CSEP 521: Applied Algorithms
Lecture 15 – Nearest neighbors

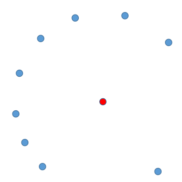
Richard Anderson, February 23, 2021

Announcements

Topics

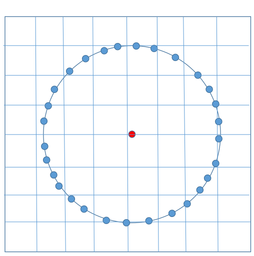
- Bad cases for quad trees
- Hashing for nearest neighbors
- High dimensional data sets
- Documents data sets
- Jaccard Similarity
- MinHash
- Dimension Reduction

Bad case for Nearest Neighbor Query



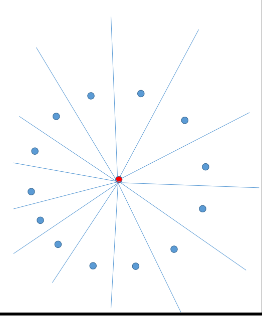
Quad tree

- Search algorithm will lead us to expand every cell containing a point
- Approximate search gives a much better results



Voronoi diagram

- In theory, Voronoi diagram should work fine
- Vertices of degree greater than three are not expected
- Numerical issues can be very challenging
- Nightmare to debug

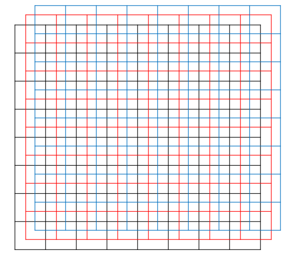


Hashing Based nearest neighbors

- Hashing to test if a query point y is with distance δ of a point in S
- Center boxes on coordinates of the form $c2^{-k}$
- Hash the boxes so that O(n) boxes are used
- Query point hashed to same boxes

Hashing based nearest neighbors

- Is y within δ of any point x in y in S?
- Construct three grids with $3\delta \times 3\delta$ squares, offset by (δ, δ) and $(2\delta, 2\delta)$
- Use hashing to record the squares containing points of S
- Lookup the squares containing y by the hash function and test if there are neighbors in S
- Use a hierarchy of values powers of two of δ for an approximate nearest neighbor



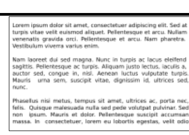
More on distance metrics

- Implement across structures with multiple types
  - Record: (int Age, string Name, enum HairColor, int Weight)
- Weighing of coordinates and monotonic functions of coordinates generally preserve being a distance function
  - Can be tuning parameters for an application
- Example data set – Health Facility Lists, Entity resolution problem
  - Problem seems like it should be easy: merge lists of health facilities from different sources
  - Fields: name, admin region, health facility type, geographic coordinates

High dimensional searching

- Many data sets are high dimensional
  - High dimension can mean a mathematical space, such as R^d, or a structure, such as bag-of-words representation of documents
- Large scale data sets – Billions of photographs, web documents, sequences
- Tree based algorithms break down for high dimensions
  - Number of points in a ball of radius B increases exponentially with dimension
  - Processing dimensions is expensive
- Idea – dimension reduction techniques
  - Is it possible to reduced N-dimensional data to K-dimensional data, K << N, that approximately preserves distances

Document databases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- Large collections of text documents
- Applications such as similarity search, plagiarism detection, classification
- Distance metrics vs. similarity measures
  - Similarity measure a function where a high value is a more similar documents
- Representation – strings, token streams, bags of words

Document representation

- Text strings
  - Edit distance as a similarity measure
- Token streams
  - Simplify words and remove punctuation or markup
- Bag of words
  - Represent the words as a set or a multiset
  - Sparse representation

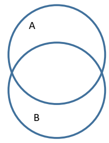
Homework
Similarity of articles from news groups
Sample of 50 articles each from 20 groups

data50.csv	Bag of bag of words
labels.csv	Association of articles to groups
groups.csv	Names of groups

data50.csv
article id, word id, multiplicity

Words have been replaced by integers into another (missing) table, so the data is not readable

Jaccard Similarity



$$\mathrm{Jaccard}(A,B) = \frac{A \cap B}{A \cup B}$$

Let X be the characteristic vector for A where x_j is the multiplicity of item j and Y be the characteristic vector for B where y_j is the multiplicity of item j.

$$\mathrm{Jaccard}(A,B) = \frac{\sum_j \min(x_j, y_j)}{\sum_j \max(x_j, y_j)}$$

Cosine Similarity

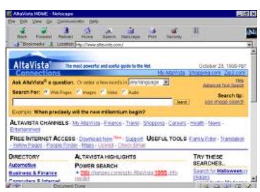
$$\mathrm{CS}(A,B) = \frac{\sum_j x_j y_j}{\|X\|_2 \|Y\|_2}$$

X is the characteristic vector for A where x_j is the multiplicity of item j
Y is the characteristic vector for B where y_j is the multiplicity of item j.

Used in programming assignment – but not a focus of the lecture

AltaVista search engine problem

- Avoid returning (near) duplicate items in search results
- Can fingerprinting techniques apply?
  - Fingerprinting is usually applied to detect or amplify small changes



Representation scheme

- Tokenize document
- Break document into shards
- Hash each shard into a domain of size 2^{64} (unsigned long)
- Treat as a bag of words
- Use Jaccard Similarity measure

far out in the uncharted backwaters of the unfashionable end of the western spiral arm of the galaxy lies a small unregarded yellow sun

1. far out in the uncharted
2. out in the uncharted backwaters
3. in the uncharted backwaters of
4. the uncharted backwaters of the
5. uncharted backwaters of the unfashionable
6. backwaters of the unfashionable end
7. of the unfashionable end of
8. the unfashionable end of the
9. unfashionable end of the western
10. end of the western spiral
11. of the western spiral arm
12. the western spiral arm of

Aside – Rabin Fingerprinting

- n-bit message $m_0, \ldots, m_{n-1}$ viewed as polynomial over Z_2
  - $f(x) = m_0 + m_1 x + m_2 x^2 + \ldots + m_{n-1} x^{n-1}$
- Pick a random irreducible polynomial p(x) of degree k (k = 64) and the fingerprint is f(x) mod p(x)
- Suitable for domain of size 2^k
- Efficient implementation with bit operations including shifts
- Rolling hash that can reuse computation from shard
- Cool algebra for math majors

Similarity testing

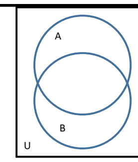
- Identify document pairs that have high similarity by doing pairwise comparison
- Precompute hashes of shards – n shards for document of n tokens
- Cost of comparison is O(n)

- How to improve this: reduce the amount of information stored per document

MinHash

- U is the domain (in this case, the hash of the shards, $[0 \ldots 2^{64})$
- Choose a random permutation π on U
- Let $A \subseteq U$
- MinHash(A) = $\text{argmin}_{x \in A}\, \pi(x)$
  - MinHash is the smallest element of A under the random permutation

An amazing result



$$\Pr[\text{MinHash}(A) = \text{MinHash}(B)] = \frac{|A \cap B|}{|A \cup B|} = \text{Jaccard}(A, B)$$

Using the MinHash

- Identify document pairs where Jaccard(A,B) $\geq$ 0.95
- Run MinHash with k independent permutations
- Number of times MinHash(A)=MinHash(B) is a good estimate of Jaccard Similarity
- Compute the k MinHashes for each documents as a sketch
- Comparison of documents requires k comparisons