The Basics of ZPL

Like sequential computation with its C programming language and von Neumann model of computation explaining the performance of programs, parallel computation needs a language calibrated to the CTA model. ZPL is the only such language.

Thread of CSE596

The reasoning bringing us to this point is:

- – Model of computation: We cannot write fast programs without having some idea of how they will perform when they execute … CTA
- – Shared memory (PRAM) seems like a natural programming generalization of sequential computation, but …
  - It “hides” performance-critical info (= locality) at “log cost”
  - Concurrency on shared memory is complex
  - Coherent shared memory OK for SMP, … but beyond???
- – Only a global view of the computation is required
- – Invent new abstractions for a global view … ZPL

ZPL -- A Practical Parallel Language

- ZPL was designed and built using the CTA model, so like C
    - Semantics are defined relative to the model
    - Compiler and run-time system assume the model

  … so ZPL programs are efficient for any CTA computer
- ZPL designed from “1st principles” meaning…
    - ZPL is not an extension of existing language -- it’s new
    - Careful analysis of programming task: XYZ-levels
    - No programming “fads”: functional, OO, “miracle” solutions
    - Search for new ideas that *help* parallel programmers
    - Focus on “user needs,” e.g. scientific computation

ZPL is the third attempt -- Spot and Orca “failed”

ZPL ...

Is an array language -- whole arrays are manipulated with primitive operations

- Requires new thinking strategies --
    - Forget one-operation-a-time scalar programming
    - Think of the computation globally -- make the global logic work efficiently and leave the details to the compiler
- Is parallel, but there are no parallel constructs in the language; the compiler...
    - Finds all concurrency
    - Performs all interprocessor communication
    - Implements all necessary synchronization (almost none)
    - Performs extensive parallel and scalar optimizations

A Sample of ZPL Code

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;

region     R = [1..n, 1..n];
        BigR = [0..n+1,0..n+1];
direction  N = [-1, 0];   S = [ 1, 0];
           E = [ 0, 1];   W = [ 0,-1];
var     Temp : [R] float;
           A : [BigR] float;
         err : float;

procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```

ZPL is an imperative array language with the usual datatypes and operators, the familiar statement forms, and a few new concepts added

Is a mix of Pascal, C and new syntax

A Sample of ZPL Code

```
program Jacobi;
config var n : integer = 512;
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region     R = [1..n, 1..n];
        BigR = [0..n+1,0..n+1];
direction  N = [-1, 0];   S = [ 1, 0];
           E = [ 0, 1];   W = [ 0,-1];
var     Temp : [R] float;
           A : [BigR] float;
         err : float;

procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```
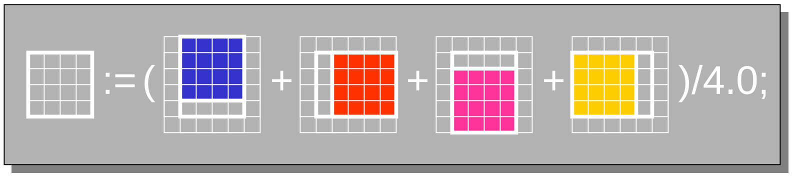

New features
config vars
region
direction
prefixing [ ]
assist with the global view of computation

Jacobi Iteration: How does it work?

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;
```

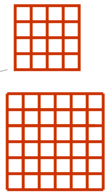


Think of averaging the 4 nearest neighbors as whole array operations

```
procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```

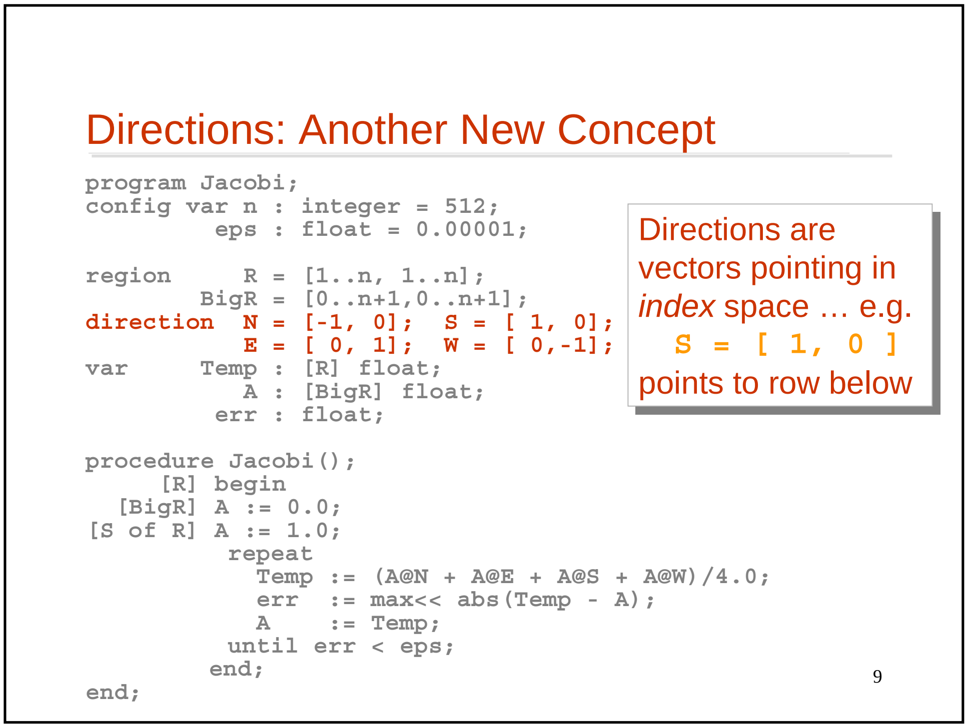
Regions: A New Concept

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;

region     R = [1..n, 1..n];
        BigR = [0..n+1,0..n+1];
direction  N = [-1, 0];  S = [ 1, 0];
           E = [ 0, 1];  W = [ 0,-1];
var     Temp : [R] float;
           A : [BigR] float;
         err : float;
```



Regions are index sets -- like arrays but with no data; used for declarations and execution control

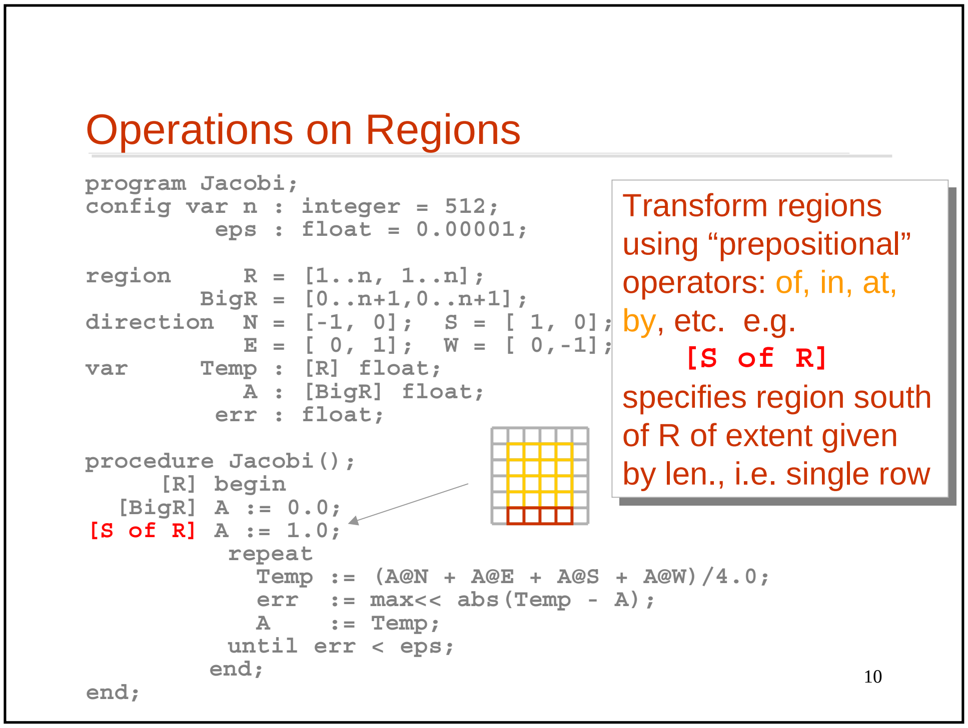
```
procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```



Directions: Another New Concept

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;

region     R = [1..n, 1..n];
        BigR = [0..n+1,0..n+1];
direction  N = [-1, 0];  S = [ 1, 0];
           E = [ 0, 1];  W = [ 0,-1];
var     Temp : [R] float;
           A : [BigR] float;
         err : float;

procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```

Directions are vectors pointing in *index* space … e.g.
`S = [ 1, 0 ]`
points to row below



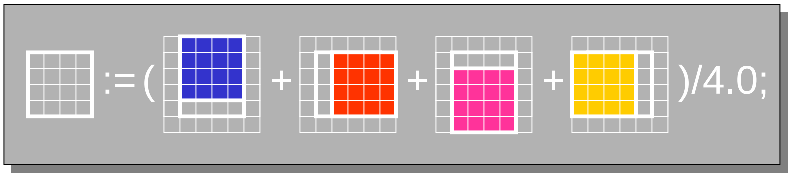
Operations on Regions

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;

region     R = [1..n, 1..n];
        BigR = [0..n+1,0..n+1];
direction  N = [-1, 0];  S = [ 1, 0];
           E = [ 0, 1];  W = [ 0,-1];
var     Temp : [R] float;
           A : [BigR] float;
         err : float;

procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```

Transform regions using "prepositional" operators: of, in, at, by, etc. e.g.
`[S of R]`
specifies region south of R of extent given by len., i.e. single row

Referencing 4 Nearest Neighbors

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;
```



```
procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```

@d shifts applicable region in d direction

The “High Level” Logic Of J-Iteration

```
program Jacobi;
config var n : integer = 512;
         eps : float = 0.00001;

region     R = [1..n, 1..n];
        BigR = [0..n+1,0..n+1];
direction  N = [-1, 0];  S = [ 1, 0];
           E = [ 0, 1];  W = [ 0,-1];
var     Temp : [R] float;
           A : [BigR] float;
         err : float;

procedure Jacobi();
     [R] begin
  [BigR] A := 0.0;
[S of R] A := 1.0;
          repeat
            Temp := (A@N + A@E + A@S + A@W)/4.0;
            err  := max<< abs(Temp - A);
            A    := Temp;
          until err < eps;
         end;
end;
```

Compute new averages
Find the largest error
Update array
… until convergence

ZPL In Detail ...

ZPL has the usual stuff

- Datatypes: `boolean`, `float`, `double`, `quad`, `complex`, signed and unsigned integers: `byte`, `ubyte`, `integer`, `uinteger`, `char`, …
- Operators:
  - Unary: `+, -, !`
  - Binary: `+, -, *, /, ^, %, &, |`
  - Relational: `<, <=, =, !=, >=, >=`
  - Bit Operations: `bnot(), band(), bor(), bxor(), bsl(), bsr()`
  - Assignments: `:=, +=, -=, *=, /=, %=, &=, |=`
- Control Structures: if-then-[elseif]-else, repeat-until, while-do, for-do, exit, return, continue, halt, begin-end

---

ZPL Detail (continued)

- White space ignored
- All statements are terminated by semicolon (;)
- Comments are

`--`	to the end of the line
`/* */`	all text within pairs including newlines

- All variables must be declared using `var`
- Names are case sensitive
- Programs begin with

```
program <name>;
```

the procedure with <name> is the entry point

ZPL Detail (continued)

- The unary global operation reduction (`<<`) “reduces” an entire array to a single value using an associative operator: `+<<`, `*<<`, `max<<`, `min<<`, `&<<`, `|<<`
- For example, `+<<` is summation (Σ) and `max<<` is global maximum

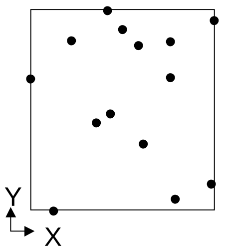
```
err := max<< abs(Temp - A);
```

Global sum was solved the first day with a tree algorithm; global maximum was solved with the tournament algorithm … primitive in ZPL

Bounding Box

- Let X,Y be 1-dimensional *n* element arrays such that (x_i, y_i) is a position in the plane
- The bounding box is the extreme coordinates in each dimension

```
[1..n] begin
          rightedge := max<< X;
          topedge  := max<< Y;
          leftedge  := min<< X;
          bottomedge := min<< Y;
       end
```



Alternative Data Representation

- ZPL allows programmers to define a type
- Rather than using X and Y arrays, define

```
type cartPoint = record
             x : integer; -- x coordinate
             y : integer; -- y coordinate
     end;
...
var Pts : [1..n] cartPoint; -- an array of points
             rightedge  := max<< Pts.x;
             topedge    := max<< Pts.y;
             leftedge   := min<< Pts.x;
             bottomedge := min<< Pts.y;
```

ZPL Inherits from C

- ZPL is translated into C
- Mathematical functions come from math.h
- ZPL's Input and Output follow C conventions and formatting, though the behavior on parallel machines can differ

Configuration variables (config vars) are a list of command line assignable variables with specified defaults … cannot be reset

```
config var prob_size : integer = 64;
```

Mean and Standard Deviation ...

Find μ and σ for array of Sample values

```
program Sample_Stats;
  config var n : integer = 100;
  region     V = [1..n];

procedure Sample_Stats();
  var Sample : [V] float;
     mu,sigma: float;
  [V] begin
        read(Sample);
        mu := +<<Sample/n;
        sigma := sqrt(+<<((Sample-mu)^2)/n);
        write ("Mean: ", mu,"S.D. :", sigma);
      end;
```

$$\sigma = \sqrt{\frac{\sum (Sample_i - \mu)^2}{n}}$$

$$\mu = \frac{\sum Sample_i}{n}$$

Basically, a direct translation into imperative form

Regions

- Regions are named sets of index tuples
- Regions are declared with syntax

```
region <name> = [<ll>..<ul> {, <ll>..<ul>}* ]
```

- For example

```
region R = [1..n, 1..n];    -- Std 2-dim region
region V = [0..m-1];        -- 0-origin
```

- Short names common; caps by convention
- Specify stride with `by` following the limits,

```
region Evens = [0..n by 2]; -- 0, 2, 4, ...
```

Declaring Variables

- Variable declarations have the form of a list followed by colon (:) followed by a datatype

```
var x, y, z : double;
```

- The type of an array is a pair

```
[<region>] <data type>
```

- The region can be named or explicit

```
var A, B, C : [R] double;
 Small_data : [1..n] byte;
```

- Arrays passed as parameters must have this type given in the formal parameter

---

Regions Controlling Array Stmt Execution

Regions specify the indices over which computation will be performed

- Specify region in brackets as statement prefix

```
[1..n,1..n]    A := B;
```

- The n^2 elements of the region are replaced in **`A`** by their corresponding elements in **`B`**
- Regions are scoped

```
[1..n,1] begin -- Work on first column only
               A := 0;
               B := 2*C;
         end;
```

More About Regions

- With explicit indices leave a dimension blank to inherit from enclosing scope

```
[1..n, 1] begin
                X := Y; -- replace first column
          [ , 2]  X += X; -- double second column
            end;
```

- *Arrays must "conform" in rank and both define elements for indices of region*
- "Applicable region" for assignments are (generally) the most tightly enclosing region of the rank of the *left hand side*

Directions

- Directions are vectors pointing in index space
- Declare directions using

  `direction <name> = [ <tuple> ]`

  where `<tuple>` is a sequence of indices separated by commas
- For example

```
direction northwest = [-1, -1];
              right = [1];
```

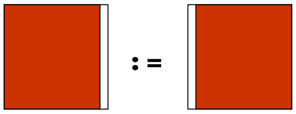
- Short names are common and preferred

The @ Operator

The **@** operator takes as operands an array variable and a direction, and returns an array whose values come from the given array offset from the prevailing region by direction

```
[1..n,1..n-1]  A := B@e;  -- assume e = [0,1]
```

- Assign `A[r,s]` the value `B[r,s+1]`
- That is, `B@e` contains the last n-1 columns of `B`, which are assigned to the first n-1 columns of `A`



The @ must reference defined values

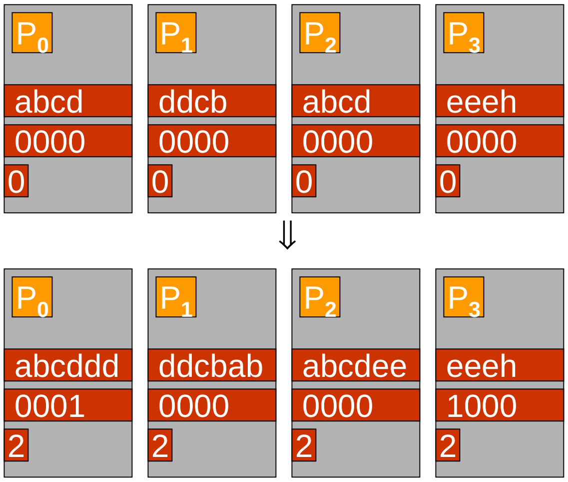
---

3 Identical Values In Sequence

```
region    V = [1..n];
var Letters : [V] char;
        Seq : [V] boolean;
    triples : integer;
direction r = [1]; r2 = [2];
…
[1..n-2] begin
            Seq     := (Letters = Letters@r)
                        & (Letters = Letters@r2);
            triples := +<< Seq;
         end;
```

What Happens

- Send left
- Compare +1
- Compare +2
- Local +
- Accum Tree
- Bdcast Tree



P_0	P_1	P_2	P_3
abcd	ddcb	abcd	eeeh
0000	0000	0000	0000
0	0	0	0

⇓

P_0	P_1	P_2	P_3
abcddd	ddcbab	abcdee	eeeh
0001	0000	0000	1000
2	2	2	2

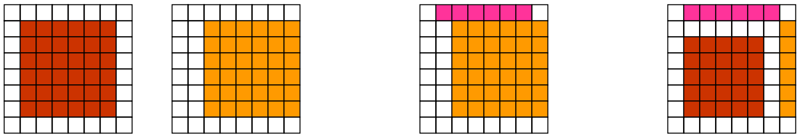
Region Operators

ZPL has region operators taking as operands a region and a direction, and producing a region

- at translates the region's index set in the direction
- of defines a new region adjacent to the given region along direction edge and of direction extent

```
region R = [1..8,1..8];
       C = [2..7,2..7];
var X, Y : [R] byte;
```

```
Direction e = [ 0,1];
          n = [-1,0];
         ne = [-1,1];
```



`[C] X:=`🟥 `[C at e] Y:=`🟧 `[n of C] Y:=`🟪 `[C] Y:=X@ne`

Index1 ...

- ZPL comes with “constant arrays” of any size
- Index*i* means indices of the ith dimension

```
[1..n,1..n]  begin
               Z := Index1; -- fill with first index
               P := Index2; -- fill with second index
               L := Z=P;    -- define identity array
             end;
```

- These array -- of arbitrary dimension -- are compiler created using no space

Scan

- Scan is the parallel prefix operation for associative operators: `+, *, min, max, &, |`
- Scan is like reduction, but uses ||
- Prefix sum from the first lecture is **+||**

```
       A ⇔  2  4   6   8   0
     +||A ⇔  2  6  12  20  20
```

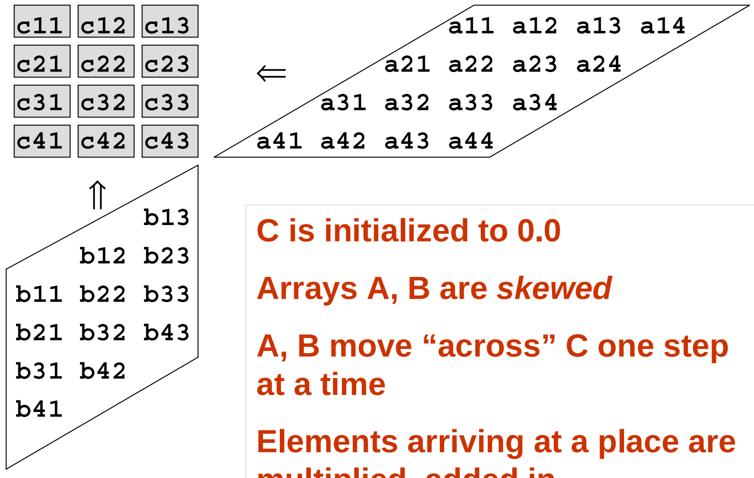
- Yes, “or scan” is ||| as in

```
             B   ⇔ 0   0   0   1   1   0   1   1
     Run:=|||B   ⇔ 0   0   0   1   1   1   1   1
     [2..n] Run := (Run != Run@w)*Index1;
            pos := max<< Run;
```

Think globablly

Break

Recall Cannon's Algorithm



c11 c12 c13
c21 c22 c23
c31 c32 c33
c41 c42 c43

⇐

a11 a12 a13 a14
a21 a22 a23 a24
a31 a32 a33 a34
a41 a42 a43 a44

⇑

b13
b12 b23
b11 b22 b33
b21 b32 b43
b31 b42
b41

C is initialized to 0.0

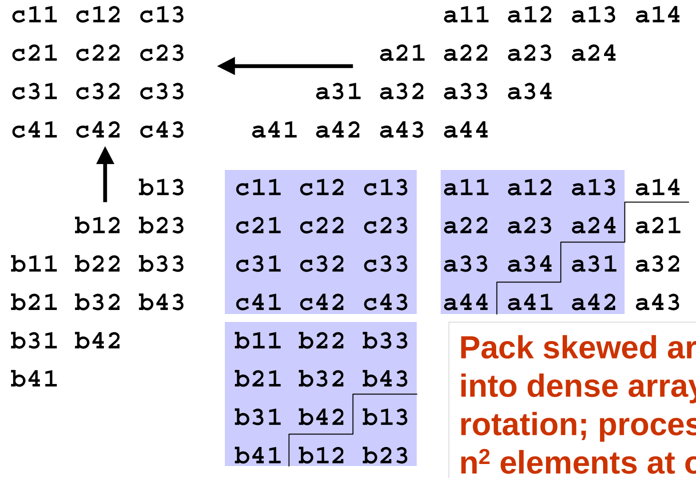
Arrays A, B are *skewed*

A, B move "across" C one step at a time

Elements arriving at a place are multiplied, added in

Programming Cannon's In ZPL

- Step 1: Handle the skewed arrays



```
c11 c12 c13                a11 a12 a13 a14
c21 c22 c23            a21 a22 a23 a24
c31 c32 c33        a31 a32 a33 a34
c41 c42 c43    a41 a42 a43 a44

        b13   c11 c12 c13  a11 a12 a13 a14
    b12 b23   c21 c22 c23  a22 a23 a24 a21
b11 b22 b33   c31 c32 c33  a33 a34 a31 a32
b21 b32 b43   c41 c42 c43  a44 a41 a42 a43
b31 b42       b11 b22 b33
b41           b21 b32 b43
              b31 b42 b13
              b41 b12 b23
```

Pack skewed arrays into dense arrays by rotation; process all n^2 elements at once

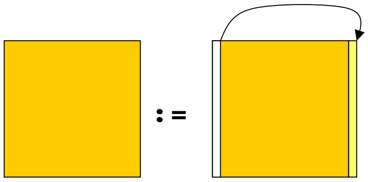
---

Wrap-@

The @-operator has (recently) been extended to automatically wrap-around an array rather than "falling off" -- excellent for "periodic boundaries":

"Falling off" relative to the declared dimensions

```
var A : [1..n,1..n] double; -- array of doubles
…
  A := A@^east; -- rotate columns left
```



Skewing Computation for A

Skew by incrementally shifting last of array left, finishing 1 row / step

- Assume declarations
- `region Lop = [1..m, 1..n];`
- `direction right = [0,1];`

```
a11 a12 a13 a14
a22 a23 a24 a21
a33 a34 a31 a32
a44 a41 a42 a43
```

Intended Result

```
             for i := 2 to m do
[i..m, 1..n]    A := A@^right; -- Shift last i rows left
             end;
```

Four Steps of Skewing A

```
             for i := 2 to m do
 [i..m, 1..n] A := A@^right;  -- Shift last m-i rows left
             end;
```

Skew B vertically

```
a11 a12 a13 a14
a21 a22 a23 a24
a31 a32 a33 a34
a41 a42 a43 a44
```

Initial

```
a11 a12 a13 a14
a22 a23 a24 a21
a32 a33 a34 a31
a42 a43 a44 a41
```

i = 2 step

```
a11 a12 a13 a14
a22 a23 a24 a21
a33 a34 a31 a32
a43 a44 a41 a42
```

i = 3 step

```
a11 a12 a13 a14
a22 a23 a24 a21
a33 a34 a31 a32
a44 a41 a42 a43
```

i = 4 step

Cannon’s Declarations

For completeness, when A is m×n and B is n×p, the declarations are …

```
region       Lop = [1..m, 1..n];
             Rop = [1..n, 1..p];
             Res = [1..m, 1..p];
direction right = [ 0, 1];
          below = [ 1, 0];
var            A : [Lop] double;
               B : [Rop] double;
               C : [Res] double;
```

Cannon’s Algorithm

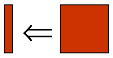
Skew A, Skew B, {Multiply, Accumulate, Rotate}

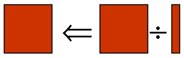
```
           for i := 2 to m do -- Skew A
  [i..m, 1..n] A := A@^right;
           end;
           for i := 2 to p do -- Skew B
  [1..n, i..p] B := B@^below;
           end;

         [Res] C := 0.0;       -- Initialize C
           for i := 1 to n do -- For common dim
         [Res] C := C + A*B;  -- For product
         [Lop] A := A@^right; -- Rotate A
         [Rop] B := B@^below; -- Rotate B
           end;
```

Combining Arrays of Different Ranks

An apparent limitation of ZPL (so far) is: Only arrays of like rank can be combined

- Element-wise operators combine corresponding elements: `[R] A := B+C;`
- Sometimes combining arrays of different rank is needed. E.g. Scale the elements of each row by the row maximum
  - Find the row maximum: 2 dimension reduced to 1 dimension
  - Divide each element of the row by its row max: 1 dimension applied to 2 dimensions





Two cases

Don’t Change Rank

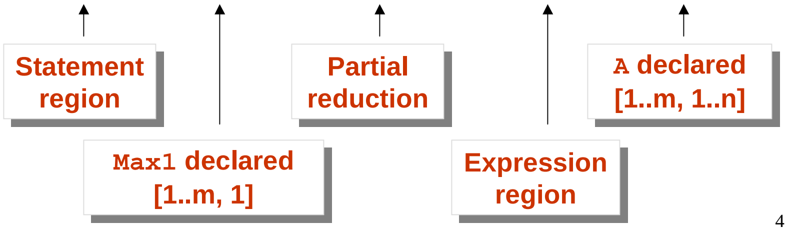
- Rather than change rank, use “singleton” values to collapse dimensions for lower rank
- For a region `R = [1..m, 1..n]`, the rank 2 arrays `R1 = [1..m, 1]` and `R2 = [1, 1..n]` are regions corresponding to the first column and row

- ZPL is designed to exploit the similarity between an array with collapsed dimensions and a corresponding array of lower rank

The Reduction Case

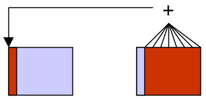
- Partial reduction applies reduction to an array to produce a subarray … two regions needed
  - “Statement region” specifies the shape of the result
  - “Expression region” specifies the shape of the operand
  - The subarray that is combined is the subarray of the operand formed by the singleton dimensions

```
[1..m,1] Max1 := max<< [1..m,1..n] A;
```



Partial Reduction (continued)

- All associative operators can be used in partial form: `+, *, max, min, &, |`
- The “singleton” dimension is “meaningful” in that the values are stored with like indices
  - E.g. `[1..m,1]` is stored with other first column values
- Arrays can have arbitrary dimension values if they have the right rank & elements defined…
  - E.g. Add row elements 2..n and store sum in 1st position
  - `[1..m,1] A := +<< [1..m, 2..n] A;`

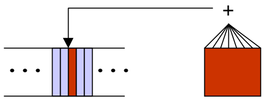


Flood Regions and Arrays

Flood regions recognize that reducing into a specific column *over specifies* the situation

Need a *generic* column -- or a column that does not have a specific position … use ‘*’ as value

```
region FlCol = [1..m, *];   -- Flood regions
       FlRow = [*, 1..n];
var   MaxCol : [FlCol] double; -- An m length col
         Row : [FlRow] double; -- An n length row
[1..m,*] MaxCol := max<< [1..m,1..n] A; -- Better
```



Think of column in every position

Flood (continued)

Since flood arrays have unspecified dimensions, they can be “promoted” in those dimensions, i.e logically replicated

- The computation is completed …

```
   [1..m,*] MaxCol := max<< [1..m,1..n] A;
[1..m,1..n]      A := A / MaxCol;     --Scale A;
```

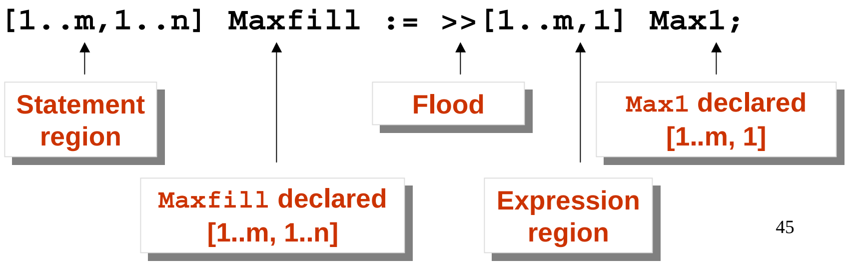
Flood makes combining different ranks “element-wise”

The promotion of flooded arrays is only logical

The Flood Operator (>>)

An alternative is to save flooded vals -- dumb

- Flood (>>) replicates values of a subarray to fill a larger array
  - “Statement region” specifies the shape of the result
  - “Expression region” specifies the shape of the operand
  - The subarray is replicated in all of the operand’s singleton dimensions



```
[1..m,1..n] Maxfill := >>[1..m,1] Max1;
```

Explicit Solution vs Logical Solution

- Reducing into a specific position and then flooding works, but it explicitly replicates values

```
   [1..m,1]     Max1 := max<<[1..m,1..n] A;--Save Col
[1..m,1..n] Maxfill := >>[1..m,1] Max1;   --Flood Col
[1..m,1..n]       A := A/Maxfill; --Div by col array
```

- Flood logically replicates values...and it’s easier

```
   [1..m,*] MaxCol := max<< [1..m,1..n] A;
[1..m,1..n]      A := A / MaxCol; --Scale A;
                       -- or --
[1..m,1..n] A := A/(>>[1..m,*]  max<< [1..m,1..n] A);
```

Partial Scan

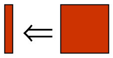
- Partial scan would seen to be an easy generalization of partial reduce, but since it doesn't "collapse" dimensions, it is not necessary to specify a region, only a dimension

```
A := Index1;
B := +||[2] A;
```

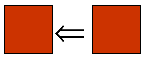
```
A:          B:
1 1 1       1 2 3
2 2 2       2 4 6
3 3 3       3 6 9
```

Remembering Reduce, Scan & Flood

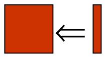
- The operators for reduce, scan and flood are suggestive …
  - Reduce << produces a result of smaller size



  - Scan || produces a result of the same size

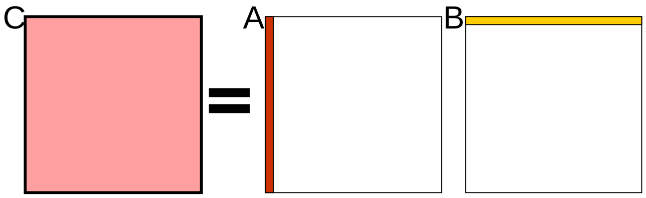


  - Flood >> produces a result of greater size



To Illustrate Computing With Flood

- Recall the SUMMA Algorithm



	b_{11}	b_{12}
a_{11}	$a_{11}b_{11}$	$a_{11}b_{12}$
a_{21}	$a_{21}b_{11}$	$a_{21}b_{12}$

Switch Orientation -- By using a *column* of A and a *row* of B broadcast to all, compute the "next" terms of the dot product

SUMMA Algorithm

- A column broadcast is simply a column flood and similarly for row broadcast is a row flood
- Define variables

```
var   Col : [1..m,*] double; -- Col flood array
      Row : [*,1..p] double; -- Row flood array
        A : [1..m,1..n] double;
        B : [1..n,1..p] double;
        C : [1..m,1..p] double;
```

SUMMA Algorithm (continued)

For each col-row in the common dimension, flood the item and combine it

```
[1..m,1..p]     C := 0.0;         -- Initialize C
            for k := 1 to n do
   [1..m,*]   Col := >>[ ,k] A; -- Flood kth col of A
   [*,1..p]   Row := >>[k, ] B; -- Flood kth row of B
[1..m,1..p]     C += Col*Row;     -- Combine elements
            end;
```

SUMMA is the easiest MM algorithm to program in ZPL

SUMMA, The First Step

```
c11 c12 c13   a11 a12 a13 a14
c21 c22 c23   a21 a22 a23 a24
c31 c32 c33   a31 a32 a33 a34
c41 c42 c43   a41 a42 a43 a44
b11 b12 b13
b21 b22 b23
b31 b32 b33          Col              Row
b41 b42 b43           a11 a11 a11      b11 b12 b13
                      a21 a21 a21      b11 b12 b13
                      a31 a31 a31  ×   b11 b12 b13
                      a41 a41 a41      b11 b12 b13

                                     C
                                      a11b11 a11b12 a11b13
                                      a21b11 a21b12 a21b13
                                      a31b11 a31b12 a31b13
                                      a41b11 a41b12 a41b13
```

Cannon’s or SUMMA?

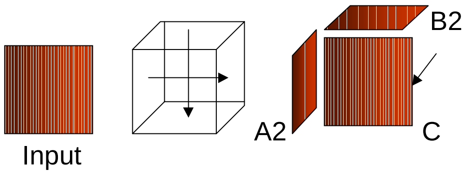
Which algorithm is better for MM?

- Cannon’s algorithm uses the simpler concepts and simpler operations
- SUMMA is conceptually cleaner, but requires ideas like flood arrays
- We will analyze the two algorithms when we have a performance model defined

Still Another MM Algorithm

If flooding is so good for columns/rows, why not use it for whole planes?

```
region IK = [1..n,*,1..n]
       JK = [*,1..n,1..n];
       IJ = [1..n,1..n,*];
      IJK = [1..n,1..n,1..n];
[IK]  A2 := <##[Index1,Index3,Index2] A;
[JK]  B2 := <##[Index3,Index2,Index1] B;
[IJ]   C := +<<[IJK](>>[IK]A2)*(>>[JK]B2);
```



ZPL Procedures

- Procedures have form:

```
procedure <name> ({<params>}) {:Type};
<statement>;
```

- Parameters are listed with their type separated by commas:

```
procedure F(A, B : [R] byte, x : float): float;
```

- Values are returned with `return … ;`
- Parameters are “called by value” as the default, but by prefixing with `var` they can be called by reference

```
procedure G(var A : [R] integer, big : integer);
```

A Procedure for Matrix Multiplication

```
procedure MM (n: integer,
          var A:[1..m,1..n] double,
          var B:[1..n,1..p] double,
          var C:[1..m,1..p] double);
var i : integer;
[1..m,1..p]  begin
                for k := 1 to n do
                    C += (>>[ ,k] A)*(>>[k, ]B);
                end;
             end;

MM(n, E, F, G);
```

Explicit values in the parameter list force specific global variables to be used

Rank Defined Formal Parameters

It is sufficient for the compiler to know the ranks of the arrays, not their specific dimensional values … rank-defined parameters

- Use commas to imply the rank (r-1)
- Call the procedure in the context of the proper region

```
procedure MM(n:integer, var A,B,C :[ , ] double);
    var i : integer;
    begin
       for k := 1 to n do
           C += (>>[ ,k] A)*(>>[k, ]B);
       end;
    end;
```

Inherit regions … call is

`[1..m,1..p] MM(n,E,F,G);`

Promoting Scalar Procedures

- Procedures that only use scalar parameters, operations, etc. can be promoted to arrays
  - Cannot use regions, array operations or other array-based notation

```
procedure sign (x: double) : integer;
    if x < 0 then return -1
             elseif x = 0 then return 0
                          else return 1;

[1..m, 1..n] A := sign(B);
```

Importing scalar computations from C is an application

Shattered Control Flow

ZPL logically executes one instruction at a time

- There is a natural generalization in which statements are controlled by arrays rather than scalars

```
if A < 0 then A := -A;  -- define absolute
```

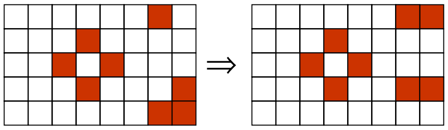
- Convenient for iterations

Let N and Nfact be defined [1..n]

```
Nfact := 1;
for i := 2 to N do
   Nfact := Nfact * i;  -- Compute N!
end;
```

Exercise: Game of Life

- Write a ZPL program for the game of life on a toroidal world, i.e. top wraps to bottom, left wraps to right
- The world is populated by organisms -- bits
    - Any 1 bit with exactly 2 neighbors in this generation lives on in the next generation; all other 1s go to 0
    - Any 0 bit with exactly 3 neighbors is born in the next generation; all other 0s stay 0



Expect a homework assignment via email

Summary

- ZPL is an array programming language
- Array programming emphasizes large operations in which the compiler specifies the looping and indexing
- One new idea is the region -- set of indices
- Programming in ZPL emphasizes thinking about the task at a high level rather than at the detailed scalar level