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Homework # 6
Due November 10

1. Prove that the first-fit algorithm for binpacking uses at most twice the optimal number of bins. (*The binpacking problem:* Let x_1, x_2,...,x_n be a set of real numbers each between 0 and 1. Partition the numbers into as few subsets (bins) as possible so that the sum of numbers in each subset is at most one. *The first-fit algorithm:* Put x_1 into the first bin, and then, for each i, put x_i in the first bin that has room for it, or start a new bin if there is no room in any of the used bins and put x_i in it.)

2. Set Cover: An instance of the set cover problem consists of a finite set of elements X and a family $\mathcal{F}$ of subsets of X. (For example X could be elements $\{1, 2, 3, 4, 5\}$ and $\mathcal{F}$ could be $(\{1, 3\}, \{1, 2, 4, 5\}, \{2, 3\}, \{2, 5\})$. The set cover problem is to find a minimum-size subset $\mathcal{C} \subseteq \mathcal{F}$ whose members contain all the elements of X. (In the example, the first and second sets in $\mathcal{F}$ $(\{1, 3\}, \{1, 2, 4, 5\})$ form a set cover because all 5 elements are included in at least one of these sets.)

   - Give a 0/1 integer programming formulation of this problem.
   - Describe (or write pseudo-code for) a branch-and-bound procedure for finding a minimum set cover that uses the linear programming relaxation of the integer program as a basis for pruning the search.

3. **Will not be graded.** An *Euler tour* of a connected, undirected graph $G = (V, E)$ is a cycle that traverses each edge of G exactly once, although it may visit a vertex more than once.

   (a) Show that G has an Euler tour if and only if every node has even degree.

   (b) Give an efficient algorithm for finding an Euler tour of a graph if one exists. What is the running time of your algorithm?

4. One can prove that the Vertex Cover problem is NP-hard by showing that Clique polynomially reduces to it as follows. Given an instance of Clique, a graph $G = (V, E)$ and an integer k (such that we're asking if G contains a clique of size at least k), construct the complement graph $G^c = (V, E^c)$. (The edge set E^c of G^c consists of precisely those edges which don't exist in E, i.e. $E^c = \{(u, v)$ such that (u, v) not in $E\}$.) The output of the reduction is an instance G^c, $V - k$ of the vertex cover problem. (Is there a vertex cover of size at most $V - k$ in G^c?). [*Convince yourself that this reduction works!*]

   In class we saw an approximation algorithm for vertex cover that finds a vertex cover of size at most twice the minimum size. Does this relationship imply that there is an approximation algorithm with constant ratio bound for the clique problem? *Justify your answer!*