CSEP 521 – Applied Algorithm
Spring 2003
Homework 5.
Due date: 5/7/03 (see submission instructions in course web-page).

In all the questions below you need to reduce one problem to another. You have to describe a polynomial-time reduction, and to explain its correctness in short. For one of the reductions (your choice) you should provide a full proof. Please state clearly at the top of your homework for which reduction you provide a full proof.

1. (20 pts.) The decision-version of the clique problem takes a graph *G* and an integer *k* and decides if *G* has a clique of size *k* or not. The optimization problem takes a graph *G*, and returns the size and participating vertices in some largest clique in *G*. Show that if the decision problem has a polynomial time algorithm then the optimization problem also has a polynomial time algorithm.

2. (20 pts.) The decision-version of SAT takes a CNF formula and decides if there is a true/false assignment to the variables such that the formula is satisfied. The reporting problem of SAT takes a CNF formula, returns a satisfying assignment, or announces that no such assignment exists. Show that if the decision problem has a polynomial time algorithm then the reporting problem also has a polynomial time algorithm.

3. (20 pts.) Consider the following variant of the spanning tree problem. The input consists of an undirected graph *G* (with no weights). The problem is to find a spanning tree *T* of *G* such that the degree of each node in *T* is at most 3, or report that no such tree exists. Show that if this problem has a polynomial time algorithm then the Hamiltonian path problem has a polynomial time algorithm. The Hamiltonian path problem asks you to determine whether a graph has a simple path that spans the vertices.

4. (20 pts.) A *dominating set* in an undirected graph is a collection *S* of vertices with the property that every vertex *v* in *G* is either in *S*, or there is an edge between a vertex in *S* and *v*. Show that the problem of finding a minimal size *dominating set* is NP-hard using a reduction from *vertex cover*. Specifically, show that if the decision version of dominating set has a polynomial time algorithm then the decision version of vertex cover also has a polynomial time algorithm. Hint: Given an instance $G=(V,E)$ for VC, the input $G'=(V',E')$ for DS has $|V'|=|V|+|E|$ and $|E'|=3|E|$.

(The 20 missing points are for the proof).