Computability and Complexity — Winter 2009

Problem Set #3 — Instructor: Anna Karlin

Due on **February 6, 2009** by email to `ncthach@cs.washington.edu`

Instructions: You are allowed to discuss the problems with fellow students taking the class. However, you must write up your solutions completely on your own. Moreover, if you do discuss the problems with someone else, I am asking, on your honor, that you do not take any written material away from the discussion. In addition, for each problem on the homework, I ask that you acknowledge the people you discussed that problem with, if any.

The problems have been carefully chosen for their pedagogical value and hence might be similar or identical to those given out in past offerings of this course at UW, or similar courses at other schools. Using any pre-existing solutions from these sources, from the Web or other textbooks constitutes a violation of the academic integrity expected of you and is strictly prohibited.

Most of the problems require only one or two key ideas for their solution – spelling out these ideas should give you most of the credit for the problem even if you err in some finer details. So, make sure you clearly write down the main idea(s) behind your solution.

A final piece of advice: Begin work on the problem set early and don't wait till the deadline is a day or two away.

Readings: Arora/Barak Chapter 2, Sipser Chapter 7

1. Arora/Barak Chapter 1, Problem 1.

   `http://www.cs.princeton.edu/theory/complexity/modelchap.pdf`, p.1.21(29)

2. Suppose you're helping to organize a summer camp and the following problem comes up. The camp is supposed to have at least one counselor who's skilled at each of the n sports covered by the camp (baseball, volleyball, etc.). They have received applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is: For a given number $k < m$, it is possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports? We'll call this the *Efficient Recruiting Problem.*

   Show that Efficient Recruiting is NP-complete.

3. Consider the following scheduling problem we call *Multiple Interval Scheduling.*

   There is a single processor available to run jobs for some period of time (say 9am to 5pm).

   People submit jobs to run on the processor: the processor can only work on one job at any single point in time. The jobs are somewhat complicated, in that each job needs a *set* of intervals of time during which it needs exclusive use the processor. Thus, for example, a single job could require the processor from 10am to 11am and then again from 2pm to 3pm. If you accept this job, it ties up your processor during those two hours, but you could still accept jobs that need any other time periods (including the hours 11am to 2pm).

   The input to the problem is a set of n jobs, each specified by a set of time intervals, and you want to answer the following question: For a given number k, is it possible to accept at least k of the jobs so that no two of the accepted jobs have any overlap in time?

Show that Multiple Interval Scheduling is NP-complete.