Quiz #2 Study Guide

Curves and particle systems

Prepared by: Brian Curless and Steve Seitz

Quiz #2 will be on Monday, June 4. It will cover shading, ray tracing, texturing, curves, and particle systems. The quiz will last approx 30 minutes and will be closed book. This study guide is intended to give you practice thinking in depth about material that was not covered by homework #2.

Problem 1 A Bézier curve of degree n, which (for the purposes of this problem) we'll denote by $Q^n(u)$, can be defined in terms of the locations of its $n+1$ control points $\{V_0, \ldots, V_n\}$:

$$Q^n(u) = \sum_{i=0}^{n} V_i \binom{n}{i} u^i (1-u)^{n-i}$$

a) Use de Casteljau's algorithm to find the (approximate) position of the Bézier curves $Q^3(u)$ and $Q^4(u)$ defined by the two control polygons below at $u = 1/3$:

True or false:

b) Every Bézier curve $Q^1(u)$ is a line segment (assuming no repeated control points).

c) Every Bézier curve $Q^2(u)$ lies in a plane.

d) Moving one control point on a Bézier curve generally changes the whole curve.

Problem 2 More complex curves can be designed by piecing together different Bézier curves to make mathematical "splines." Two popular splines are the B-spline and the Catmull-Rom spline. If $\{B_0, B_1, B_2, B_3\}$ and $\{C_0, C_1, C_2, C_3\}$ are cubic B-spline and Catmull-Rom spline control points, respectively, then the corresponding Bézier control points $\{V_0, V_1, V_2, V_3\}$ can be constructed by the following identity:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix}\begin{bmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 6 & 0 & 0 \\ -1 & 6 & 1 & 0 \\ 0 & 1 & 6 & -1 \\ 0 & 0 & 6 & 0 \end{bmatrix}\begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

<u>True or false:</u>

a) B-splines and Catmull-Rom splines both have C^2 continuity.

b) Neither B-splines nor Catmull-Rom splines interpolate their control points.

c) B-splines and Catmull-Rom splines both provide local control.

Problem 2 (continued)

d) The points $\{B_0, B_1, B_2, B_3\}$ below are control points for a cubic B-spline. Construct, as carefully as you can on the diagram below, the Bézier control points $\{V_0, V_1, V_2, V_3\}$ corresponding to the same curve. Do not extend the curve to the endpoints.

e) The points $\{C_0, C_1, C_2, C_3\}$ below are control points for a cubic Catmull-Rom spline. Construct, as carefully as you can on the diagram below, the Bézier control points $\{V_0, V_1, V_2, V_3\}$ corresponding to the same curve. Do not extend the curve to the endpoints.

Problem 3

In class we described a force update model that can be used to simulate a damped spring. Now suppose that you wanted to simulate the "point-circle" spring, shown below, which applies forces to pull a point $\mathbf{p}$ onto a circle with center $\mathbf{c}$ and radius r. At each point in time, the force on the particle should pull it toward the closest point on the circle. Similarly, the circle should be pulled towards $\mathbf{p}$. For the following questions, you may want to use the spring model as a reference.

a) Give the equation for the force $\mathbf{f_p}$ applied to $\mathbf{p}$. Ignore damping for now.

b) Give the equation for the force $\mathbf{f_c}$ applied to $\mathbf{c}$. Ignore damping for now.

c) Specify a force damping term for the point $\mathbf{p}$ that acts to slow down the velocity of $\mathbf{p}$ towards (or away from) the circle, but does not affect $\mathbf{p}$'s velocity in other directions.

d) How is the point-circle spring different, if at all, from the standard point-point spring presented in class? Explain the reason for the *similarity* between point-point and point-circle springs.