CSEP 521 – Applied Algorithm
Spring 2003
Homework 4.
Due date: 4/30/03 (see submission instructions in course web-page).

1. (30 pts.)
1.a. An edge is *upward critical* if increasing the capacity of this edge increases the maximum flow value. Does every network have an upward critical edge? Describe an algorithm for identifying all upward critical edges in a network. The worst-case complexity of your algorithm should be substantially better than that of solving m maximum flow problems.

1.b An edge is *downward critical* if decreasing the capacity of this edge decreases the maximum flow value. Is the set of upward critical edges the same as the set of downward critical edges? Prove or give a counter example. If not, describe an algorithm for identifying all downward critical edges; analyze your algorithm's worst-case complexity.

2. (25 pts.) Networks with node capacities: In some networks, in addition to edge capacities, each node i other than the source and target, might have an upper bound, say w(i), on the amount of flow that can pass through it. In these networks we are interested in determining the maximum flow satisfying both the edge and node capacities. Transform this problem to the standard maximum flow problem. From the perspective of worst-case complexity, is the maximum flow problem with node capacities more difficult to solve than the standard maximum flow problem?

3. (20 pts.) Let $U=\{1,2,\ldots,n\}$. Given sets $S_1,S_2,\ldots,S_k$ such that for all i, $S_i \subseteq U$, describe an algorithm to select representatives $r_1,r_2,\ldots,r_k$, such that for all i, $r_i \in Si$ and for all i,j $r_i \neq r_j$. If no such selection exists the algorithm should announce it. Prove that your algorithm is correct and analyze its time complexity.

4. There are three groups of participants in a party:
A - Girls
B - Boys with blue hats
C - Boys with green hats
For each girl $a \in A$ we know the sets $B(a) \subseteq B$ and $C(a) \subseteq C$ of boys with blue and green hats with whom she is ready to dance. Each boy can dance with at most one girl.

4a. (15 pts.) In the first dance a girl can dance with one boy, or with two boys having different hat colors, or not dance at all. Give an algorithm to match the party participants in a way that maximizes the number of dancing boys. Explain in short why your algorithm is correct and analyze its time complexity (in terms of $|A|$,$|B|$, and $|C|$ - the group sizes).

4b. (10 pts., difficult) Repeat the above for another dance, in which a girl can dance with two boys having different hat colors, or not dance at all (but cannot dance with a single boy).