_3, h_3, l_3)$$



Animation

The above examples are called **articulated models**:

- ◆ rigid parts
- ◆ connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.

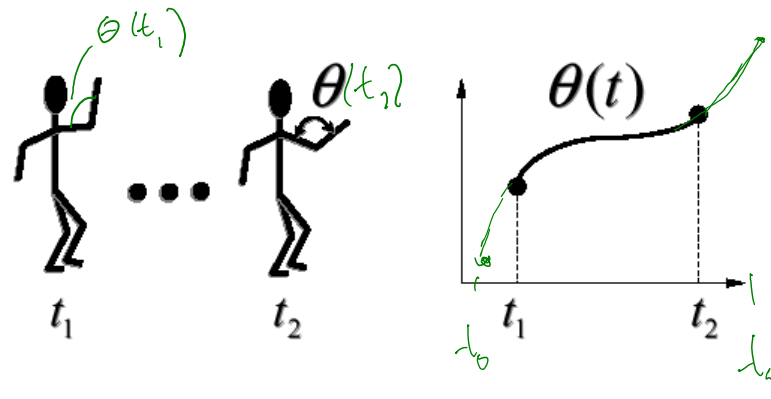
Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- ◆ Each joint specified at various **key frames** (not necessarily the same as other joints)
- ◆ System does interpolation or **in-betweening**

Doing this well requires:

- ◆ A way of smoothly interpolating key frames: **splines**
- ◆ A good interactive system
- ◆ A lot of skill on the part of the animator

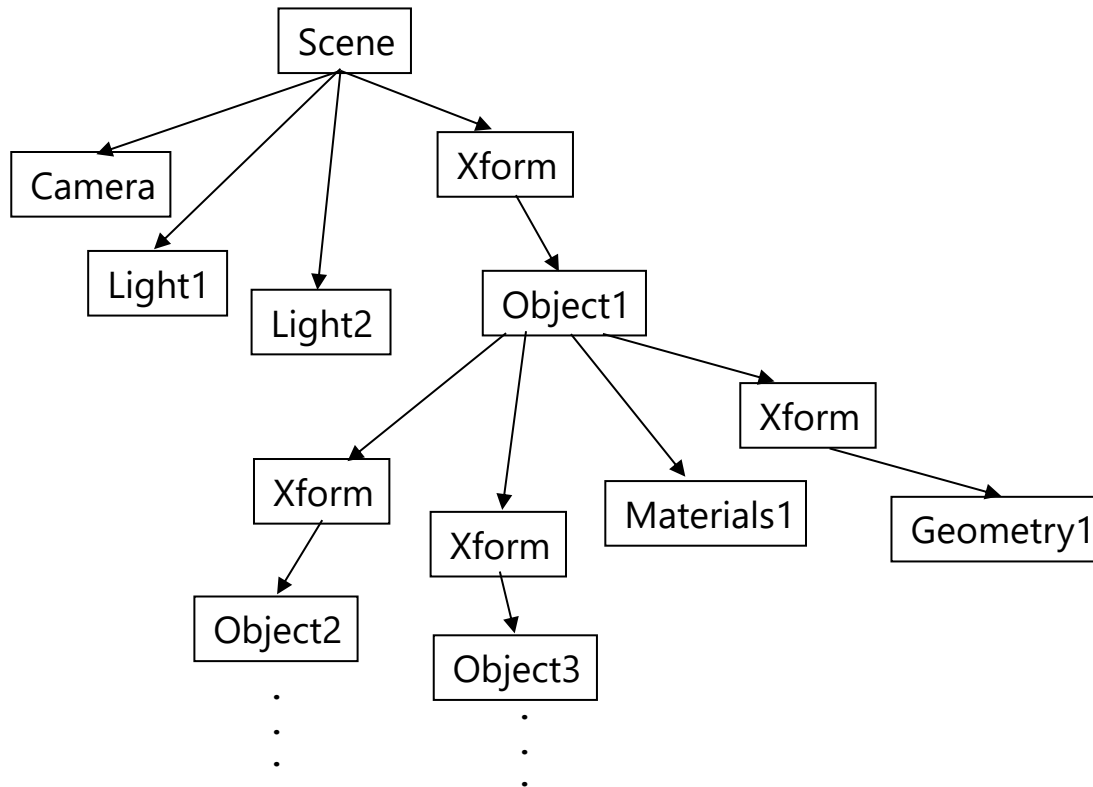


Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- ◆ many different objects
- ◆ lights
- ◆ camera position

This is called a **scene tree** or **scene graph**.



Summary

Here's what you should take home from this lecture:

- ◆ All the **boldfaced terms**.
- ◆ How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- ◆ How the notion of a model tree or DAG can be extended to entire scenes.
- ◆ How OpenGL transformations can be used in hierarchical modeling.
- ◆ How keyframe animation works.