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Homework # 4
Due 4pm, October 27

Reading: see slides

Problems

1. Give a linear program for the optimal pipeline problem discussed in lecture 4 – A D bit data object must be sent through a pipeline of n stages. The time it takes an x bit packet to get through stage i is $o_i + \frac{x}{b_i}$, where o_i is the overhead of the i-th stage and b_i is the bandwidth of the i-th stage (in bits/sec). Your linear program should determine how the data should be broken up into k pieces, not necessarily of equal size, so as to minimize the time through the pipeline.

2. We discussed the problem of finding a maximum flow in a graph $G = (V, E)$ with capacities $c(v, w)$ on each edge $(v, w) \in E$. Formulate this problem as a linear programming problem.

3. Drawing graphs nicely is a problem that arises constantly in applications. Consider the problem of drawing a tree. Some characteristics that would be desirable in the drawing are:

   - All nodes on the same level in the tree should line up horizontally.
   - There should be a minimum horizontal spacing between nodes at the same level.
   - The vertical distance between a node and it's children in the tree should not be less than some minimum value m.
   - All nodes should lie within a certain window on the screen.
   - The parent of a set of nodes should be centered over those nodes in the horizontal direction.
   - The height and width of the tree drawing should be small.

   How would you formulate the problem of placing the tree nodes in the drawing using linear programming?

4. **(Revised)** Let $G = (V, E)$ be a flow network. Show that there is a sequence of at most E argmenting paths such that augmenting along these paths in the specified order produces a maximum flow. (Hint: Determine the paths after finding the maximum flow.)