

Subdivision curves and surfaces

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Reading

Recommended:

- ♦ Stollnitz, DeRose, and Salesin. *Wavelets for Computer Graphics: Theory and Applications*, 1996, section 6.1-6.3, 10.2, A.5.

Note: there is an error in Stollnitz, et al., section A.5. Equation A.3 should read:

$$MV = V\Lambda$$

This is already fixed in the handout.

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Subdivision curves

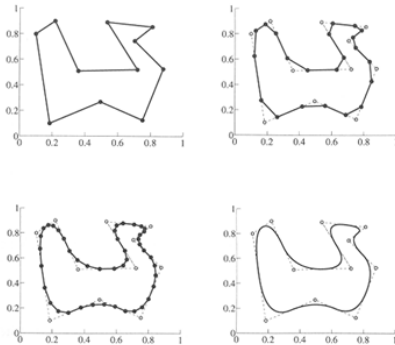
Idea:

- ♦ repeatedly refine the control polygon

$$p^1 \rightarrow p^2 \rightarrow p^3 \rightarrow \dots$$

- ♦ curve is the limit of an infinite process

$$Q = \lim_{j \rightarrow \infty} p^j$$

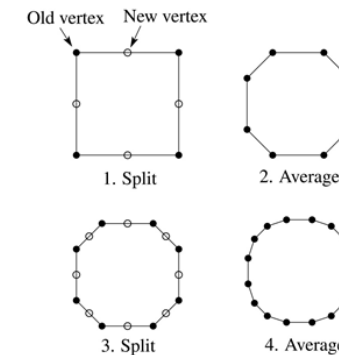


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Chaikin's algorithm

Chakin introduced the following "corner-cutting" scheme in 1974:

- ♦ Start with a piecewise linear curve
- ♦ Insert new vertices at the midpoints (the **splitting step**)
- ♦ Average each vertex with the "next" (clockwise) neighbor (the **averaging step**)
- ♦ Go to the splitting step



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Averaging masks

The limit curve is a quadratic B-spline!

Instead of averaging with the nearest neighbor, we can generalize by applying an **averaging mask** during the averaging step:

$$r = (\dots, r_{-1}, r_0, r_1, \dots)$$

In the case of Chaikin's algorithm:

$$r = \left(0, \frac{1}{2}, \frac{1}{2}\right)$$

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Lane-Riesenfeld algorithm (1980)

Use averaging masks from Pascal's triangle:

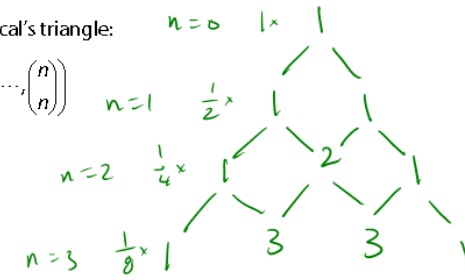
$$r = \frac{1}{2^n} \binom{n}{0} r_0 \binom{n}{1} r_1 \dots \binom{n}{n} r_n$$

Gives B-splines of degree $n+1$.

linear n=0: (1)

quadratic n=1: $\left(0, \frac{1}{2}, \frac{1}{2}\right)$

cubic n=2: $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$



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Subdivide ad nauseum?

After each split-average step, we are closer to the **limit curve**.

How many steps until we reach the final (limit) position?

∞

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Recipe for subdivision curves

Can we push a vertex to its limit position without infinite subdivision? Yes!

After subdividing and averaging a few times, we can push each vertex to its limit position by applying an **evaluation mask**.

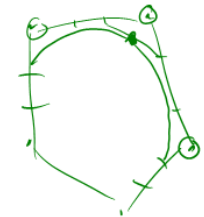
Each subdivision scheme has its own evaluation mask, mathematically determined by analyzing the subdivision and averaging rules.

For Lane-Riesenfeld cubic B-spline subdivision, we get:

$$\frac{1}{6} \begin{pmatrix} 1 & 4 & 1 \end{pmatrix}$$

Now we can cook up a simple procedure for creating subdivision curves:

- ◆ Subdivide (split+average) the control polygon a few times. Use the averaging mask.
- ◆ Push the resulting points to the limit positions. Use the evaluation mask.



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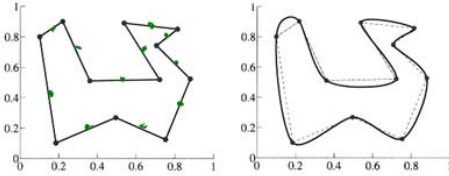
DLG interpolating scheme (1987)

Slight modification to subdivision algorithm:

- ♦ splitting step introduces midpoints
- ♦ averaging step *only changes midpoints*

For DLG (Dyn-Levin-Gregory), use:

$$r_{\text{old}} = (1) \quad r_{\text{new}} = \frac{1}{16}(-2, 5, 10, 5, -2)$$

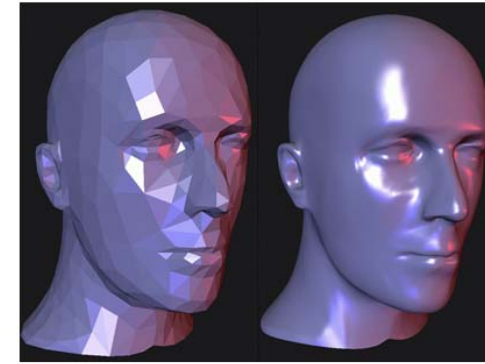


Since we are only changing the midpoints, the points after the averaging step do not move.

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Building complex models

We can extend the idea of subdivision from curves to surfaces...



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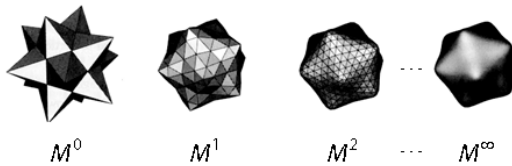
Subdivision surfaces

Chaikin's use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface

$$S = \lim_{j \rightarrow \infty} M^j$$

using splitting and averaging steps.

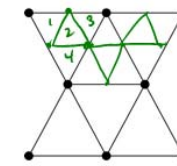


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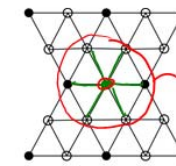
Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:



Original



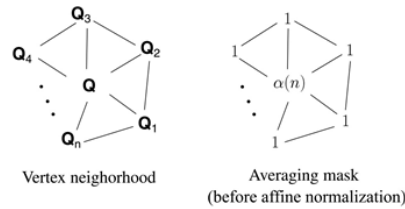
After splitting

1-ring neighborhood
valence = # of edges touching a vertex

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Loop averaging step

Once again we can use **masks** for the averaging step:



$$\mathbf{Q} \leftarrow \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \dots + \mathbf{Q}_n}{\alpha(n) + n}$$

where

$$\alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi/n))^2}{32}$$

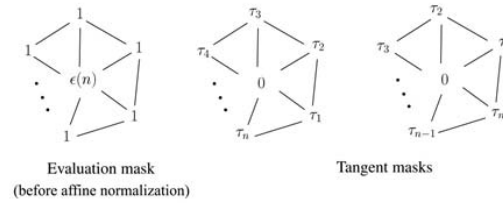
These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as G^1 continuity for surfaces.

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Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



$$\mathbf{Q}^\infty = \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \dots + \mathbf{Q}_n}{\alpha(n) + n}$$

$$\mathbf{T}_1^\infty = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \dots + \tau_n(n)\mathbf{Q}_n$$

$$\mathbf{T}_2^\infty = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \dots + \tau_{n-1}(n)\mathbf{Q}_n$$

where

$$\alpha(n) = \frac{3n}{\beta(n)} \quad \tau_i(n) = \cos(2\pi i/n)$$

How do we compute the normal?

$$\mathbf{N} \sim \mathbf{T}_1^\infty \times \mathbf{T}_2^\infty$$

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Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!

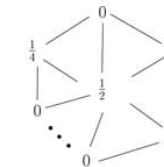
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Adding creases without trim curves

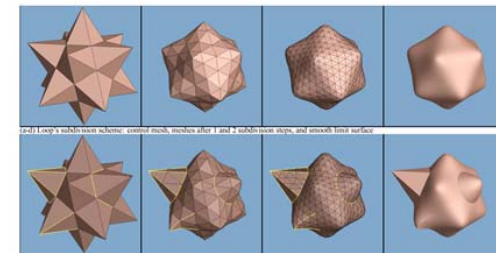
In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.



For subdivision surfaces, we can just modify the subdivision mask:



This gives rise to G^0 continuous surfaces (i.e., having positional but not tangent plane continuity)



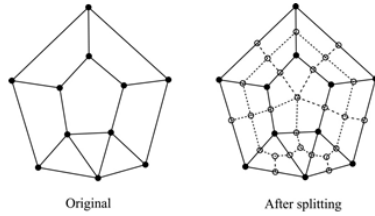
(c) © Loop's subdivision scheme: control mesh, maskes after 1 and 2 subdivision steps, and smooth limit surface
(d) © Our piecewise smooth subdivision scheme: tagged control mesh, maskes after 1 and 2 subdivision steps, and piecewise smooth limit surface

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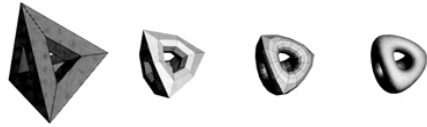
Catmull-Clark subdivision

4:1 subdivision of triangles is sometimes called a **face scheme** for subdivision, as each face begets more faces.

An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:



Catmull-Clark subdivision:



Note: after the first subdivision, all polygons are quadrilaterals in this scheme.

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Creases without trim curves, cont.

Here's an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):



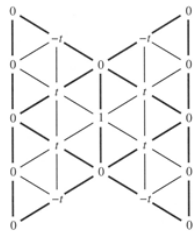
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Interpolating subdivision surfaces

Interpolating schemes are defined by

- ♦ splitting
- ♦ averaging only new vertices

The following averaging mask is used in **butterfly subdivision**:



Setting $t=0$ gives the original polyhedron, and increasing small values of t makes the surface smoother, until $t=1/8$ when the surface is provably G^1 .

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Summary

What to take home:

- ♦ The meanings of all the **boldfaced** terms.
- ♦ How to perform the splitting and averaging steps on subdivision curves.
- ♦ How to perform mesh splitting steps for subdivision surfaces, especially Loop.
- ♦ How to construct and render subdivision surfaces from their averaging masks, evaluation masks, and tangent masks.

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