CSEP 521 – Applied Algorithm
Spring 2003
Homework 1.

Due date: 4/9/03 (see submission instructions in course web-page).

1. (12 points) True of False? Prove or give a counter example. The functions f,g,h are positive and monotonically increasing.

a. if $f(n)=\Omega(g(n))$ and $g(n)=O(h(n))$ then $f(n)=\Theta(h(n))$

b. if $f(n)=\Theta(g(n))$ then for any h, $h(f(n))=\Theta(h(g(n)))$

2. (18 points) True or False? Give a brief explanation.

a. $\sum_{k=1}^{n} k = O(n)$

b. $\sum_{k=1}^{n} k = \Omega(n)$

c. $2^n = \Theta(3^n)$

d. $3n^2 + n + n \cdot \log(n) = \Omega(n^2)$

e. $3n^2 + n + n \cdot \log(n) = \Omega(n \cdot \log(n))$

f. $\frac{n^2}{2^n} = O(1)$

3. (12 points) For each of the following questions, briefly explain your answer.

a. If I prove that an algorithm takes $O(n^2)$ worst-case time, is it possible that it takes $O(n)$ on some inputs?
b. If I prove that an algorithm takes $O(n^2)$ worst-case time, is it possible that it takes $O(n)$ on all inputs?
c. If I prove that an algorithm takes $\Theta(n^2)$ worst-case time, is it possible that it takes $O(n)$ on some inputs?
d. If I prove that an algorithm takes $\Theta(n^2)$ worst-case time, is it possible that it takes $O(n)$ on all inputs?

4. (14 points) Use induction to show that for any n>0

$$(1 + 2 + \ldots + n)^2 = 1^3 + 2^3 + \ldots + n^3$$

5. (22 points) In class we argued that the Traditional Marriage Algorithm will never require more than n^2 days to terminate. In fact, it is possible to prove a tighter upper bound, $n^2 - 2n + 2$, on the maximum number of days until the algorithm terminates. Describe a set of preference lists that requires $n^2 - 2n + 2$ days to terminate.

6. (22 points) Let A and B be two different stable pairings of n boys with n girls. Consider the following way to build a new pairing C from A and B. For each boy, A pairs him with some girl g_A and B pairs him with some girl g_B; to construct C we give him his favorite of the two girls g_A and g_B. (g_A and g_B might be the same girl, which is fine.) It is not even obvious that C is a pairing-perhaps some girl will get matched with two boys. Strangely, C is not only a pairing, it is stable! Prove that it is a pairing and that it is stable.