Surfaces of Revolution

Brian Curless
CSEP 557
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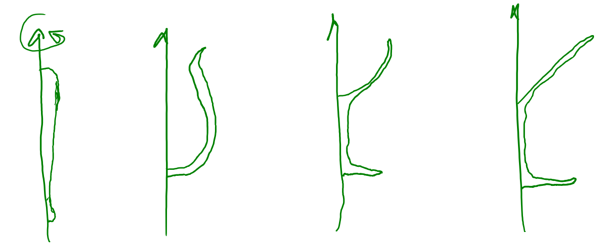
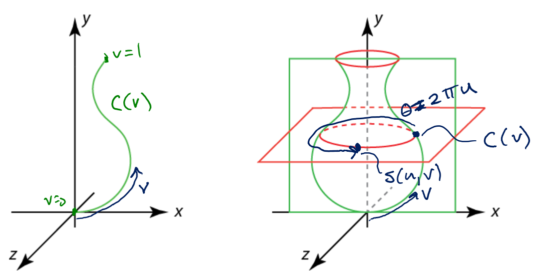
Surfaces of revolution



Idea: rotate a 2D **profile curve** around an axis.

What kinds of shapes can you model this way?

Constructing surfaces of revolution



Given: A curve $C(v)$ in the xy-plane:

$$C(v) = \begin{bmatrix} C_x(v) \\ C_y(v) \\ 0 \\ 1 \end{bmatrix}$$

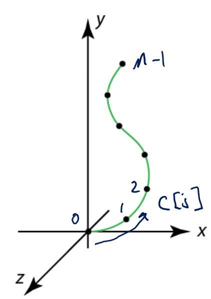
Let $R_y(\theta)$ be a rotation about the y-axis.

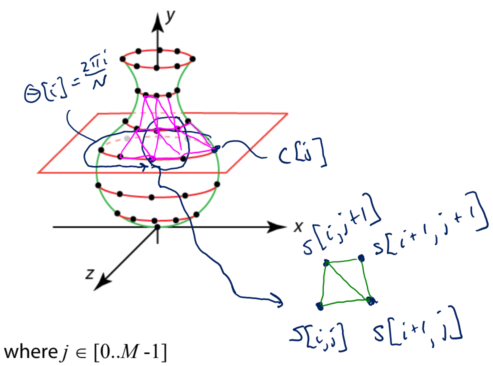
Find: A surface $S(u,v)$ which is $C(v)$ rotated about the y-axis, where $u,v \in [0, 1]$.

Solution: $S(u,v) = R_y(2\pi u)\, C(v)$

Constructing surfaces of revolution

We can sample in u and v to get a grid of points over the surface.





Suppose we sample:

- in v, to give $C[j]$ where $j \in [0..M-1]$
- in u, to give rotation angle $\theta[i] = 2\pi i / N$ where $i \in [0..N]$

We can now write the surface as:

$$S[i,j] = R_y\left(\frac{2\pi i}{N}\right) C[j]$$

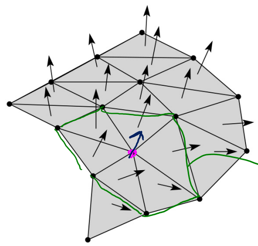
How would we turn this into a mesh of triangles?
How do we assign per-vertex normals?

Surface normals

Now that we describe the surface as a triangle mesh, we need to provide surface normals. As we'll see later, these normals are important for drawing and shading the surface (i.e., for "rendering").

One approach is to compute the normal to each triangle. How do we compute these normals?

normal ≈ average of normals of Δ's in 1-ring neighborhood



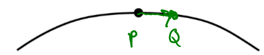
1-ring neighborhood (of the given center vertex)

# of vertices in 1-ring neighborhood = valence

Per-face normal lead to faceted appearance. We can get better-looking results with per-vertex normal (I'll explain why in the "shading" lecture).

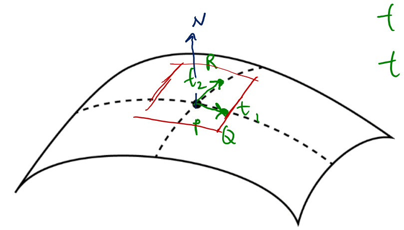
How might we compute per-vertex normals?

Tangent vectors, tangent planes, and normals



$t \approx Q - P$

$$t = \lim_{Q \to P} \frac{Q-P}{\|Q-P\|}$$

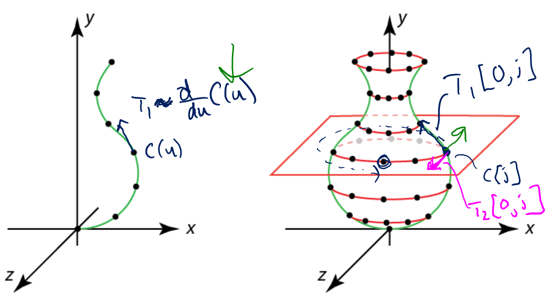


$t_1 \approx Q - P$

$t_2 \approx R - P$

$$N = \frac{t_1 \times t_2}{\|t_1 \times t_2\|}$$

Normals on a surface of revolution



We can compute tangents to the curve points in the xy-plane:

$\mathbf{T}_1[0,j] \approx C[j+1] - C[j]$

$\mathbf{T}_2[0,j] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

to get the normal in that plane:

$\hat{\mathbf{N}}[0,j] = T_1[0,j] \times T_2[0,j] \, / \, \| T_1[\dots] \times T_2[\dots] \|$

and then rotate it around:

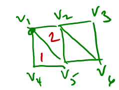
$$\hat{N}[i,j] = R\left(\frac{2\pi i}{N}\right) \hat{N}[0,j]$$

Triangle meshes

How should we generally represent triangle meshes?

~~1 $V_1, \hat{N}_1, \dots$~~
~~$V_4, \hat{N}_4, \dots$~~
~~$V_5, \hat{N}_5, \dots$~~

~~2 $V_1, \hat{N}_1, \dots$~~
~~$V_5, \hat{N}_5, \dots$~~
~~$V_2, \hat{N}_2, \dots$~~



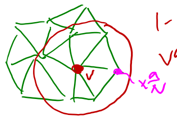
Vertex list

$V_1, \hat{N}_1, \dots$
$V_2, \hat{N}_2, \dots$
$V_3, \dots$
$V_4, \dots$
$V_5, \dots$
$V_6, \dots$
⋮

Triangle list

1: 1, 4, 5
2: 1, 5, 2

Mesh filtering



1-ring neighborhood
valence = N

$$v' \leftarrow \frac{v + \frac{a}{N} v_1 + \frac{a}{N} v_2 + \cdots \frac{a}{N} v_N}{1+a}$$

then, recompute normals .. as average of 1-ring triangle normals (and normalize this)

Summary

What to take away from this lecture:

- All the names in boldface.
- How to compute a surface of revolution given a profile curve.
- How to represent a surface of revolution as a triangle mesh.
- How to compute per-vertex normals for a surface of revolution.