CSEP 521 – Applied Algorithm
Spring 2003
Homework 8.
Due date: 5/28/03 (see submission instructions in course web-page).

1.(35 points) In the SET-COVER problem we are given a set E of n elements and a collection $S_1,\ldots, S_m$ of subsets of E. The goal is to find a minimum-size set-cover, that is, a minimum-size collection of input subsets, whose union is E (in other words 'which cover all the elements of E').

Example: E={1,2,3,4,5,6,7}
S_1={1} S_2={7} S_3={2,3,4,5,6} S_4={1,4,5,7}
Then $\{S_3,S_4\}$ is an optimal solution of size 2. The collection $\{S_1,S_2,S_3\}$ is also a cover but its size is 3 so it is not optimal.

1.a Describe SET-COVER as an integer programming problem.
1.b Relax the integer constraints to get a linear programming problem.
1.c For $e \in E$, let r(e) be the number of subsets that include e. In the above example r(2)=1, r(4)=2. Let r=max{r(e):$e \in E$}. Given a solution to the relaxed LP, we take to the set-cover all the subsets whose corresponding variables have value at least 1/r.
Show that the selected collection of subsets is a valid set-cover and that this is an r-approximation.

2. (30 points) You are given a text over 9 characters c1,…,c9. The total text length is 1000 characters. The number of appearances of the different characters in the text is 600, 200, 50, 50, 30, 30, 30, 5, 5, respectively (i.e., c1 appears 600 times, etc.). You need to decode the text in binary.
2.a Give an optimal fixed-length code. What is the length of the whole code?
2.b Give an optimal prefix-free variable-length code. What is the length of the whole code?

3. (35 points) For general graphs, we know that the vertex-cover problem is NP-hard. However, for some classes of graphs we can solve vertex-cover optimally in polynomial time. Give, prove, and analyze an **optimal** greedy algorithm that finds a minimum vertex cover in an undirected **tree**. The input is a tree T=(V,E), the output should be $C \subseteq V$, a set of vertices of minimum size, such that each edge has at least one endpoint in C.

4. (extra credit) Write one or two questions for the exam, it should be of the form 'is the following claim true? justify or give a counter example'. Please mail your suggestions + answers (that must fit into a 5 lines space) to tami@cs.washington.edu .
I promise to include some of the good suggestions in the exam. You are free to discuss your suggestions with other students. Any reasonable, original, question will give you some credit in this HW. The real bonus will be to 'meet' your question in the exam!