Computability and Complexity Winter 2009
Problem Set #5 Instructor: Anna Karlin
Due on **March 14, 2009** by email to `ncthach@cs.washington.edu`

Instructions:

- You must do this problem set *entirely* on your own, without discussing any of the problems with any fellow students.
- The problems have been carefully chosen for their pedagogical value and hence might be similar or identical to those given out in past offerings of this course at UW, or similar courses at other schools. Using any pre-existing solutions from these sources, from the Web or other textbooks constitutes a violation of the academic integrity expected of you and is strictly prohibited.

Problems:

1. Indicate for each of the following if it is **true or false or unknown**. "Unknown" means that according to the current state of knowledge of computability and complexity theory, we do not know whether the statement is true or false. You do NOT need to justify your answer.

   - *True or False or Unknown:* $HAMPATH \leq_{\mathcal{P}} PATH$. ($HAMPATH$ is the problem: given an undirected graph, does it have a Hamiltonian path. $PATH$ is the problem: given an undirected graph G , and two specified vertices s and t, is there a path in G from s to t.
   - *True or False or Unknown:* $PATH \leq_{\mathcal{P}} HAMPATH$.
   - *True or False or Unknown:* The intersection of a decidable language with a Turing-recognizable language is decidable.
   - *True or False or Unknown:* $\{0^*1^*\}$ is PSPACE-complete.
   - *True or False or Unknown:* $P \neq NP$.
   - *True or False or Unknown:* There exists a Turing-recognizable (but not decidable) language L for which $L \leq_m \bar{L}$.
   - *True or False or Unknown:* We know of a problem in $\mathcal{NP}$ that is also in $\mathcal{P}$.
   - *True or False or Unknown:* Suppose that X and Y are both in $\mathcal{P}$. Then there is a polytime reduction from X to Y.
   - *True or False or Unknown:* If $X \in \mathcal{P}$ and Y is NP-complete, then $Y \leq_{\mathcal{P}} X$.
   - *True or False or Unknown :* Suppose that X is NP-complete and $Y \in \mathcal{NP}$. Then $Y \leq_{\mathcal{P}} X$.
   - *True or False or Unknown:* Suppose that $X \in \mathcal{NP}$ and $X \in co\mathcal{NP}$. Then $X \in \mathcal{P}$.
   - *True or False or Unknown:* Suppose that $X \leq_{\mathcal{P}} Y$. and Y is in PSPACE. Then X is in PSPACE.

2. Consider the following list of properties that might apply to a stated language:

   - The language is Turing-recognizable.
   - The language is decidable.
   - The language is in NP.
   - The language is NP-complete.
   - The language is in $\mathcal{P}$.
   - The language is in PSPACE.
   - The language is in NP $\cap$ coNP.

   For each of the following languages, list which of the previous properties are true of that language and which are false. You might not be able to do either for some properties (because we don't know). You do NOT need to justify any of your answers.

   (a) $\{(M, w)|$ Turing machine M accepts $w.\}$

   (b) $\{(M, w)|$ Turing machine M accepts w in at most $|w|$ steps.$\}$

   (c) $\{(M, w)|$ Turing machine M accepts w in at most $2^{|w|}$ steps.$\}$

   (d) $\{(M, w)|$ Turing machine M does not accept $w.\}$

   (e) $\{\phi|\ \phi$ is a 3-CNF formula which evaluates to true on some truth assignment.$\}$

   (f) $\{(\phi, x)|\ \phi$ is a 3-CNF formula which evaluates to true on truth assignment $x.\}$

   (g) $\{(x, y)|\ 1 < x < y$ and y has an integer factor f s.t. $1 < f < x.\}$