Lecture 6

Genetic Algorithms

Model Ensembles

Genetic Algorithms

- Evolutionary computation
- Prototypical GA
- An example: GABIL
- Schema theorem
- Genetic programming
- The Baldwin effect

Evolutionary Computation

1. Computational procedures patterned after biological evolution
2. Search procedure that probabilistically applies search operators to set of points in the search space

Biological Evolution

Lamarck:

- Species "transmute" over time

Darwin:

- Consistent, heritable variation among individuals in population
- Natural selection of the fittest

Mendel/Genetics:

- A mechanism for inheriting traits
- Mapping: Genotype → Phenotype

GA($Fitness, Fitness\_threshold, p, r, m$)

- *Initialize:* $P \leftarrow p$ random hypotheses
- *Evaluate:* for each h in P, compute $Fitness(h)$
- While $[\max_h Fitness(h)] < Fitness\_threshold$
  1. *Select:* Randomly select $(1-r)p$ members of P to add to P_S. $\Pr(h_i) = \frac{Fitness(h_i)}{\sum_{j=1}^{p} Fitness(h_j)}$
  2. *Crossover:* Randomly select $\frac{r \cdot p}{2}$ pairs of hypotheses from P. For each pair $\langle h_1, h_2 \rangle$, produce two offspring by crossover. Add all offspring to P_s.
  3. *Mutate:* Invert random bit in mp random hyps.
  4. *Update:* $P \leftarrow P_s$
  5. *Evaluate:* for each h in P, compute $Fitness(h)$
- Return hypothesis from P with highest fitness.

Representing Hypotheses

Represent

$$(Outlook = Overcast \vee Rain) \wedge (Wind = Strong)$$

by

Outlook	*Wind*
011	10

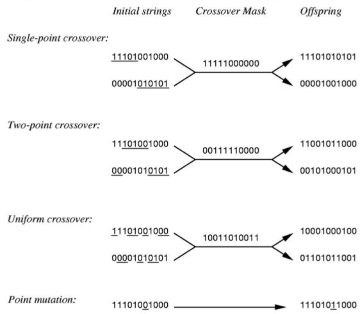
Represent

$$\text{IF } Wind = Strong \text{ THEN } PlayTennis = yes$$

by

Outlook	*Wind*	*PlayTennis*
111	10	10

Operators for Genetic Algorithms



Selecting Fittest Hypotheses

Fitness-proportionate selection:

$$\Pr(h_i) = \frac{Fitness(h_i)}{\sum_{j=1}^{p} Fitness(h_j)}$$

... can lead to *crowding*

Tournament selection:

- Pick h_1, h_2 at random with uniform probability
- With probability p, select the more fit

Rank selection:

- Sort all hypotheses by fitness
- Prob. of selection is proportional to rank

Example: The GABIL System

Learn disjunctive set of propositional rules
Competitive with C4.5

Fitness: $Fitness(h) = (correct(h))^2$

Representation:

IF $a_1 = T \wedge a_2 = F$ THEN $c = T$; IF $a_2 = T$ THEN $c = F$

represented by

a_1	a_2	c	a_1	a_2	c
10	01	1	11	10	0

Genetic operators: ???

- Want variable length rule sets
- Want only well-formed bitstring hypotheses

Crossover with Variable-Length Bitstrings

Start with

	a_1	a_2	c	a_1	a_2	c
$h_1:$	10	01	1	11	10	0
$h_2:$	01	11	0	10	01	0

1. Choose crossover points for h_1, e.g., after bits 1, 8
2. Now restrict points in h_2 to those that produce bitstrings with well-defined semantics, e.g., $\langle 1, 3\rangle$, $\langle 1, 8\rangle$, $\langle 6, 8\rangle$.

If we choose $\langle 1, 3\rangle$, result is

	a_1	a_2	c
$h_3:$	11	10	0

	a_1	a_2	c	a_1	a_2	c	a_1	a_2	c
$h_4:$	00	01	1	11	11	0	10	01	0

GABIL Extensions

Add new genetic operators, also applied probabilistically:

1. *AddAlternative*: generalize constraint on a_i by changing a 0 to 1
2. *DropCondition*: generalize constraint on a_i by changing every 0 to 1

And add new field to bitstring to determine whether to allow these

a_1	a_2	c	a_1	a_2	c	AA	DC
01	11	0	10	01	0	1	0

So now the learning strategy also evolves!

Schemas

How to characterize evolution of population in GA?

Schema = string containing 0, 1, $*$ ("don't care")

- Typical schema: 10$**$0$*$
- Instances of above schema: 101101, 100000, ...

Characterize population by number of instances representing each possible schema

- $m(s,t) = \#$ instances of schema s in pop, at time t

Consider Just Selection

- $\bar{f}(t) =$ average fitness of pop. at time t
- $m(s,t) =$ instances of schema s in pop. at time t
- $\hat{u}(s,t) =$ average fitness of instances of s at time t

Probability of selecting h in one selection step

$$\Pr(h) = \frac{f(h)}{\sum_{i=1}^{n} f(h_i)} = \frac{f(h)}{n\bar{f}(t)}$$

Probability of selecting an instance of s in one step

$$\Pr(h \in s) = \sum_{h \in s \cap p_t} \frac{f(h)}{n\bar{f}(t)} = \frac{\hat{u}(s,t)}{n\bar{f}(t)} m(s,t)$$

Expected number of instances of s after n selections

$$E[m(s,t+1)] = \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)$$

Schema Theorem

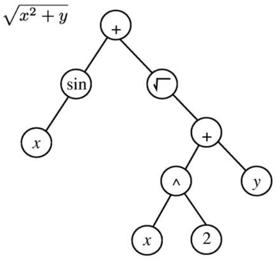
$$E[m(s,t+1)] \geq \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t) \left(1 - p_c \frac{d(s)}{l-1}\right) (1-p_m)^{o(s)}$$

- $m(s,t) =$ instances of schema s in pop at time t
- $\bar{f}(t) =$ average fitness of pop. at time t
- $\hat{u}(s,t) =$ ave. fitness of instances of s at time t
- $p_c =$ probability of single point crossover operator
- $p_m =$ probability of mutation operator
- $l =$ length of single bit strings
- $o(s)$ number of defined (non "$*$") bits in s
- $d(s) =$ dist. between left & rightmost defined bits in s

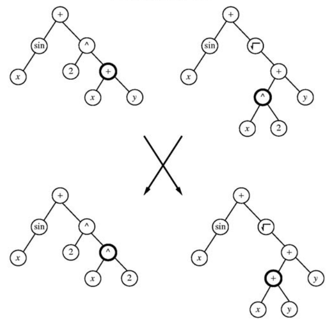
Genetic Programming

Population of programs represented by trees

E.g.: $\sin(x) + \sqrt{x^2 + y}$



Crossover



Example: Electronic Circuit Design

- Individuals are programs that transform beginning circuit to final circuit, by adding/subtracting components and connections
- Use population of 640,000, run on 64-node parallel processor
- Discovers circuits competitive with best human designs

Biological Evolution

Lamarck (19th century)

- Believed individual genetic makeup was altered by lifetime experience
- But current evidence contradicts this view

What is the impact of individual learning on population evolution?

Baldwin Effect

Assume

- Individual learning has no direct influence on individual DNA
- But ability to learn reduces need to "hard wire" traits in DNA

Then

- Ability of individuals to learn will support more diverse gene pool, because learning allows individuals with various "hard wired" traits to be successful
- More diverse gene pool will support faster evolution of gene pool

⇒ Individual learning increases rate of evolution

Baldwin Effect

Plausible example:

1. New predator appears in environment
2. Individuals who can learn (to avoid it) will be selected
3. Increase in learning individuals will support more diverse gene pool
4. Resulting in faster evolution
5. Possibly resulting in new non-learned traits such as instintive fear of predator

Computer Experiments on Baldwin Effect

Evolve simple neural networks:

- Some network weights fixed, others trainable
- Genetic makeup determines which are fixed, and their weight values

Results:

- With no individual learning, population failed to improve over time
- When individual learning allowed
  - Early generations: population contained many individuals with many trainable weights
  - Later generations: higher fitness, while number of trainable weights decreased

Genetic Algorithms: Summary

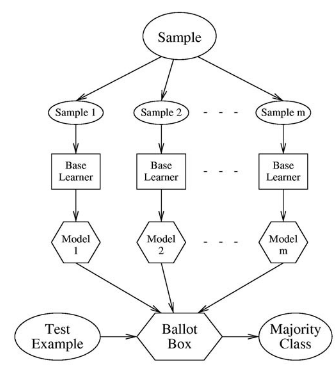
- Evolving algorithms by natural selection
- Genetic operators avoid (some) local minima
- Why it works: schema theorem
- Genetic programming
- Baldwin effect

Model Ensembles

- **Basic idea:**
  Instead of learning one model,
  Learn several and combine them
- Typically improves accuracy, often by a lot
- **Many methods:**
  - Bagging
  - Boosting
  - ECOC (error-correcting output coding)
  - Stacking
  - Etc.

Bagging

- Generate "bootstrap" replicates of training set by sampling with replacement
- Learn one model on each replicate
- Combine by uniform voting



Boosting

- Maintain vector of weights for examples
- Initialize with uniform weights
- Loop:
  - Apply learner to weighted examples (or sample)
  - Increase weights of misclassified examples
- Combine models by weighted voting

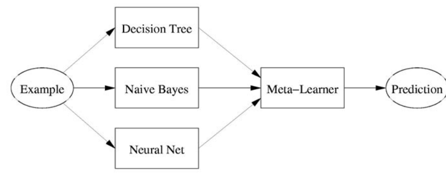
AdaBoost(S, *Learn*, k)
  S: Training set $\{(x_1, y_1), \ldots, (x_m, y_m)\}$, $y_i \in Y$
  Learn: Learner(S, weights)
  k: # Rounds
For all i in S: $w_1(i) = 1/m$
For $r = 1$ to k do
  For all i: $p_r(i) = w_r(i) / \sum_i w_r(i)$
  $h_r = Learn(S, p_r)$
  $\epsilon_r = \sum_i p_r(i)\, \mathbf{1}[h_r(i) \neq y_i]$
  If $\epsilon_r > 1/2$ then
    $k = r - 1$
    Exit
  $\beta_r = \epsilon_r / (1 - \epsilon_r)$
  For all i: $w_{r+1}(i) = w_r(i) \beta_r^{1 - \mathbf{1}[h_r(x_i) \neq y_i]}$
Output: $h(x) = \text{argmax}_{y \in Y} \sum_{r=1}^{k} (\log \frac{1}{\beta_r})\, \mathbf{1}[h_r(x) = y]$

Error-Correcting Output Coding

- **Motivation:**
  Applying binary classifiers to multiclass problems
- **Train:** Repeat L times:
  - Form a binary problem by randomly assigning classes to "superclasses" 0 and 1
    E.g.: A, B, D → 0; C, E → 1
  - Apply binary learner to binary problem
- Each class is represented by a binary vector
- **Test:**
  - Apply each classifier to test example, forming vector of predictions $\mathbf{P}$
  - Predict class whose vector is closest to $\mathbf{P}$ (Hamming)

Stacking

- Apply multiple base learners
  (e.g.: decision trees, naive Bayes, neural nets)
- Meta-learner: Inputs = Base learner predictions
- Training by leave-one-out cross-validation:
  Meta-L. inputs = Predictions on left-out examples



Model Ensembles: Summary

- Learn several models and combine them
- Bagging: Random resamples
- Boosting: Weighted resamples
- ECOC: Recode outputs
- Stacking: Multiple learners