CSEP 521 – Applied Algorithm
Spring 2003
Homework 3.
Due date: 4/23/03 (see submission instructions in course web-page).

Please submit with this HW your answer to HW2/question 4

1. (20 pts.) Is the following claim true? Prove or give a counter example.
Let G=(V,E) be an undirected graph with weights on the edges. There exists a DFS execution on G that produces a minimum spanning tree.

2. (30 pts.) Suppose we are given the minimum spanning tree T of a given graph with n vertices and m edges, and a new edge e=(u,v) of weight w that we will add to G. Give an efficient algorithm to find the minimum spanning tree of the graph G=(V,E $\cup${e}). Your algorithm should run in O(n) time.

3. (20 pts.) Show that a graph that does not include a cycle of odd length is 2-colorable.

4. (30 pts.) Let G be an undirected graph representing a network. The vertices represent processors and there is an edge between any two conflicting processors. Each of the processors needs to run for one time unit. Two processors are conflicting if they cannot run simultaneously (since they need the same resource, or any other reason). In class we showed that a coloring of G induces a legal schedule of the processors. Formally, we can schedule all the processors that are colored i in time interval [i-1,i]. For each such processor, j, we say that the completion time of j is i. A coloring of G with minimal number of colors induces a schedule in which the *maximal* completion time of some processor is minimized. In other words, a coloring in which $max_v\ c(v)$ is minimal is optimal for the objective of minimizing the *maximal completion time*.

4.1. Consider now another objective: We would like to minimize the *average* completion time of the processors.

a. Describe this problem as a graph-coloring problem; what is an optimal coloring for this goal?
b. Is it true that an optimal coloring for the 'average'-objective must minimize also the 'max'-objective? Prove or give a counter example.

4.2. Assume that each processor, *v*, is associated with a different length, *l(v)*, meaning that *v* needs to be processed for *l(v) consecutive* time units. We want to minimize the *maximal* completion time. Describe this problem as a graph-coloring problem; what is an optimal coloring?