CSE P 501 – Compilers

Loops

Hal Perkins

Autumn 2023

Agenda

- Loop optimizations
  - Dominators – discovering loops
  - Loop invariant calculations
  - Loop transformations
- A quick look at some memory hierarchy issues

  (if we have time)

- Largely based on material in Appel ch. 18, 21; similar material in other books

Loops

Much of the execution time of programs is spent inside loops

∴ worth considerable effort to make loops go faster

∴ want to figure out how to recognize loops and figure out how to “improve” them

What’s a Loop?

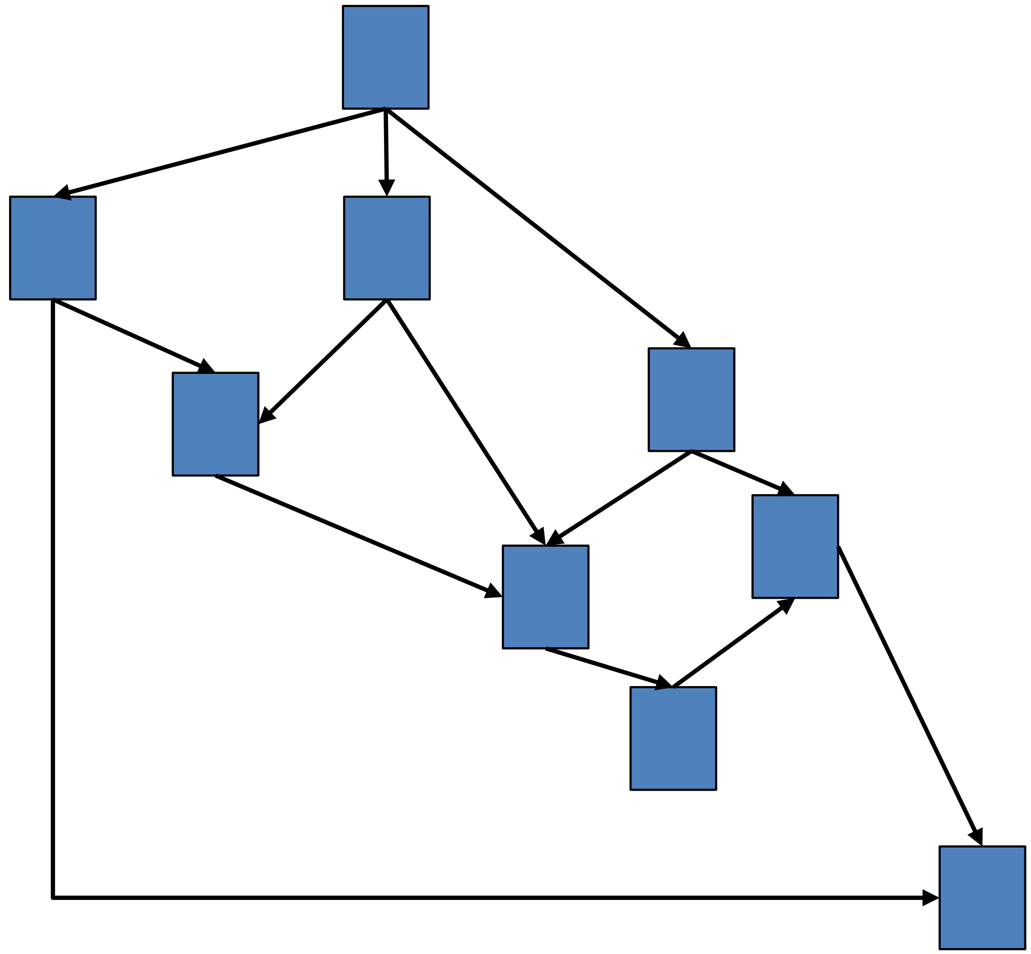
- In source code, a loop is the set of statements in the body of a for/while construct
- But, in a language that permits free use of GOTOs, how do we recognize a loop?
- In a control-flow-graph (node = basic-block, arc = flow-of-control), how do we recognize a loop?

Any Loops in this Code?

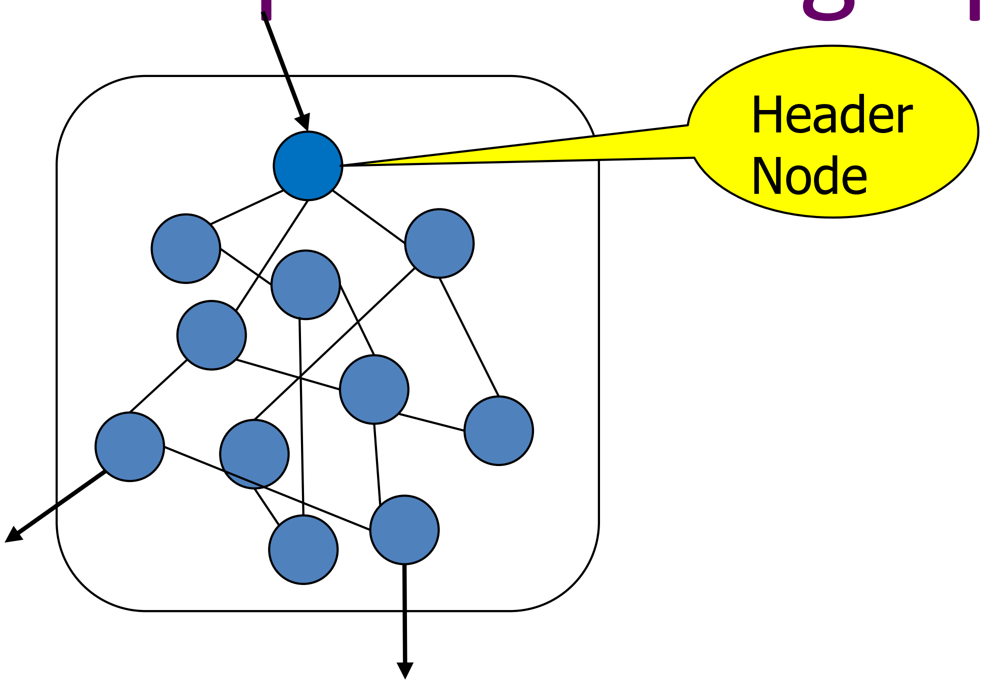
```
        i = 0
        goto L8
L7:     i++
L8:     if (i < N) goto L9
        s = 0
        j = 0
        goto L5
L4:     j++
L5:     N--
        if(j >= N) goto L3
        if (a[j+1] >= a[j]) goto L2
        t = a[j+1]
        a[j+1] = a[j]
        a[j] = t
        s = 1
L2:     goto L4
L3:     if(s != 0) goto L1 else goto L9
L1:     goto L7
L9:     return
```

Anyone recognize or guess the algorithm?

Any Loops in this Flowgraph?



Loop in a Flowgraph: Intuition



- Cluster of nodes, such that:

- There's one node called the "header"
- I can reach all nodes in the cluster from the header
- I can get back to the header from all nodes in the cluster
- Only once entrance - via the header
- One or more exits

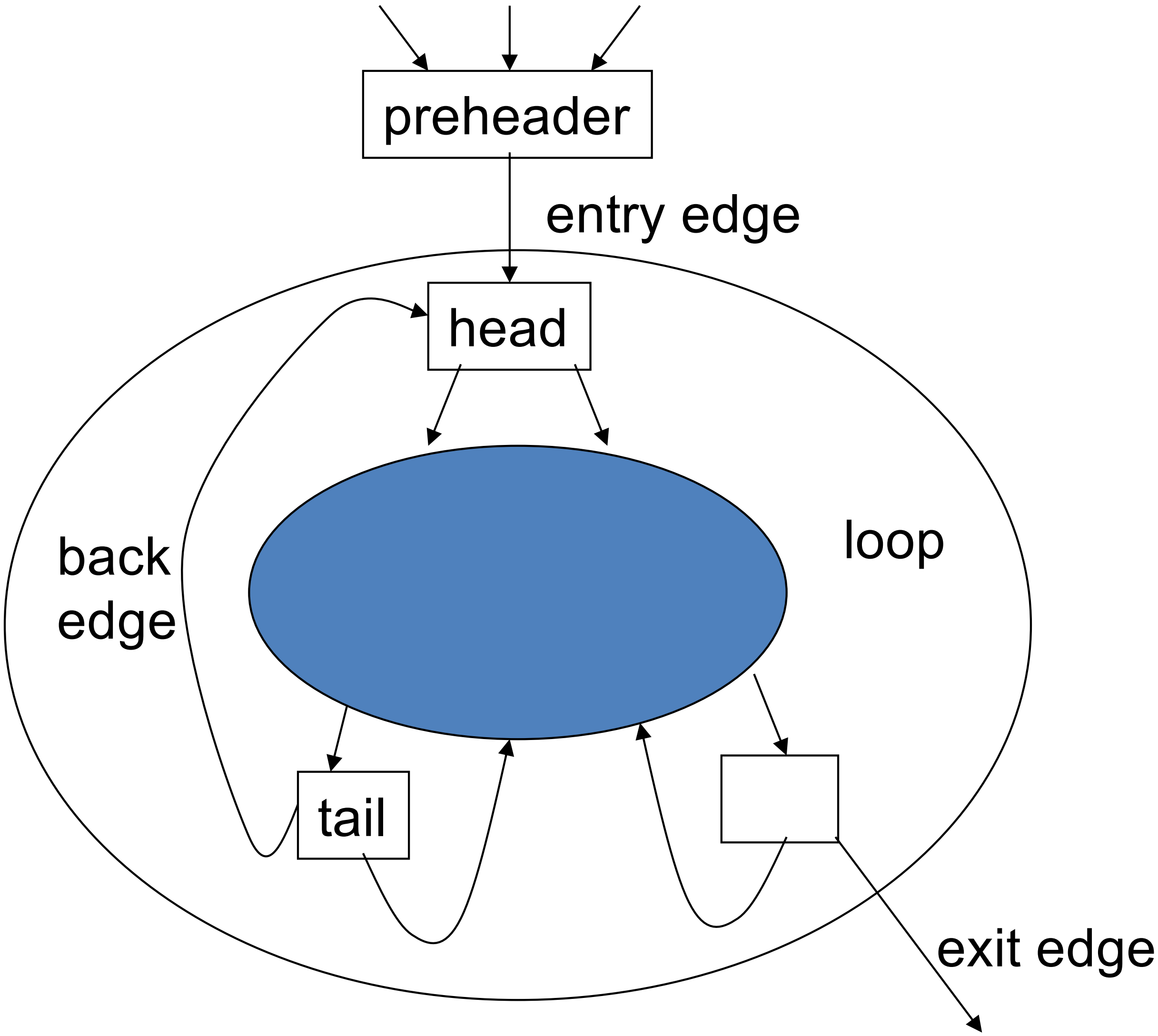
What's a Loop?

- In a control flow graph, a loop is a set of nodes S such that:
  - S includes a *header node* h
  - From any node in S there is a path of directed edges leading to h
  - There is a path from h to any node in S
  - There is no edge from any node outside S to any node in S other than h

Entries and Exits

- In a loop
  - An *entry node* is one with some predecessor outside the loop
  - An *exit node* is one that has a successor outside the loop
- Corollary: A loop may have multiple exit nodes, but only one entry node

Loop Terminology



Finding Loops in Flow Graphs

- We use *dominators* for this
- Recall:
  - Every control flow graph has a unique start node s_0
  - Node x dominates node y if every path from s_0 to y must go through x
  - A node x dominates itself

Calculating Dominator Sets

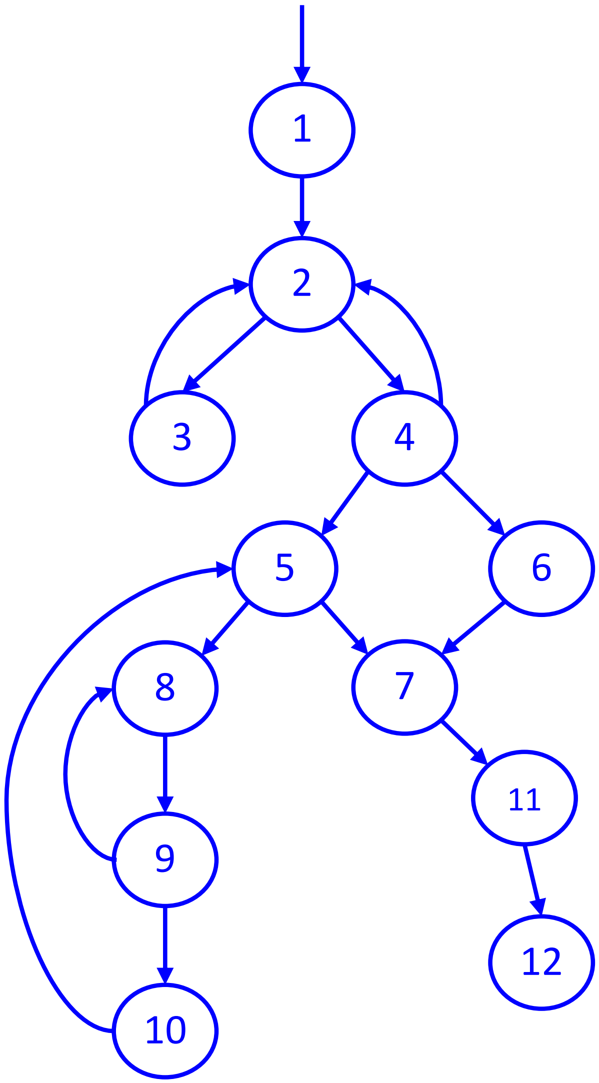
- D[n] is the set of nodes that dominate n
  - $D[s_0] = \{ s_0 \}$
  - $D[n] = \{ n \} \cup (\bigcap_{p \in pred[n]} D[p])$
- Set up an iterative analysis as usual to solve this
  - Except initially each D[n] must be all nodes in the graph – updates make these sets smaller if changed
- WARNING: this is different from the DOM(x) relationship we used with SSA – that was the set of all blocks *dominated by* x (sigh)

Immediate Dominators

- Every node n has a single *immediate dominator* idom(n)
  - idom(n) dominates n
  - idom(n) differs from n – i.e., strictly dominates
  - idom(n) does not dominate any other strict dominator of n
    - i.e., strictly dominates and is nearest dominator
- Fact (er, theorem): If a dominates n and b dominates n, then either a dominates b or b dominates a

  ∴ idom(n) is unique

Example

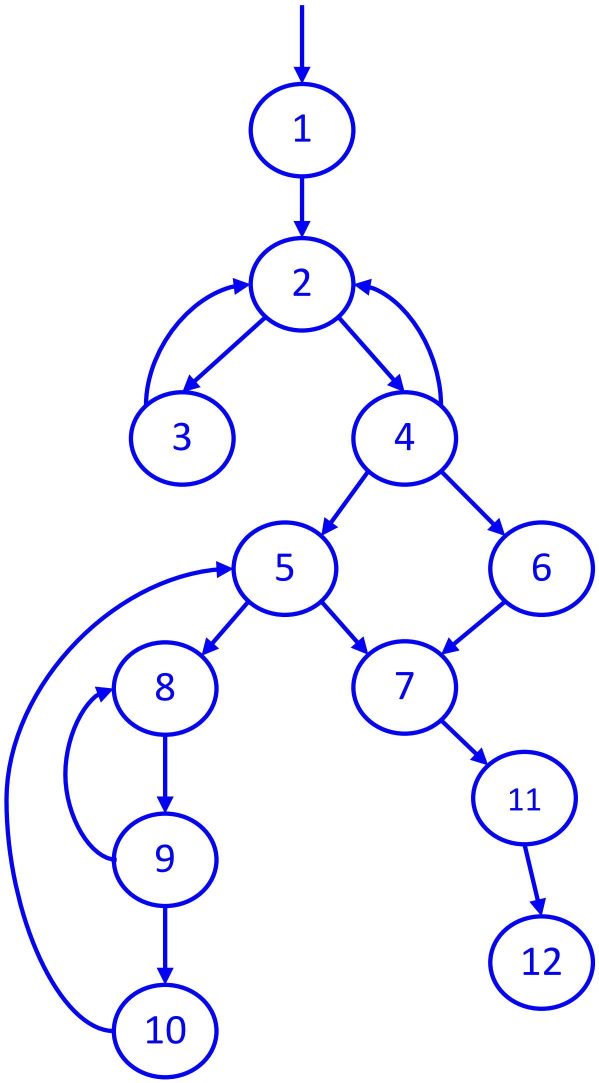


Node	D[node]	IDOM
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

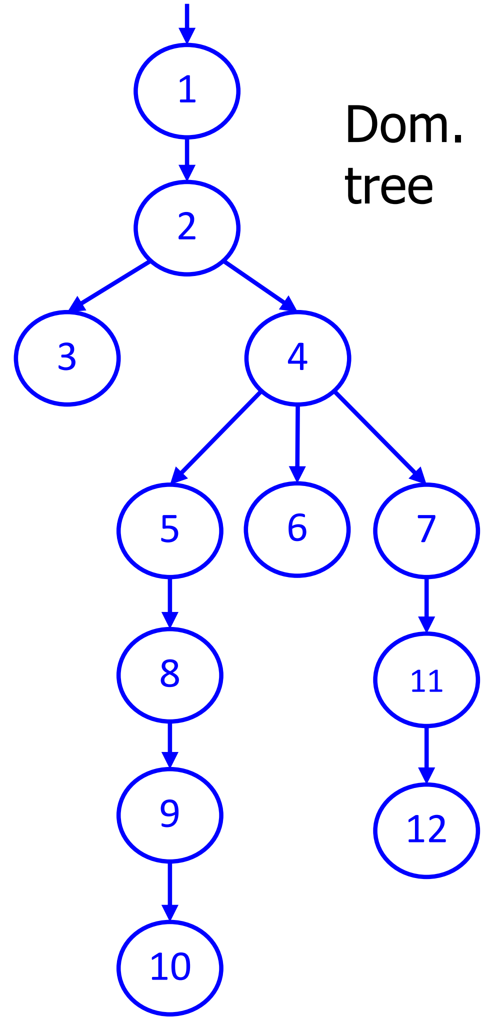
Dominator Tree

- A *dominator tree* is constructed from a flowgraph by drawing an edge between every node in n and the corresponding idom(n)
  - This will be a tree. Why?

Example



Node	D[node]	IDOM
1	1	--
2	1,2	1
3	1,2,3	2
4	1,2,4	2
5	1,2,4,5	4
6	1,2,4,6	4
7	1,2,4,7	4
8	1,2,4,5,8	5
9	1,2,4,5,8,9	8
10	1,2,4,5,8,9,10	9
11	1,2,4,7,11	7
12	1,2,4,7,11,12	11



Dom. tree

Back Edges & Loops

- A flow graph edge from a node n to a node h that dominates n is a *back edge*
  - In our example, from nodes 3 and 4 to 2; from 9 to 8; from 10 to 5
    - (And a node can have a back edge to itself! – although not in our example)
- For every back edge there is a corresponding subgraph of the flow graph that is a loop

Natural Loops

- If h dominates n and n->h is a back edge, then the *natural loop* of that back edge is the set of nodes x such that
  - h dominates x
  - There is a path from x to n not containing h
- h is the *header* of this loop
- Standard loop optimizations can cope with loops whether they are natural or not

Inner Loops

- Inner loops are more important for optimization because most execution time is expected to be spent there
- If two loops share a header, it is hard to tell which one is “inner”
  - Common way to handle this is to merge natural loops with the same header
    - Resulting loop could well not be a “natural loop”

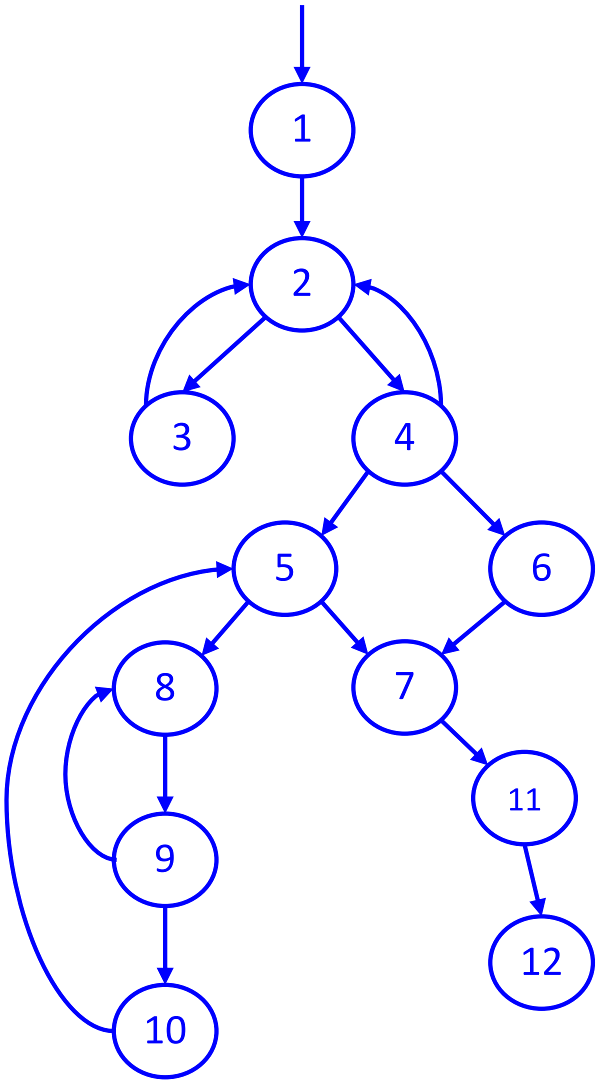
Inner (nested) loops

- Suppose
  - A and B are loops with headers a and b
  - $a \neq b$
  - b is inside A
- Then
  - The nodes of B are a proper subset of A
  - B is nested in A, or B is the *inner loop*

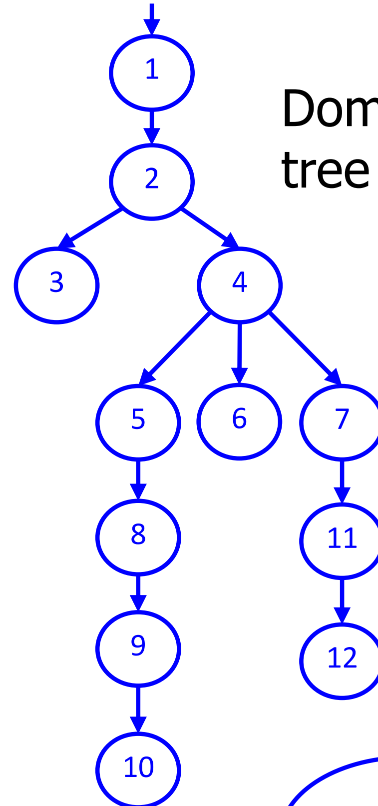
Loop-Nest Tree

- Given a flow graph G
  1. Compute the dominators of G
  2. Construct the dominator tree
  3. Find the natural loops (thus all loop-header nodes)
  4. For each loop header h, merge all natural loops of h into a single loop: loop[h]
  5. Construct a tree of loop headers s.t. h_1 is above h_2 if h_2 is in loop[h_1]

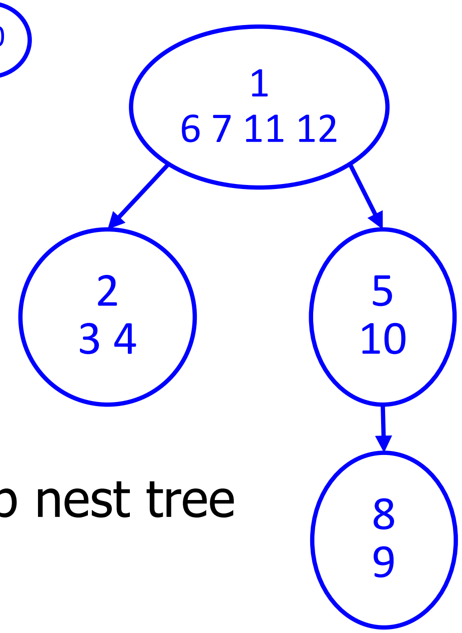
Example



Node	DOM	IDOM
1	1	--
2	1,2	1
3	1,2,3	2
4	1,2,4	2
5	1,2,4,5	4
6	1,2,4,6	4
7	1,2,4,7	4
8	1,2,4,5,8	5
9	1,2,4,5,8,9	8
10	1,2,4,5,8,9,10	9
11	1,2,4,7,11	7
12	1,2,4,7,11,12	11



Dom. tree



Loop nest tree

Loop-Nest Tree details

- Leaves of this tree are the innermost loops
- Need to put all non-loop nodes somewhere
  - Convention: lump these into the root of the loop-nest tree

Loop Preheader

- Often we need a place to park code right before the beginning of a loop
- Easy if there is a single node preceding the loop header h
  - But this isn't the case in general
- So insert a *preheader* node p
  - Include an edge p->h
  - Change all edges x->h to be x->p

Loop-Invariant Computations

- Idea: If x := a1 op a2 always does the same thing each time around the loop, we'd like to *hoist* it and do it once outside the loop
- But can't always tell if a1 and a2 will have the same value
  - Need a conservative (safe) approximation

Loop-Invariant Computations

- d: x := a1 op a2 is *loop-invariant* if for each a_i
  - a_i is a constant, or
  - All the definitions of a_i that reach d are outside the loop, or
  - Only one definition of a_i reaches d, and that definition is loop invariant
- Use this to build an iterative algorithm
  - Base cases: constants and operands defined outside the loop
  - Then: repeatedly find definitions with loop-invariant operands

Hoisting

- Assume that d: x := a1 op a2 is loop invariant. We can hoist it to the loop preheader if
  - d dominates all loop exits where x is live-out, and
  - There is only one definition of x in the loop, and
  - x is not live-out of the loop preheader
- Need to modify this if a1 op a2 could have side effects or raise an exception

Hoisting t:=a op b   Possible?

- Example 1

```
L0: t := 0
L1: i := i + 1
d:  t := a op b
    M[i] := t
    if i < n goto L1
L2: x := t
```

- Example 2

```
L0: t := 0
L1: if i ≥ n goto L2
    i := i + 1
d:  t := a op b
    M[i] := t
    goto L1
L2: x := t
```

Hoisting t:=a op b   Possible?

- Example 3

```
L0: t := 0
L1: i := i + 1
d:  t := a op b
    M[i] := t
    t := 0
    M[j] := t
    if i < n goto L1
L2: x := t
```

- Example 4

```
L0: t := 0
L1: M[j] := t
    i := i + 1
d:  t := a op b
    M[i] := t
     if i < n goto L1
L2: x := t
```

Hoisting t:=a op b Possible?

- Example 1

```
L0: t := 0
L1: i := i + 1
d:  t := a op b
    M[i] := t
    if i < n goto L1
L2: x := t
```

OK

- Example 2

```
L0: t := 0
L1: if i ≥ n goto L2
    i := i + 1
d:  t := a op b
    M[i] := t
    goto L1
L2: x := t
```

Not OK – can't hoist because loop body isn't always executed

Hoisting t:=a op b Possible?

- Example 3

```
L0: t := 0
L1: i := i + 1
d:  t := a op b
    M[i] := t
    t := 0
    M[j] := t
    if i < n goto L1
L2: x := t
```

Not OK – can’t hoist because of multiple assignments to t

- Example 4

```
L0: t := 0
L1: M[j] := t
    i := i + 1
d:  t := a op b
    M[i] := t
    if i < n goto L1
L2: x := t
```

Not OK – can’t hoist because t is used before assigned

Induction Variables

- Suppose inside a loop
  - Variable i is incremented or decremented
  - Variable j is set to i*c+d where c and d are loop-invariant
- Then we can calculate j's value without using i
  - Whenever i is incremented by a, increment j by a*c

Example

- Original

```
      s := 0
      i := 0
L1:   if i ≥ n goto L2
      j := i*4
      k := j+a
      x := M[k]
      s := s+x
      i := i+1
      goto L1
L2:
```

- To optimize, do…
  - Induction-variable analysis to discover i and j are related induction variables
  - Strength reduction to replace *4 with an addition
  - Induction-variable elimination to replace i ≥ n
  - Assorted copy propagation

Result

- Original

```
     s := 0
     i := 0
L1:  if i ≥ n goto L2
     j := i*4
     k := j+a
     x := M[k]
     s := s+x
     i := i+1
     goto L1
L2:
```

- Transformed

```
     s := 0
     k’ = a
     b = n*4
     c = a+b
L1:  if k’ ≥ c goto L2
     x := M[k’]
     s := s+x
     k’ := k’+4
     goto L1
L2:
```

Details are somewhat messy – see your favorite compiler book

Basic and Derived Induction Variables

- Variable i is a *basic induction variable* in loop L with header h if the only definitions of i in L have the form i:=i±c where c is loop invariant
- Variable k is a *derived induction variable* in L if:
  - There is only one definition of k in L of the form k:=j*c or k:=j+d where j is an induction variable and c, d are loop-invariant, *and*
  - if j is a derived variable in the family of i, then:
    - The only definition of j that reaches k is the one in the loop, *and*
    - there is no definition of i on any path between the definition of j and the definition of k

Optimizating Induction Variables

- Strength reduction: if a derived induction variable is defined with j:=i*c, try to replace it with an addition inside the loop
- Elimination: after strength reduction some induction variables are not used or are only compared to loop-invariant variables; delete them
- Rewrite comparisons: If a variable is used only in comparisons against loop-invariant variables and in its own definition, modify the comparison to use a related induction variable

Loop Unrolling

- If the body of a loop is small, much of the time is spent in the “increment and test” code
- Idea: reduce overhead by *unrolling* – put two or more copies of the loop body inside the loop

Loop Unrolling

- Basic idea: Given loop L with header node h and back edges s_i->h
  1. Copy the nodes to make loop L' with header h' and back edges s_i'->h'
  2. Change all back edges in L from s_i->h to s_i->h'
  3. Change all back edges in L' from s_i'->h' to s_i'->h

Unrolling Algorithm Results

- Before

```
L1: x := M[i]
    s := s + x
    i := i + 4
   if i<n goto L1 else L2
L2:
```

- After

```
L1: x := M[i]
    s := s + x
    i := i + 4
    if i<n goto L1’ else L2
L1’:x := M[i]
    s := s + x
    i := i + 4
   if i<n goto L1 else L2
L2:
```

Hmmmm….

- Not so great – just code bloat
- But: use induction variables and various loop transformations to clean up

After Some Optimizations

- Before

```
L1: x := M[i]
    s := s + x
    i := i + 4
    if i<n goto L1’ else L2
L1’:x := M[i]
    s := s + x
    i := i + 4
   if i<n goto L1 else L2
L2:
```

- After

```
L1: x := M[i]
    s := s + x
    x := M[i+4]
    s := s + x
    i := i + 8
    if i<n goto L1 else L2
L2:
```

Still Broken…

- But in a different, better(?) way
- Good code, but only correct if original number of loop iterations was even
- Fix: add an epilogue to handle the “odd” leftover iteration

Fixed

- Before

```
L1:  x := M[i]
     s := s + x
     x := M[i+4]
     s := s + x
     i := i + 8
   if i<n goto L1 else L2
L2:
```

- After

```
     if i<n-8 goto L1 else L2
L1:  x := M[i]
     s := s + x
     x := M[i+4]
     s := s + x
     i := i + 8
     if i<n-8 goto L1 else L2
L2:  x := M[i]
     s := s+x
     i := i+4
     if i < n goto L2 else L3
L3:
```

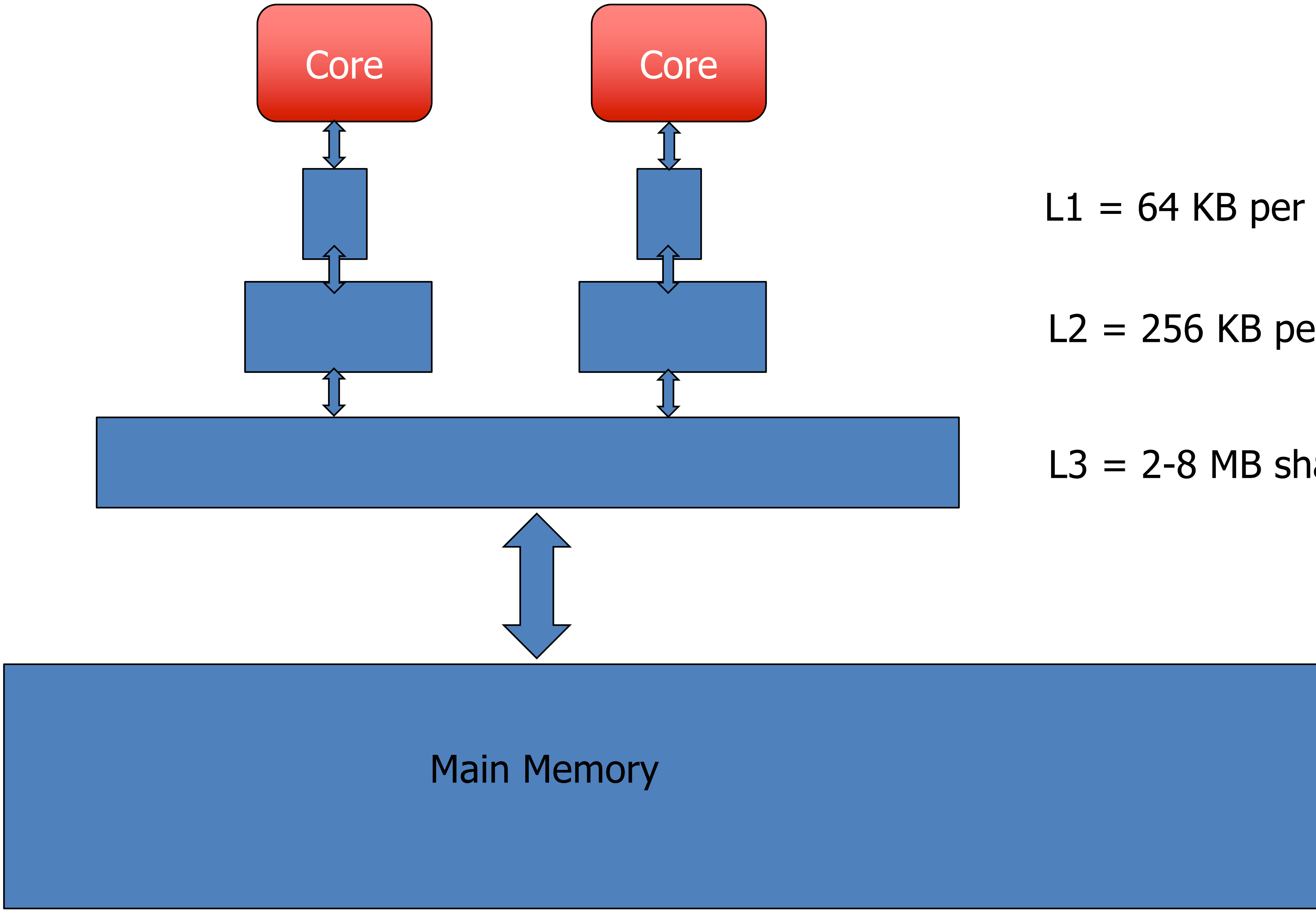
Postscript

- This example only unrolls the loop by a factor of 2
- More typically, unroll by a factor of K
  - Then need an epilogue that is a loop like the original that iterates up to K-1 times

Memory Heirarchies

- One of the great triumphs of computer design
- Effect is a large, fast memory
- Reality is a series of progressively larger, slower, cheaper stores, with frequently accessed data automatically staged to faster storage (cache, main storage, disk)
- Programmer/compiler typically treats it as one large store. (but not always the best idea)
- Hardware maintains cache coherency – most of the time

Intel Haswell Caches (typical example)



L1 = 64 KB per core

L2 = 256 KB per core

L3 = 2-8 MB shared

Just How Slow *is* Operand Access?

• Instruction	~5 per cycle
• Register	1 cycle
• L1 CACHE	~4 cycles
• L2 CACHE	~10 cycles
• L3 CACHE (unshared line)	~40 cycles
• DRAM	~100 ns

Implications

- CPU speed increases have out-paced increases in memory access times
- Memory access now often determines overall execution speed
- “Instruction count” is not the only performance metric for optimization

Memory Issues

- Byte load/store is often slower than whole (physical) word load/store
  - Unaligned access is often extremely slow
- Temporal locality: accesses to recently accessed data will usually find it in the (fast) cache
- Spatial locality: accesses to data near recently used data will usually be fast
  - "near" = in the same cache block
- But – alternating accesses to blocks that map to the same cache block will cause thrashing

Data Alignment

- Data objects (structs) often are similar in size to a cache block (≈ 64 bytes)
  - ∴ Better if objects don’t span blocks
- Some strategies
  - – Allocate objects sequentially; bump to next block boundary if useful
  - – Allocate objects of same common size in separate pools (all size-2, size-4, etc.)
- Tradeoff: speed for some wasted space

Instruction Alignment

- Align frequently executed basic blocks on cache boundaries (or avoid spanning cache blocks)
- Branch targets (particularly loops) may be faster if they start on a cache line boundary
  - Often see multi-byte nops in optimized code as padding to align loop headers
  - How much depends on architecture (typical 16 or 32 bytes)
- Try to move infrequent code (startup, exceptions) away from hot code
- Optimizing compiler can perform basic-block ordering

Loop Interchange

- Watch for bad cache patterns in inner loops; rearrange if possible
- Example

```
for (i = 0; i < m; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < p; k++)
      a[i,k,j] = b[i,j-1,k] + b[i,j,k] + b[i,j+1,k]
```

  – b[i,j+1,k] is reused in the next two iterations, but will have been flushed from the cache by the k loop

Loop Interchange

- Solution for this example: interchange j and k loops

```
for (i = 0; i < m; i++)
  for (k = 0; k < p; k++)
    for (j = 0; j < n; j++)
      a[i,k,j] = b[i,j-1,k] + b[i,j,k] + b[i,j+1,k]
```

  - Now b[i,j+1,k] will be used three times on each cache load
  - Safe here because loop iterations are independent

Loop Interchange

- Need to construct a data-dependency graph showing information flow between loop iterations
- For example, iteration (j,k) depends on iteration (j’,k’) if (j’,k’) computes values used in (j,k) or stores values overwritten by (j,k)
  - If there is a dependency and loops are interchanged, we could get different results – so can’t do it

Blocking

- Consider matrix multiply

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++) {
    c[i,j] = 0.0;
    for (k = 0; k < n; k++)
      c[i,j] = c[i,j] + a[i,k]*b[k,j]
  }
```

- If a, b fit in the cache together, great!
- If they don't, then every b[k,j] reference will be a cache miss
- Loop interchange (i<->j) won't help; then every a[i,k] reference would be a miss

Blocking

- Solution: reuse rows of A and columns of B while they are still in the cache
- Assume the cache can hold 2*c*n matrix elements ($1 < c < n$)
- Calculate $c \times c$ blocks of C using c rows of A and c columns of B

Blocking

- Calculating c × c blocks of C

```
for (i = i0; i < i0+c; i++)
  for (j = j0; j < j0+c; j++) {
    c[i,j] = 0.0;
    for (k = 0; k < n; k++)
      c[i,j] = c[i,j] + a[i,k]*b[k,j]
  }
```

Blocking

- Then nest this inside loops that calculate successive c × c blocks

```
for (i0 = 0; i0 < n; i0+=c)
  for (j0 = 0; j0 < n; j0+=c)
    for (i = i0; i < i0+c; i++)
      for (j = j0; j < j0+c; j++) {
        c[i,j] = 0.0;
        for (k = 0; k < n; k++)
          c[i,j] = c[i,j] + a[i,k]*b[k,j]
      }
```

Parallelizing Code

- There is a large literature about how to rearrange loops for better locality and to detect parallelism
- Some starting points
  - Latest edition of *Dragon book*, ch. 11
  - Allen & Kennedy *Optimizing Compilers for Modern Architectures*
  - Wolfe, *High-Performance Compilers for Parallel Computing*