Lecture 11: Bloom Filters, Final Review

December 7, 2011

Lecture on Bloom Filters

Not described in the textbook !

Lecture based in part on:

- Broder, Andrei; Mitzenmacher, Michael (2005), "Network Applications of Bloom Filters: A Survey", Internet Mathematics 1 (4): 485–509
- Bloom, Burton H. (1970), "Space/time trade-offs in hash coding with allowable errors", Communications of the ACM 13 (7): 422–42

Example

Users(uid, name, age)
Pages(uid, url)

Compute this query on Map/Reduce

```
SELECT Pages.url, count(*) as cnt
FROM Users, Pages
WHERE Users.age in [18..25]
      and Users.uid = Pages.uid
GROUP BY Pages.url
ORDER DESC count(*)
```

Example

- Relational algebra plan:

  T1 ← SIGMA{age in [18,25]} (Users) JOIN Pages

  Answer ← GAMMA{url,count(*)}(T1)

- Map/Reduce program: has one MR job for each line above.

Example

Map-Reduce job 1

- Map tasks 1: User where age in [18,25]) → (uid,User)
- Map tasks 2: Page → (uid, Page)
- Reduce task:
  (uid, [User, Page1, Page2, …]) → url1, url2, …
  (uid, [Page1, Page2, …]) → null

Map-Reduce job 2

- Map task: url → (url, 1)
- Reduce task: (url, [1,1,…]) → (url, count)

Example

Problem: many Pages, but only a few visited by users with age 18..25

- How can we reduce the number of Pages sent during MR Job 1?

Hash Maps

- Let $S = \{x_1, x_2, . . ., x_n\}$ be a set of elements
- Let $m > n$
- Hash function $h : S \rightarrow \{1, 2, \ldots, m\}$

$$S = \{x_1, x_2, . . ., x_n\}$$

$$\downarrow$$

	1	2										m
H=	0	0	1	0	1	1	0	0	1	1	0	0

0	0	1	0	1	1	0	0	0	1	0	1

Hash Map = Dictionary

The hash map acts like a dictionary

- Insert(x, H) = set bit h(x) to 1
  - Collisions are possible
- Member(y, H) = check if bit h(y) is 1
  - False positives are possible
- Delete(y, H) = not supported !
  - Extensions possible, see later

0	0	1	0	1	1	0	0	0	1	0	1

Example (cont'd)

- Map-Reduce job 1a
  - Map task: Set of Users → hash map H of User.uid where age in [18..25]
  - Reduce task: combine all hash maps using OR. One single reducer suffices
  - Note: there is a unique key (say =1) produce by Map
- Map-Reduce job 1b
  - Map tasks 1: User where age in[18,25] → (uid, User)
  - Map tasks 2: Page where uid in H → (uid, Page)
  - Reduce task: do the join

Why don't we lose any Pages?

Analysis

- Let $S = \{x_1, x_2, \ldots, x_n\}$
- Let j = a specific bit in H ($1 \leq j \leq m$)
- What is the probability that j remains 0 after inserting all n elements from S into H ?
- Will compute in two steps

0	0	0	0	1	0	0	0	0	0	0	0

Analysis

- Recall |H| = m
- Let's insert only x_i into H

- What is the probability that bit j is 0 ?

0	0	0	0	1	0	0	0	0	0	0	0

Analysis

- Recall |H| = m
- Let's insert only x_i into H

- What is the probability that bit j is 0 ?

- Answer: p = 1 – 1/m

0	0	1	0	1	1	0	0	0	1	0	1

Analysis

- Recall $|H| = m$, $S = \{x_1, x_2, \ldots, x_n\}$
- Let's insert all elements from S in H

- What is the probability that bit j remains 0 ?

0	0	1	0	1	1	0	0	0	1	0	1

Analysis

- Recall $|H| = m$, $S = \{x_1, x_2, \ldots, x_n\}$
- Let's insert all elements from S in H

- What is the probability that bit j remains 0 ?

- Answer: $p = (1 - 1/m)^n$

0	0	1	0	1	1	0	0	0	1	0	1

Probability of False Positives

- Take a random element y, and check member(y,H)
- What is the probability that it returns *true* ?

0	0	1	0	1	1	0	0	0	1	0	1

Probability of False Positives

- Take a random element y, and check member(y,H)
- What is the probability that it returns *true* ?

- Answer: it is the probability that bit h(y) is 1, which is $f = 1 – (1 – 1/m)^n \approx 1 – e^{-n/m}$

0	0	1	0	1	1	0	0	0	1	0	1

Analysis: Example

- Example: m = 8n, then

  $f \approx 1 - e^{-n/m} = 1-e^{-1/8} \approx 0.11$

- A 10% false positive rate is rather high…
- Bloom filters improve that (coming next)

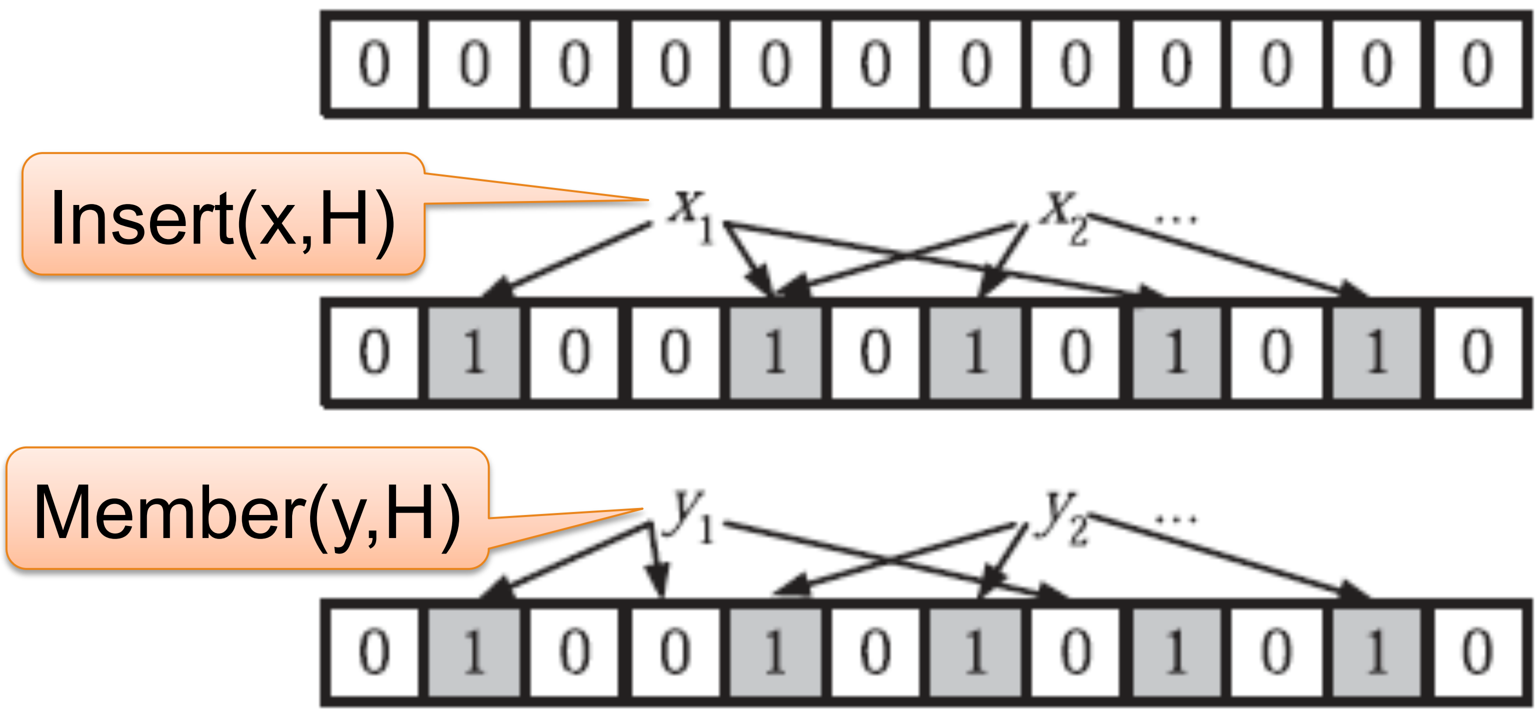
Bloom Filters

- Introduced by Burton Bloom in 1970
- Improve the false positive ratio
- Idea: use k independent hash functions

Bloom Filter = Dictionary

- Insert(x, H) = set bits $h_1(x), \ldots, h_k(x)$ to 1
  - Collisions between x and x' are possible
- Member(y, H) = check if $h_1(y), \ldots, h_k(y)$ are 1
  - False positives are possible
- Delete(z, H) = not supported !
  - Extensions possible, see later

Example Bloom Filter k=3



y_1 = is not in H (why ?); y_2 may be in H (why ?)

Choosing k

Two competing forces:

- If k = large
  - Test more bits for member(y,H) ➔ lower false positive rate
  - More bits in H are 1 ➔ higher false positive rate
- If k = small
  - More bits in H are 0 ➔ lower positive rate
  - Test fewer bits for member(y,H) ➔ higher rate

0	0	1	0	1	1	0	0	0	1	0	1

Analysis

- Recall |H| = m, #hash functions = k
- Let's insert only x_i into H

- What is the probability that bit j is 0 ?

0	0	1	0	1	1	0	0	0	1	0	1

Analysis

- Recall |H| = m, #hash functions = k
- Let's insert only x_i into H

- What is the probability that bit j is 0 ?

- Answer: $p = (1 – 1/m)^k$

0	0	1	0	1	1	0	0	0	1	0	1

Analysis

- Recall $|H| = m$, $S = \{x_1, x_2, . . ., x_n\}$
- Let's insert all elements from S in H

- What is the probability that bit j remains 0 ?

0	0	1	0	1	1	0	0	0	1	0	1

Analysis

- Recall $|H| = m$, $S = \{x_1, x_2, \ldots, x_n\}$
- Let's insert all elements from S in H
- What is the probability that bit j remains 0 ?
- Answer: $p = (1 - 1/m)^{kn} \approx e^{-kn/m}$

Probability of False Positives

- Take a random element y, and check member(y,H)
- What is the probability that it returns *true* ?

Probability of False Positives

- Take a random element y, and check member(y,H)
- What is the probability that it returns *true* ?

- Answer: it is the probability that all k bits $h_1(y)$, …, $h_k(y)$ are 1, which is:

$$\boxed{f = (1-p)^k \approx (1 - e^{-kn/m})^k}$$

Optimizing k

- For fixed m, n, choose k to minimize the false positive rate f
- Denote $g = \ln(f) = k \ln(1 - e^{-kn/m})$
- Goal: find k to minimize g

$$\frac{\partial g}{\partial k} = \ln\left(1 - e^{-\frac{kn}{m}}\right) + \frac{kn}{m}\frac{e^{-\frac{kn}{m}}}{1 - e^{-\frac{kn}{m}}}$$

$$\boxed{k = \ln 2 \times m / n}$$

Bloom Filter Summary

Given $n = |S|$, $m = |H|$,
choose $k = \ln 2 \times m/n$ hash functions

Probability that some bit j is 1 $\boxed{p \approx e^{-kn/m} = ½}$

Expected distribution $\boxed{m/2 \text{ bits } 1,\ m/2 \text{ bits } 0}$

Probability of false positive

$$\boxed{f = (1-p)^k \approx (½)^k = (½)^{(\ln 2)m/n} \approx (0.6185)^{m/n}}$$

Bloom Filter Summary

- In practice one sets $m = cn$, for some constant c
  - Thus, we use c bits for each element in S
  - Then $f \approx (0.6185)^c$ = constant

- Example: $m = 8n$, then
  - $k = 8(\ln 2) = 5.545$ (use 6 hash functions)
  - $f \approx (0.6185)^{m/n} = (0.6185)^8 \approx 0.02$ (2% false positives)
  - Compare to a hash table: $f \approx 1 - e^{-n/m} = 1\text{-}e^{-1/8} \approx 0.11$

The reward for increasing m is much higher for Bloom filters

Set Operations

Intersection and Union of Sets:

- Set S ➔ Bloom filter H
- Set S' ➔ Bloom filter H'

- How do we computed the Bloom filter for the intersection of S and S' ?

Set Operations

Intersection and Union:

- Set S ➔ Bloom filter H
- Set S' ➔ Bloom filter H'

- How do we computed the Bloom filter for the intersection of S and S' ?
- Answer: bit-wise AND: $H \wedge H'$

Counting Bloom Filter

Goal: support delete(z, H)

Keep a counter for each bit j

- Insertion ➔ increment counter
- Deletion ➔ decrement counter
- Overflow ➔ keep bit 1 forever

Using 4 bits per counter:

Probability of overflow $\leq 1.37\ 10^{-15} \times m$

Application: Dictionaries

Bloom originally introduced this for hyphenation

- 90% of English words can be hyphenated using simple rules
- 10% require table lookup
- Use “bloom filter” to check if lookup needed

Application: Distributed Caching

- Web proxies maintain a cache of (URL, page) pairs
- If a URL is not present in the cache, they would like to check the cache of other proxies in the network
- Transferring all URLs is expensive !
- Instead: compute Bloom filter, exchange periodically

Final Review

The Final

- Take-home final; Webquiz

- Posted: Thursday, Dec. 8, 2011, 8pm
- Closed: Saturday, Dec. 10, 10pm

The Final

- No software required (no postgres, no nothing)
- Open books, open notes

- No communication with your colleagues about the final

The Final

- You will receive an email on Thursday night with two things:
  - A url with the online version of the final
  - A pdf file with your final (for your convenience, so you can print it)
- Answer the quiz online
- When done: submit, receive confirmation code

The Final Content (tentative)

1. SQL (including views, constraints, datalog, relational calculus)
2. Conceptual design (FDs, BCNF)
3. Transactions
4. Indexes
5. Query execution and optimization
6. Statistics
7. Parallel query processing
8. Bloom filters

Final Comments

- All the information you need to solve the final can be found in the lecture notes
- No need to execute the SQL queries
- To write a Relational Algebra Plan, use the notation on slide 4
- You may save, and continue
- Questions? Send me an email, but note that I will be offline all day on Friday