CSEP 521
Applied Algorithms

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Course Objective

- Introduce a toolkit of modern algorithmic techniques
- Theory of algorithms developed around developing efficient algorithms for a natural class of problems with respect to a standard model of computation (1950s through 1980s)
- Newer work in algorithms focuses on using different models (changing the rules) and bringing in a new set of mathematical tools

My background

- PhD, Stanford (1985)
  - Thesis: *The Complexity of Parallel Algorithms*
- Post Doc (1985-86) Mathematical Science Research Institute, Berkeley
- University of Washington (since 1986)
  - Broad range of work: Algorithms, Software Engineering, Educational Technology, Computing for Development
- Sabbatical 1993-1994
  - Indian Institute of Science, Bangalore
  - Parallel Algorithms
- Sabbatical 2001-2002
  - Microsoft Research, Redmond
  - Learning Science and Technology
- Sabbatical 2008-2009
  - PATH, Seattle
  - Digital solutions for global health

Preview of Course Topics

- Distinct from (my) undergraduate courses [see CSE 417/421]
- Work in progress (but drawing on Karlin/Lee CSEP 521)

- Randomized Algorithms and Average Case Analysis
  - Randomization is a powerful technique and worst case analysis sometimes misses the point
- Hashing
- Streaming Algorithms
- High dimensional searching
- Linear Algebra Techniques

PMP Course Philosophy

- Students working full time in industry
- Varying backgrounds and length of time from prerequisite courses
- Interests in both relevance and broadening

- Aim for relatively uniform workload and clear expectations
- Ability to work independently and figure things out and seek out resources and assistance

- We've got to make the best of remote teaching

Course mechanics

- Zoom
- Recorded lectures
- Course website: https://courses.cs.washington.edu/courses/csep521/21wi/
- Office hours, zoom links on website
  - RJA: Monday 11am, Friday 2pm
  - Oscar: Wednesay 11 am, Friday 11 am
- Homework Assignments
  - Weekly assignments
  - Electronic turn in on gradescope
  - Mix of written problems and programming experiments
  - Programming assignments
    - Your choice of language (Java, C#, Python) and environment
- No exams, no course project

Making the best of Zoom

- Attendance is class sessions strongly encouraged
- Slides will be available before class
- Feel free to ask questions
- Oscar will monitor chat
  - Useful for clarification questions
- You will need to tolerate some technical glitches
  - "I think the instructor can't hear us"
- Turning your camera on is optional
- We will use break out rooms for discussion questions
- The course will have office hours, four hours per week

Policies

- Weekly homework assignments, due Tuesdays, 11:59 PM
  - Homework posted on the website (HW1 is available)
  - Late homework accepted with penalty
    - 10% per day
    - Maximum 50% reduction
    - 5 free late days
    - Late day computation will be done at the end of the course
  - Homework received after grading has started may not receive feedback
  - Course grade based on top 9 of 10 assignments
- Collaboration policy
  - Its fine to work together (but not required)
  - Independent write ups
  - Acknowledge collaborators

Models of computation

- Problem: Instance, Solution, Constraints, Value
- Computation Model: Idealized Computer, Unit Cost per Instruction
- Runtime function (for Algorithm A)
  - Runtime on instance I, $T_A(I)$, number of steps to compute solution to I
  - Runtime function, $T_A(n)$, maximum runtime over all instances of length n

Asymptotic Analysis

- Ignore constant factors
  - Constant factors are arbitrary and tedious
- Express run time as O(f(n))
- T(n) is O(f(n)) [T : $Z^+ \rightarrow R^+$]
  - If n is sufficiently large, T(n) is bounded by a constant multiple of f(n)
  - Exist c, n_0, such that for $n > n_0$, $T(n) < c\, f(n)$

- Emphasize algorithms with slower growth rates

Randomized Computation

- Assume a source of random bits
- Compute using the random bits
- Multiple types of results are possible
  - Always correct, just the run time varies
  - One sided error
  - Two sided error
- Amplification of probability of correctness
  - Try multiple times

Why randomization

- Actually, lots of reasons
  - Foiling an adversary
  - Random sampling
  - Witnesses
  - Fingerprinting
  - Hashing
  - Re-ordering to avoid bad inputs
  - Load balancing
  - Convergence in Markov chains
  - Symmetry breaking
  - The Probabilistic Technique

Breakout Groups!

- I'm going to try to use breakout groups for discussions / problems, but this first one is just a chance to get to know each other. Introduce yourselves!

Generating a random permutation

- Permutation: Bijection from [1..n] to [1..n]
  - [2, 5, 8, 1, 10, 3, 4, 7, 6, 9]
- n! permutations on [1..n]
- Random permutation – generate permutations with each having probability of exactly 1/n!

Generating a random permutation

```
public static int[] Permutation(int n, Random rand) {
    int[] arr = IdentityPermutation(n);

    for (int i = 1; i < n; i++) {
        int j = rand.Next(0, i + 1);      // Random number in range 0..i
        int temp = arr[i];                // Swap arr[i], arr[j]
        arr[i] = arr[j];
        arr[j] = temp;
    }
    return arr;
}
```

Correctness proof

- Invariant: arr[0] . . . arr[i] is a random permutation of 0 . . i at the end of the loop
- Base case: i = 0
- Induction: Suppose arr[0] … arr[i-1] is a random permutation of 0..i-1 at the start of the loop

Let P be a permutation on i+1 elements, show Prob[P] = 1/(i+1)! at end of loop

Suppose P[i] = x and P[j] = i

P is created from P’ (on i elements), by moving x from location j to location i

Prob[Swapping i and j] = 1/(i+1)

Prob[P’] = 1/i! By the induction hypothesis

Hence Prob[P] = 1/(i+1)!

Quicksort

QSort(A){

    If |A| <= 1 return A

    Choose element x from A

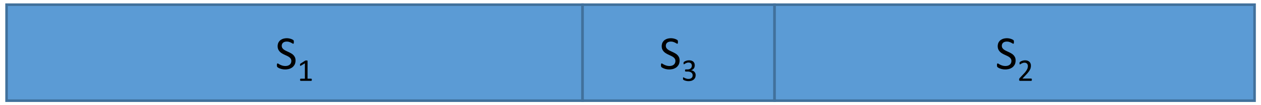
    $S_1 = \{y \text{ in } A \mid y < x\}$

    $S_2 = \{y \text{ in } A \mid y > x\}$

    $S_3 = \{y \text{ in } A \mid y = x\}$

    return QSort(S1) + S3 + QSort(S2)

}



Basic QS facts

- Considered one of best sorts in practice with careful implementation
- Usual runtime O(n log n), Worst case $O(n^2)$
- Sort {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . ., N}
- Avoiding the worst case
  - Change pivot selection
  - Randomly permute before sorting
- Compute average case run time

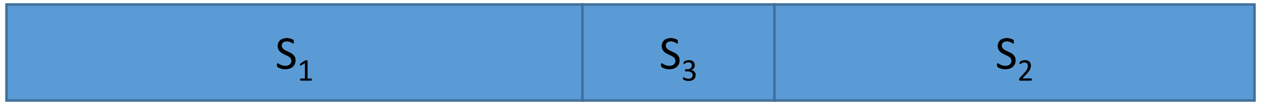
- Mathematically tractable, but not easy

Finding the k-th largest

- Given n numbers and an integer k, find the k-th largest
- If k = n/2 this is computing the median
- Obviously, we can solve this problem by sorting (in time O(n log n)) but can we do better
- Quicksort idea, but only one recursive call
- This will still have the same pathological case

QSelect(A, k)

QSelect(A, k){
  Choose element x from A
  $S_1 = \{y \text{ in } A \mid y < x\}$
  $S_2 = \{y \text{ in } A \mid y > x\}$
  $S_3 = \{y \text{ in } A \mid y = x\}$
  if ($|S_2| >= k$)
    return QSelect(S_2, k)
  else if ($|S_2| + |S_3| >= k$)
    return x
  else
    return QSelect(S_1, $k - |S_2| - |S_3|$)
}



Quick Select

- Same worst case as Quick Sort
- Random pivot will give an O(n) solution (Expected time)
- Deterministic solution (due to BFPRT) not practical

- Exact evaluation of Quick Select recurrence is very difficult

Evaluating Quick Select

- Deterministic recurrence: $T(1) = 1$; $T(N) = T(aN) + bN$ for $a < 1$

- What is the chance that for a random pivot, the subproblem is of size at most aN (for an appropriate $a < 1$)

Analysis

- A random pivot reduces the problem to less then ¾ the size with probability at least ½
- Recurrence $T(n) <= \frac{1}{2} T(3n/4) + \frac{1}{2} T(n) + cn$
- Recurrence for expected number of steps

- This recurrence gives an upper bound on the number of steps for randomized selection

Selection results

- O(n) expected time median algorithm
- Practical algorithm

- How many comparison to find the median?