Propositional Logic

With $\rightarrow$ for implies, $+$ for inclusive-or and $*$ for and:

$$\begin{array}{lcl} p & ::= & b \mid p \rightarrow p \mid p * p \mid p + p \\ \Gamma & ::= & \cdot \mid \Gamma, p \end{array}$$

$\boxed{\Gamma \vdash p}$

$$\frac{\Gamma \vdash p_1 \qquad \Gamma \vdash p_2}{\Gamma \vdash p_1 * p_2} \qquad \frac{\Gamma \vdash p_1 * p_2}{\Gamma \vdash p_1} \qquad \frac{\Gamma \vdash p_1 * p_2}{\Gamma \vdash p_2}$$

$$\frac{\Gamma \vdash p_1}{\Gamma \vdash p_1 + p_2} \qquad \frac{\Gamma \vdash p_2}{\Gamma \vdash p_1 + p_2}$$

$$\frac{\Gamma \vdash p_1 + p_2 \quad \Gamma, p_1 \vdash p_3 \quad \Gamma, p_2 \vdash p_3}{\Gamma \vdash p_3}$$

$$\frac{p \in \Gamma}{\Gamma \vdash p} \qquad \frac{\Gamma, p_1 \vdash p_2}{\Gamma \vdash p_1 \rightarrow p_2} \qquad \frac{\Gamma \vdash p_1 \rightarrow p_2 \qquad \Gamma \vdash p_1}{\Gamma \vdash p_2}$$

Guess what!!!!

That's *exactly* our type system, erasing terms and changing each τ to a p

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (e_1, e_2) : \tau_1 * \tau_2} \qquad \frac{\Gamma \vdash e : \tau_1 * \tau_2}{\Gamma \vdash e.1 : \tau_1} \qquad \frac{\Gamma \vdash e : \tau_1 * \tau_2}{\Gamma \vdash e.2 : \tau_2}$$

$$\frac{\Gamma \vdash e : \tau_1}{\Gamma \vdash \mathsf{A}(e) : \tau_1 + \tau_2} \qquad \frac{\Gamma \vdash e : \tau_2}{\Gamma \vdash \mathsf{B}(e) : \tau_1 + \tau_2}$$

$$\frac{\Gamma \vdash e : \tau_1 + \tau_2 \quad \Gamma, x{:}\tau_1 \vdash e_1 : \tau \quad \Gamma, y{:}\tau_2 \vdash e_2 : \tau}{\Gamma \vdash \textsf{match } e \textsf{ with A}x.\ e_1 \mid \textsf{B}y.\ e_2 : \tau}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.\ e : \tau_1 \rightarrow \tau_2} \qquad \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_1}$$