CSEP 521 – Applied Algorithm
Spring 2003
Homework 7.

Due date: 5/21/03 (see submission instructions in course web-page).

1. (25 points) You are given a sequence of n positive integers $x_1, x_2, \ldots, x_n$, alternating with n-1 arithmetic operations, either + or * . The problem is to fully parenthesize the arithmetic expression so that the RESULT is maximized.
Example: for the input 3 * 2 + 1, by ((3*2)+1) you get 7, and by (3*(2+1)) you get 9, which is better. Write a program based on dynamic programming to calculate the maximal possible result. Explain shortly and analyze time and space complexity.
Hint: define MAXP (i; j) as the maximum result achievable by fully parenthesizing the partial expression $x_i, \ldots, x_j$.

2. (30 points) You need to drive along a highway, using rental cars. There are n rental car agencies 1, 2, ..., n along the highway. At any of the agencies, you can rent a car that can be returned at any other agency down the road. You cannot backtrack, i.e., you can drive only in one direction along the highway. So a car rented at agency i can be returned only at some agency j > i. For each pair of agencies i, j with j > i, the cost c[i,j] of the i-to-j car rental is known. For example, c[2,5]=10 means that the cost of renting a car in agency 2 and returning it in agency 5 is 10$. The costs c[i,j] are positive integers, but you can not assume anything about them, in particular, it might be that c[2,5]<c[2,4].

2.a (10 pts.) A greedy algorithm for this driving/renting problem might always choose the lowest-cost rental from the current location. i.e., at location i, always choose the i-to-j rental with minimum cost. Give (and explain) a counterexample showing that the greedy algorithm is not optimal.

2.b (20 pts.) Give an O(n^3) time dynamic programming algorithm that takes the matrix of i-to-j rental costs as input, and returns the minimum cost of traveling from agency 1 to agency n. (i.e., you must rent your first car at 1, and you must return your last car at n.) Explain the recursive formula and justify the time complexity bound.

3. (20 points) Consider the following LP.

maximize $3x + 5y$
subject to $x \le 4$
$2y \le 12$
$3x+2y \le 18$
$x,y \ge 0$

Draw the feasible set and determine graphically the solution.

4. (25 pts.) WheelsR'Us is a company that manufactures bikes, wheelbarrows, and firefighter cars for kids. Each bike requires two wheels and 2 lights, and can be sold for 40$. Each wheelbarrow requires one wheel and 2 lights, and can be sold for 30$. Each firefighter car requires four wheels and six lights, and can be sold for 150$. There are 60 wheels and 100 lights in the company storage. The company's goal is to produce toys with highest potential revenue (assume that all other resources and materials required for manufacturing are not limited).

4.a Describe this problem as an LP optimization problem.

4.b Write the dual problem. What is the 'story' describing the dual problem?. What can we learn from the Duality Theorem (explain in words, you don't have to solve the LP)?