Hierarchical Modeling

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Reading

Optional:
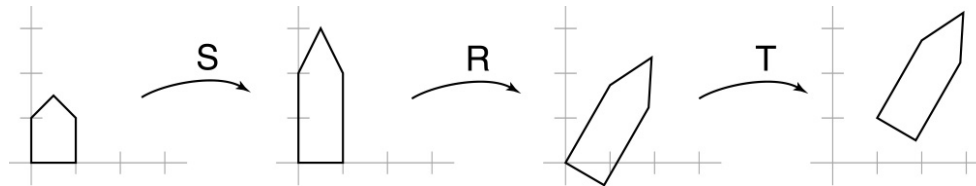
* Angel, sections 8.1 – 8.6, 8.8

Further reading:
* *OpenGL Programming Guide*, chapter 3

Symbols and instances

Most graphics APIs support a few geometric **primitives**:

- ◆ spheres
- ◆ cubes
- ◆ cylinders

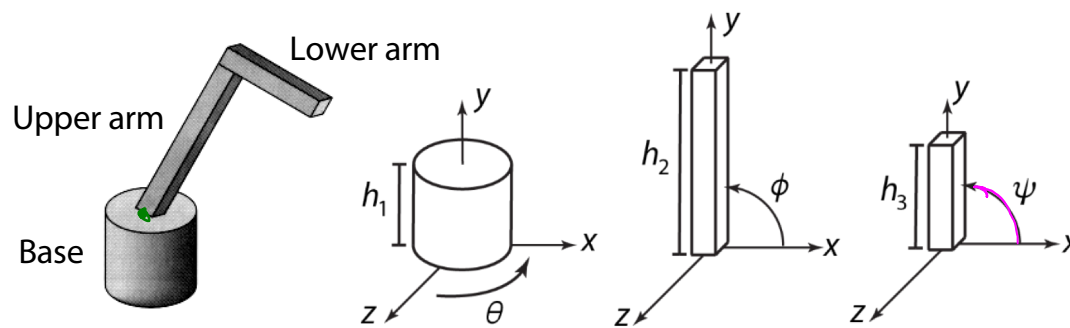These symbols are **instanced** using an **instance transformation**.

Q: What is the matrix for the instance transformation above?

$$M = TRS$$

3D Example: A robot arm

Let's build a robot arm out of a cylinder and two cuboids, with the following 3 degrees of freedom:

- ◆ Base rotates about its vertical axis by θ
- ◆ Upper arm rotates in its xy-plane by ϕ
- ◆ Lower arm rotates in its xy-plane by ψ

$$T(0, h_1, 0) R_z(\phi)$$

Upper arm

Lower arm

Base

[Angel, 2011]
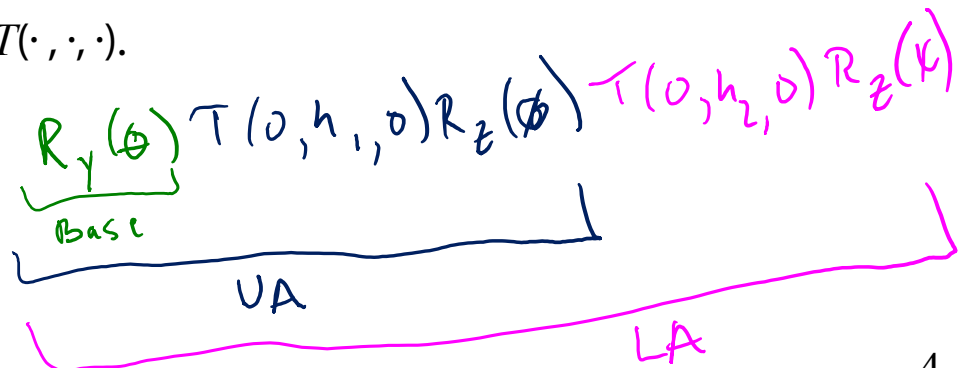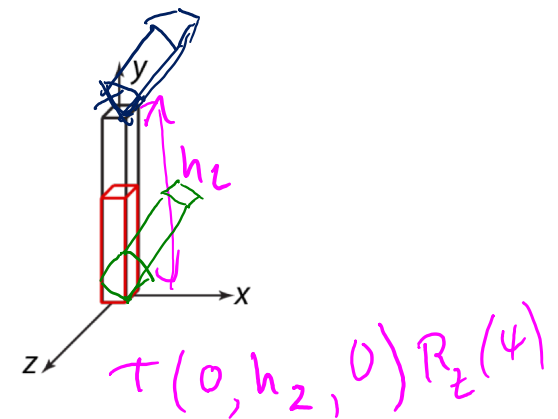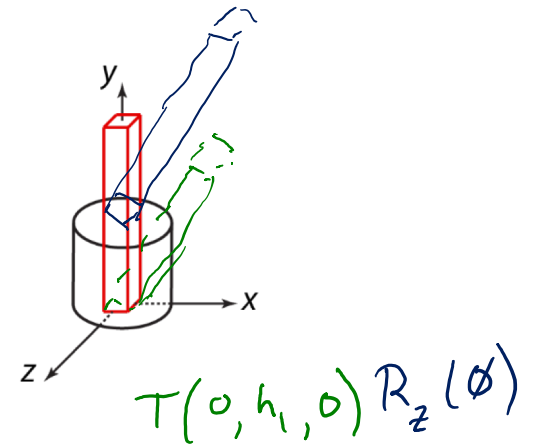
$$T(0, h_2, 0) R_z(\psi)$$

(Note that the angles are set to zero in the figures on the right; i.e., the parts are shown in their "default" positions.)

Suppose we have transformations $R_x(\cdot)$, $R_y(\cdot)$, $R_z(\cdot)$, $T(\cdot\,, \cdot\,, \cdot)$.

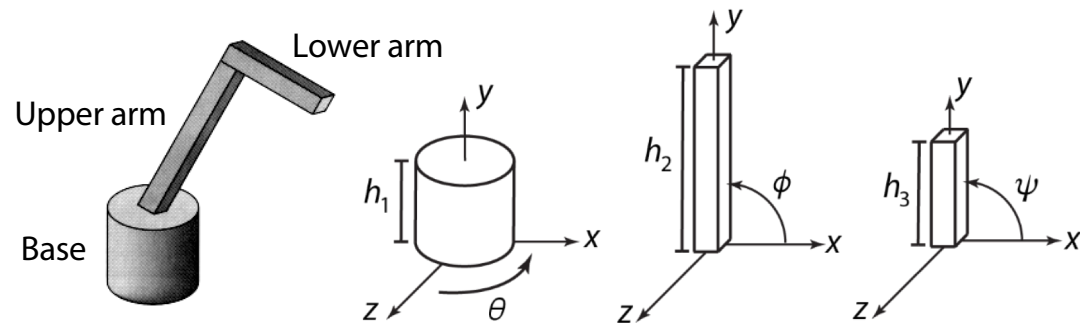Q: What matrix do we use to transform the base?

Q: What matrix product for the upper arm?

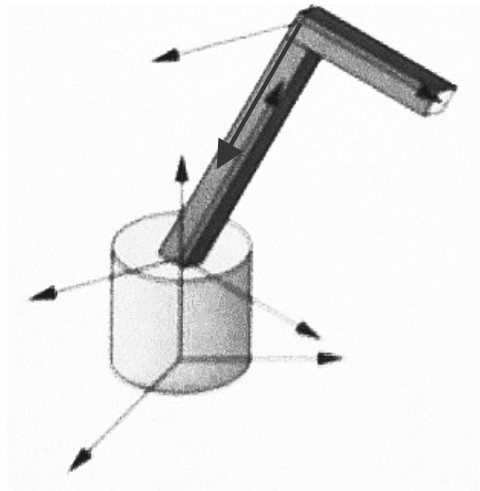Q: What matrix product for the lower arm?

$$\underbrace{R_y(\theta)}_{\text{Base}} \underbrace{T(0, h_1, 0) R_z(\phi)}_{\text{UA}} \underbrace{T(0, h_2, 0) R_z(\psi)}_{\text{LA}}$$
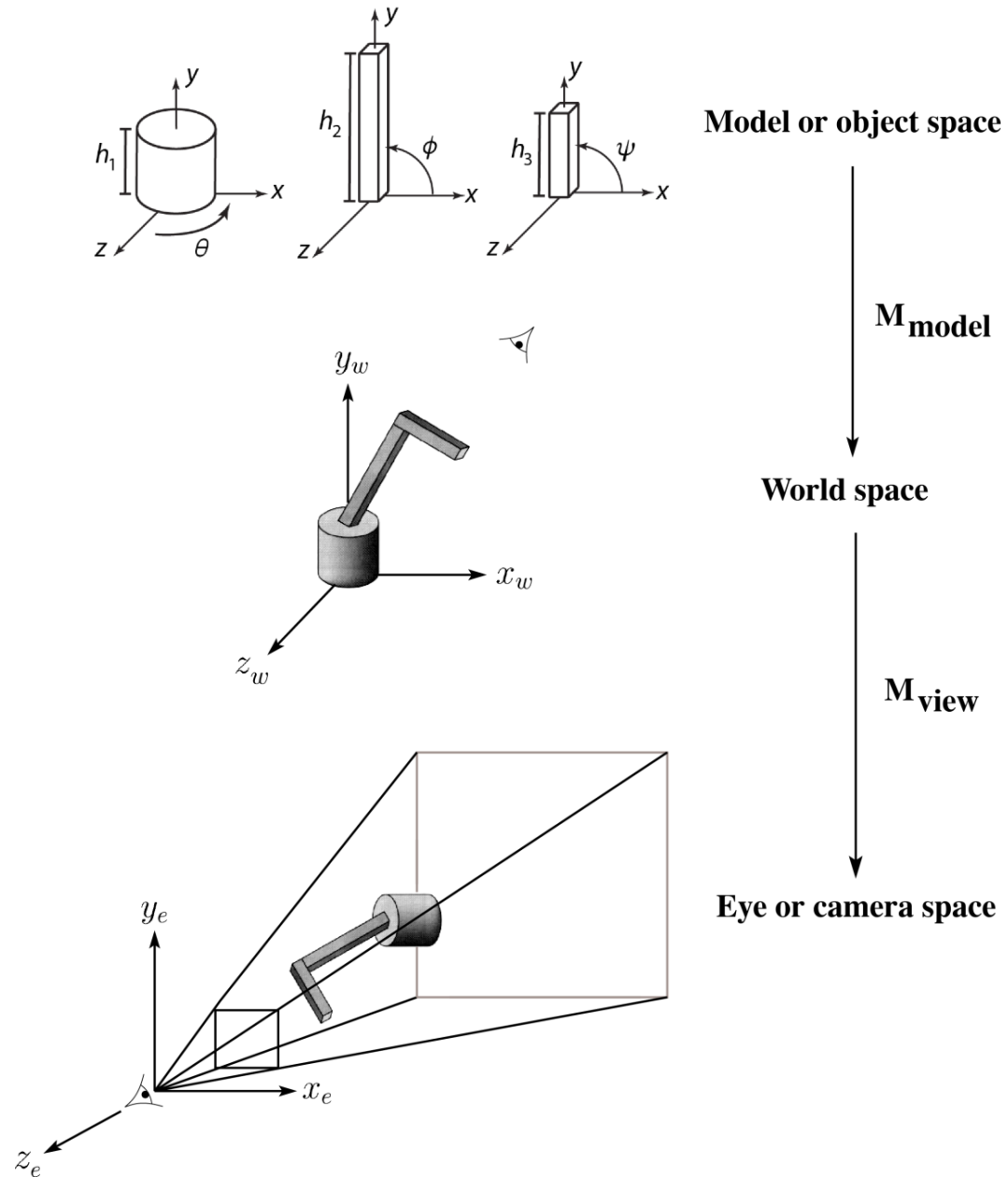
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3D Example: A robot arm

An alternative interpretation is that we are taking the original coordinate frames…

…and translating and rotating them into place:

From parts to model to viewer

Model or object space

$\mathbf{M_{model}}$

World space

$\mathbf{M_{view}}$

Eye or camera space

Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M, M_model, M_view;


main()
{
    . . .
    M_view = compute_view_transform();
    robot_arm();
    . . .
}


robot_arm()
{
    M_model = R_y(theta);
    M = M_view*M_model;
    base();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);
    M = M_view*M_model;
    upper_arm();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)*T(0,h2,0)*R_z(psi);
    M = M_view*M_model;
    lower_arm();
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place* by concatenating matrices on the right:
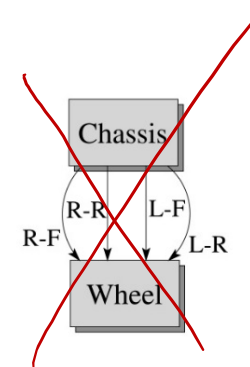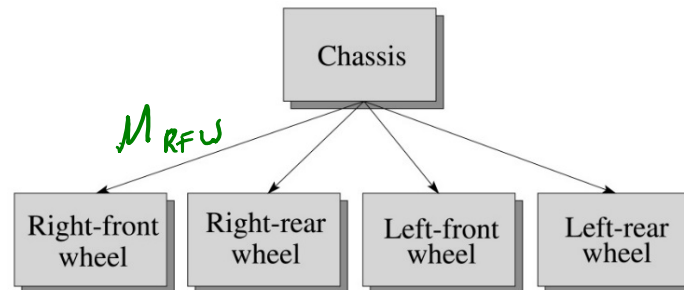
```
Matrix M_modelview;


main()
{
    . . .

    M_modelview = compute_view_transform();

    robot_arm();

    . . .

}


robot_arm()
{
    M_modelview *= R_y(theta);

    base();

    M_modelview *= T(0,h1,0)*R_z(phi);

    upper_arm();

    M_modelview *= T(0,h2,0)*R_z(psi);

    lower_arm();

}
```
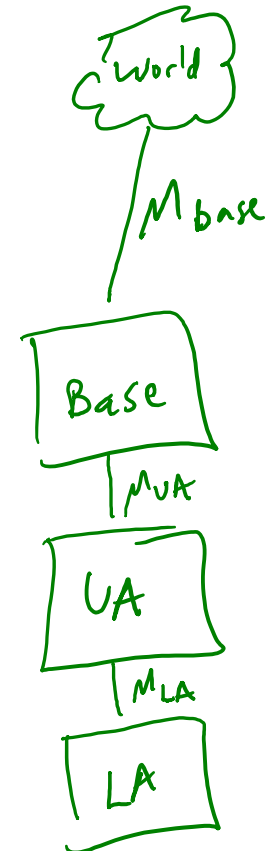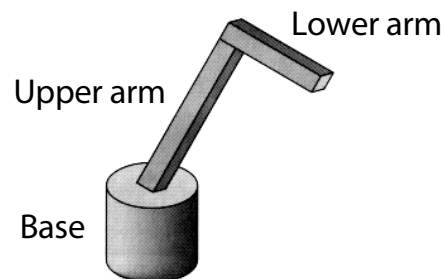
Hierarchical modeling

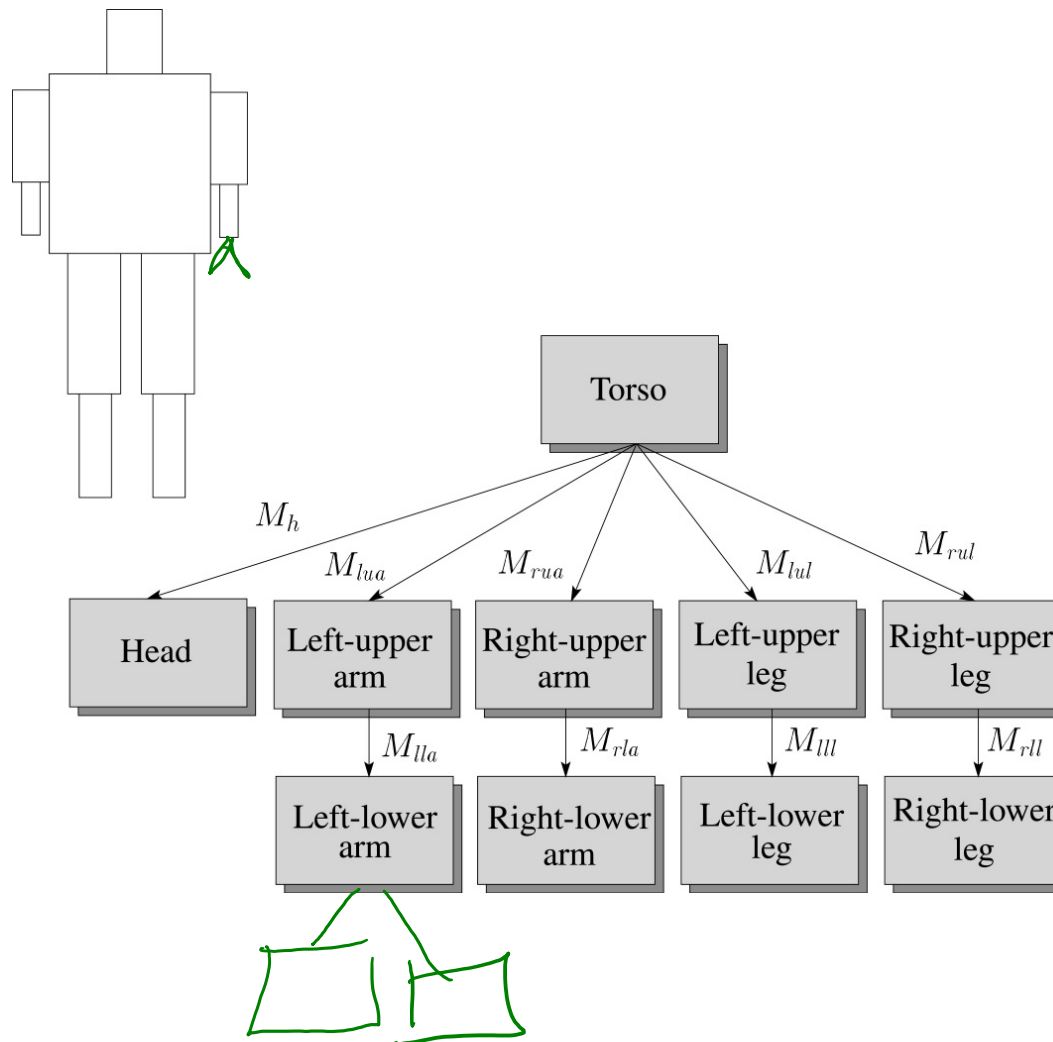Hierarchical models can be composed of instances using trees or DAGs:

- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

We will use trees for hierarchical models.

How might we draw the tree for the robot arm?

A complex example: human figure

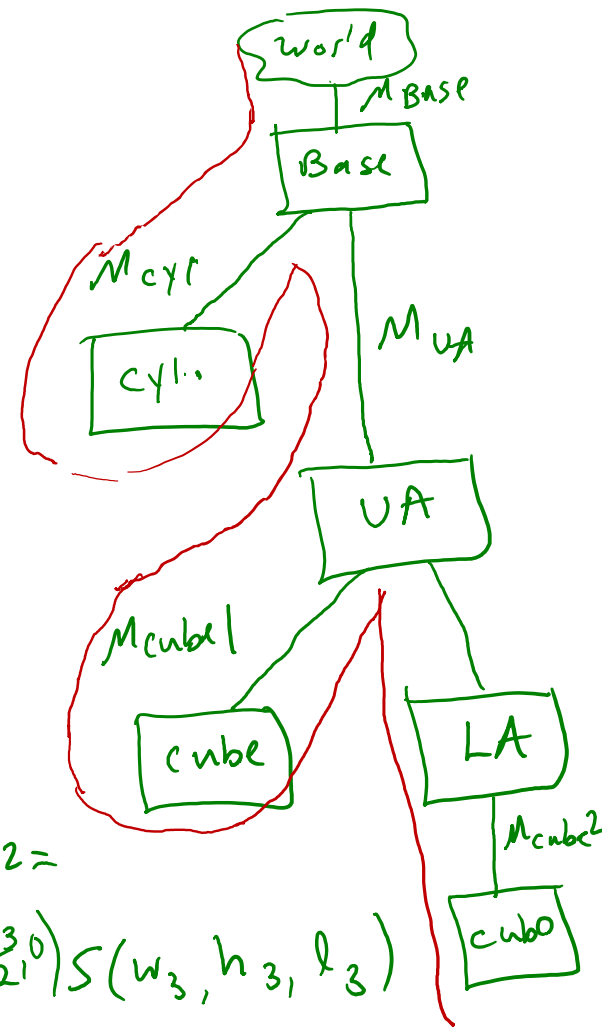**Q:** What's the most sensible way to traverse this tree?

depth first w/ slack
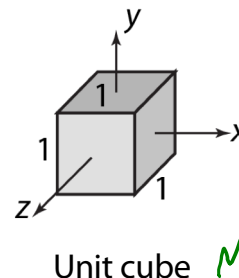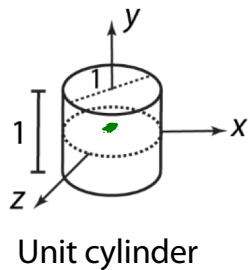
Using canonical primitives

Consider building the robot arm again, but this time the building blocks are canonical primitives like a unit cylinder and a unit cube. We can use transformations like $T(t_x, t_y, t_z)$, $S(s_x, s_y, s_z)$, $R_y(\theta)$, etc.

What additional transformations are needed?
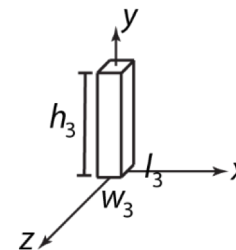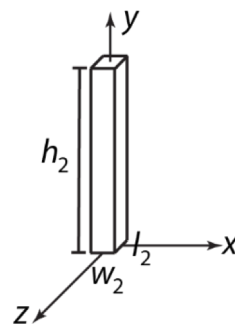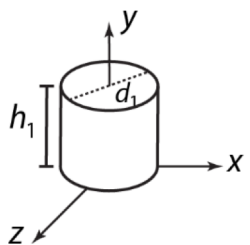
What does the hierarchy look like now?

Canonical primitives

Unit cylinder

$M_{cyl} = T(0, \frac{h_1}{2}, 0) \, S(d_1, h_1, d_1)$

M_{cube1}

$T(0, \frac{h_2}{2}, 0) \, S(w_2, h_2, l_2)$

Unit cube

$M_{cube2} = T(0, \frac{h_3}{2}, 0) \, S(w_3, h_3, l_3)$

Upper arm

Lower arm

Base

world

M_{Base}

Base

M_{cyl}

cyl.

M_{UA}

UA

M_{cube1}

cube

LA

M_{cube2}

cube

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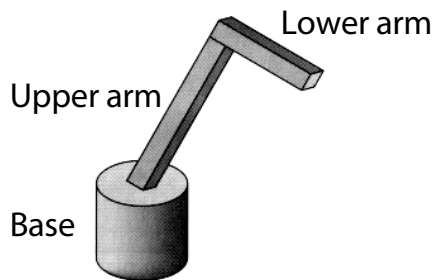
Animation

The above examples are called **articulated models**:

- ◆ rigid parts
- ◆ connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.

Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- Each joint specified at various **key frames** (not necessarily the same as other joints)
- System does interpolation or **in-betweening**

Doing this well requires:

- A way of smoothly interpolating key frames: **splines**
- A good interactive system
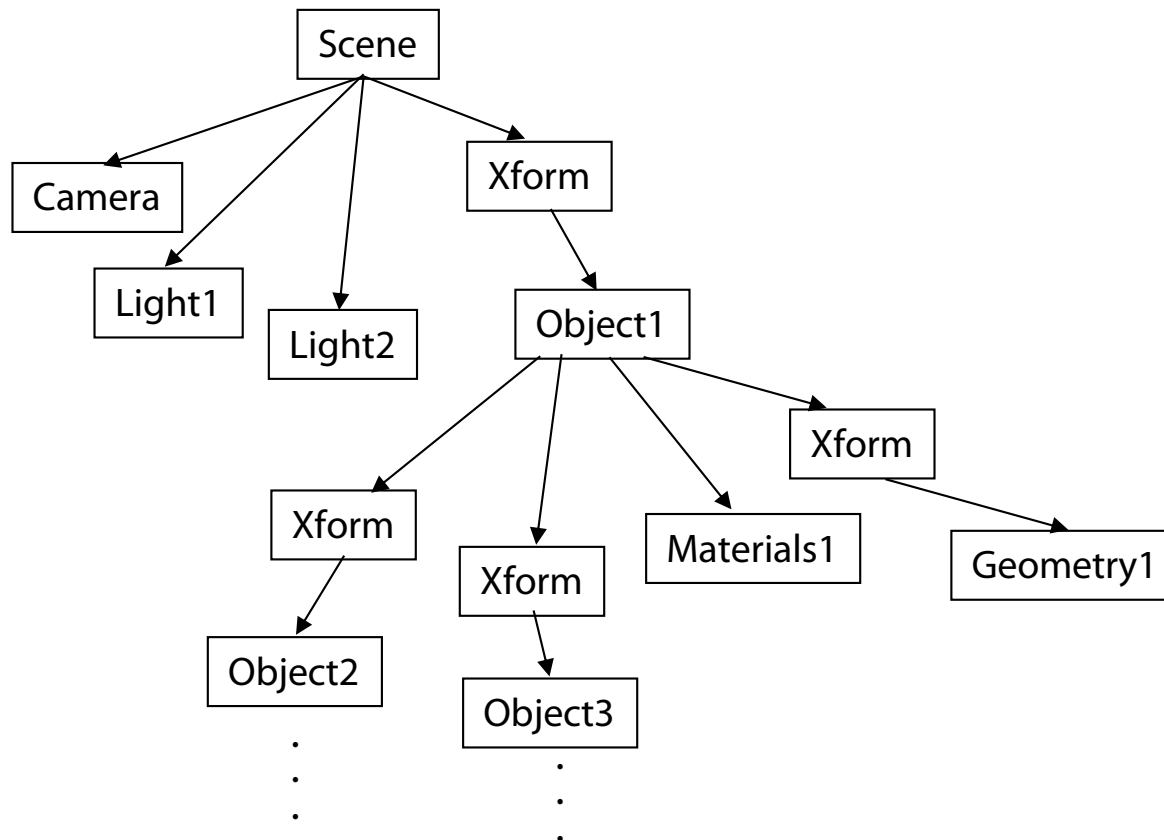- A lot of skill on the part of the animator
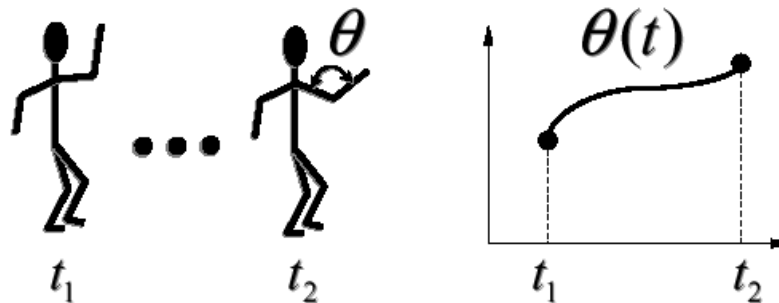
Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- ◆ many different objects
- ◆ lights
- ◆ camera position

This is called a **scene tree** or **scene graph**.

Summary

Here's what you should take home from this lecture:

- All the **boldfaced terms**.
- How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- How the notion of a model tree or DAG can be extended to entire scenes.
- How OpenGL transformations can be used in hierarchical modeling.
- How keyframe animation works.