Paxos
The Part-Time Parliament

- Parliament determines laws by passing sequence of numbered decrees
- Legislators can leave and enter the chamber at arbitrary times
- No centralized record of approved decrees—instead, each legislator carries a ledger
Government 101

- No two ledgers contain contradictory information
- If a majority of legislators were in the Chamber and no one entered or left the Chamber for a sufficiently long time, then
  - any decree proposed by a legislator would eventually be passed
  - any passed decree would appear on the ledger of every legislator
Paxos legislature is non-partisan, progressive, and well-intentioned

Legislators only care that something is agreed to, not what is agreed to

We’ll take care of Byzantine legislators later
Back to the future

- A set of processes that can propose values
- Processes can crash and recover
- Processes have access to stable storage
- Asynchronous communication via messages
- Messages can be lost and duplicated, but not corrupted
The Players

- Proposers
- Acceptors
- Learners
Timeline
The Game: Consensus

SAFETY

- Only a value that has been proposed can be chosen
- Only a single value is chosen
- A process never learns that a value has been chosen unless it has been

LIVENESS

- Some proposed value is eventually chosen
- If a value is chosen, a process eventually learns it
Choosing a value

Use a single acceptor
What if the acceptor fails?

Choose only when a “large enough” set of acceptors accepts

Using a majority set guarantees that at most one value is chosen

6 is chosen!
Accepting a value

- Suppose only one value is proposed by a single proposer.
- That value should be chosen!
- First requirement:

  P1: An acceptor must accept the first proposal that it receives

- ...but what if we have multiple proposers, each proposing a different value?
P1 + multiple proposers

No value is chosen!
Handling multiple proposals

- Acceptors must accept more than one proposal
- To keep track of different proposals, assign a natural number to each proposal
  - A proposal is then a pair \((\text{psn}, \text{value})\)
  - Different proposals have different \(\text{psn}\)
- A proposal is chosen when it has been accepted by a majority of acceptors
- A value is chosen when a single proposal with that value has been chosen
Choosing a unique value

P2. If a proposal with value $\nu$ is chosen, then every higher-numbered proposal that is chosen has value $\nu$
It’s up to the Acceptors!

**P2.** If a proposal with value $\nu$ is chosen, then every higher-numbered proposal that is chosen has value $\nu$

We strengthen it to:

**P2a.** If a proposal with value $\nu$ is chosen, then every higher-numbered proposal accepted by any acceptor has value $\nu$
What about P1?

Do we still need P1?

YES, to ensure that some proposal is accepted

How well do P1 and P2a play together?

Asynchrony is a problem...

6 is chosen!

How does $a_1$ know it should not accept?

5

(2,7)

$\begin{array}{c}
5 \\
7 \\
6 \\
2 \\
\end{array}$

(1,6)

$\begin{array}{c}
6 \\
\end{array}$

(1,6)

$\begin{array}{c}
2 \\
\end{array}$
It’s up to the Proposers!

Recall P2a:

**P2a.** If a proposal with value $\nu$ is chosen, then every higher-numbered proposal accepted by any acceptor has value $\nu$

We strengthen it to:

**P2b.** If a proposal with value $\nu$ is chosen, then every higher-numbered proposal issued by any proposer has value $\nu$
What to propose

**P2b:** If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

Suppose $p$ wants to issue a proposal numbered $n$.

- If $p$ can be certain that no proposal numbered $n' < n$ has been chosen then $p$ can propose any value!
  - If a proposal numbered $n' < n$ has been chosen, then it has been accepted by a majority set $S$
  - Any majority set $S'$ must intersect $S$
  - If $p$ can find one $S'$ in which no acceptors has accepted a proposal numbered $n' < n$, then no such proposal can have yet been chosen!
  - If no such $S'$, a proposal numbered $n' < n$ may have been chosen...
  - *Then what?*
What to propose

**P2b:** If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

Suppose $p$ wants to issue a proposal numbered $n$.

- If $p$ can be certain that no proposal numbered $n' < n$ has been chosen then $p$ can propose any value!

- If not, $p$ should propose the chosen value. But how?
  - What about using induction...
  - Say proposal numbered $m$ with value $v$ is chosen: some majority-set $C$ of acceptors has accepted it
  - Suppose every proposal issued with number $m...n-1$ has value $v$
  - *Consider proposal $n$: since* every majority set $S'$ intersects with $C$ and every proposal accepted by any acceptor with sequence number in $m...n-1$ has value $v$, *then*
  - $p$ should propose the highest numbered proposal among all proposals, numbered less than $n$, accepted by some majority set $S$
Example
It’s up to an invariant!

P2b: If a proposal with value $v$ is chosen, then every higher-numbered proposal issued by any proposer has value $v$

Achieved by enforcing the following invariant

P2c: For any $v$ and $n$, if a proposal with value $v$ and number $n$ is issued, then there is a set $S$ consisting of a majority of acceptors such that either:

- no acceptor in $S$ has accepted any proposal numbered less than $n$, or
- $v$ is the value of the highest-numbered proposal among all proposals numbered less than $n$ accepted by the acceptors in $S$
$v$ is the value of the highest-numbered proposal among all proposals numbered less than $n$ and accepted by the acceptors in $S$. 

- $a_1$ with proposal $(4,8)$ 
- $a_2$ with proposal $(3,2)$ 
- $a_3$ with proposal $(5,2)$
P2c in action

\( \nu \) is the value of the highest-numbered proposal among all proposals numbered less than \( n \) and accepted by the acceptors in \( S \)

The invariant is violated
Future telling?

- $p$ must learn the highest-numbered proposal with number less than $n$, if any, that has been or will be accepted by each acceptor in some majority of acceptors.

- Avoid predicting the future by extracting a promise from a majority of acceptors not to subsequently accept any proposals numbered less than $n$. 
The proposer’s protocol (I)

- A proposer chooses a new proposal number \( n \) and sends a request to each member of some set of acceptors, asking it to respond with:
  
  a. A promise never again to accept a proposal numbered less than \( n \), and
  
  b. The accepted proposal with highest number less than \( n \) if any.

...call this a \textit{prepare request} with number \( n \)
The proposer’s protocol (II)

- If the proposer receives a response from a majority of acceptors, then it can issue a proposal with number $n$ and value $v$, where $v$ is
  - the value of the highest-numbered proposal among the responses, or
  - is any value selected by the proposer if responders returned no proposals

A proposer issues a proposal by sending, to some set of acceptors, a request that the proposal be accepted.
...call this an accept request.
The acceptor’s protocol

- An acceptor receives **prepare** and **accept** requests from proposers.
  - It can always respond to a **prepare** request
  - It can respond to an **accept** request, accepting the proposal, iff it has not promised not to, e.g.

P1a: An acceptor can accept a proposal numbered $n$ iff it has not responded to a prepare request having number greater than $n$

...which subsumes P1.
Small optimizations

- If an acceptor receives a \texttt{prepare} request $r$ numbered $n$ when it has already responded to a \texttt{prepare} request for $n' > n$, then the acceptor can simply ignore $r$.

...so an acceptor needs only remember the highest numbered proposal it has accepted and the number of the highest-numbered \texttt{prepare} request to which it has responded.
Choosing a value:
Phase 1

- A proposer chooses a new $n$ and sends $<\text{prepare}, n>$ to a majority of acceptors.

- If an acceptor $a$ receives $<\text{prepare}, n'>$ where $n' > n$ of any $<\text{prepare}, n>$ to which it has responded, then it responds to $<\text{prepare}, n'>$ with:
  - a promise not to accept any more proposals numbered less than $n'$
  - the highest numbered proposal (if any) that it has accepted.
Choosing a value:

Phase 2

- If the proposer receives a response to \(<\text{prepare}, n>\) from a majority of acceptors, then it sends to each \(<\text{accept}, n, v>\), where \(v\) is either
  - the value of the highest numbered proposal among the responses
  - any value if the responses reported no proposals
- If an acceptor receives \(<\text{accept}, n, v>\), it accepts the proposal unless it has in the meantime responded to \(<\text{prepare}, n'>\), where \(n' > n\)
Learning chosen values (I)

Once a value is chosen, learners should find out about it. Many strategies are possible:

i. Each acceptor informs each learner whenever it accepts a proposal.

ii. Acceptors inform a distinguished learner, who informs the other learners

iii. Something in between (a set of not-quite-as-distinguished learners)
Questions

What are the liveness properties of Paxos?
What do you do when nodes fail?
Question

Are there any advantages/disadvantages to having a designated leader?
Implementing State Machine Replication

Implement a sequence of separate instances of consensus, where the value chosen by the $i^{th}$ instance is the $i^{th}$ message in the sequence.

Each server assumes all three roles in each instance of the algorithm.

Assume that the set of servers is fixed
The role of the leader

In normal operation, elect a single server to be a leader. The leader acts as the distinguished proposer in all instances of the consensus algorithm.

- Clients send commands to the leader, which decides where in the sequence each command should appear.
- If the leader, for example, decides that a client command is the $k^{th}$ command, it tries to have the command chosen as the value in the $k^{th}$ instance of consensus.
A new leader is elected...

Since leader is a learner in all instances of consensus, it should know most of the commands that have already been chosen. For example, it might know commands 1-10, 13, and 15.

- It executes phase 1 of instances 11, 12, and 14 and of all instances 16 and larger.

- This might leave, say, 14 and 16 constrained and 11, 12 and all commands after 16 unconstrained.

- leader then executes phase 2 of 14 and 16, thereby choosing the commands numbered 14 and 16.
Stop-gap measures

All replicas can execute commands 1-10, but not 13-16 because 11 and 12 haven't yet been chosen.

leader can either take the next two commands requested by clients to be commands 11 and 12, or can propose immediately that 11 and 12 be no-op commands.

leader runs phase 2 of consensus for instance numbers 11 and 12.

Once consensus is achieved, all replicas can execute all commands through 16.
leader can efficiently execute phase 1 for infinitely many instances of consensus! (e.g. command 16 and higher)

- leader just sends a message with a sufficiently high proposal number for all instances

- An acceptor replies non trivially only for instances for which it has already accepted a value
Question

What are the costs to using Paxos? Is it practical?