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Homework # 5
Due 4pm, November 3

Reading: see slides

Problems

1. Consider the following LP.

   $$maximize\ 3x + 5y$$

   subject to $x \leq 4,\ 2y \leq 12,\ 3x + 2y \leq 18,\ x, y \geq 0$.

   - Draw the feasible set.
   - Determine graphically the solution.
   - Which vertices of the feasible set would the simplex algorithm visit en route to the solution?

2. What is the dual problem of the following linear program?

   $$maximize\ 4x + 7y + 2z$$

   subject to $x + 2y + z \leq 10,\ 2x + 3y + 3z \leq 10,\ x, y, z \geq 0$.

3. Prove that the subgraph isomorphism problem is NP-complete. In this problem, you are given two graphs $G = (V_1, E_1)$ and $H = (V_2, E_2)$, and you are asked if G contains a subgraph isomorphic to H, i.e., does there exist a subset $V \subseteq V_1$ and a subset $E \subseteq E_1$, such that (a) V and V_2 have the same size, (b) E and E_2 have the same size, and (c) there exists a one to one function $f : V_2 \rightarrow V$ satisfying $(u, v) \in E_2$ if and only if $(f(u), f(v)) \in E$?

4. You are a traveling salesperson of the new millenium. Your boss has asked you to visit n cities, each exactly once, and you are planning your flight itinerary. You do not care about the cost of travel – the company is going to pay your expenses anyway. What you want to maximize is the benefits you will receive as a frequent flyer! In other words, you want to *maximize* the length of the entire trip. Do not worry about choosing different airlines. You just want to maximize the total mileage. Prove that this problem is NP-complete, and suggest heuristic approaches to it.

5. Find, by either searching the web, searching the literature, or talking to people a *real-life important* problem that is NP-complete. It should not be one of the problems that's been mentioned in class. How is the problem solved in practice? (Either find out, or come up with your own idea.)

6. **Extra Credit** Consider the following closest-point heuristic for building an approximate traveling-salesman tour. Begin with a trivial cycle consisting of a single arbitrarily chosen vertex. At each step, identify the vertex u that is not on the cycle but whose distance to any vertex on the cycle is minimum. Suppose that the vertex on the cycle that is nearest u is vertex v. Extend the cycle to include u by inserting u just after v. Repeat until all vertices are on the cycle. Prove that this heuristic returns a tour whose total cost is not more than twice the cost of an optimal tour.