CSE P 501 – Compilers

Dataflow Analysis

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Agenda

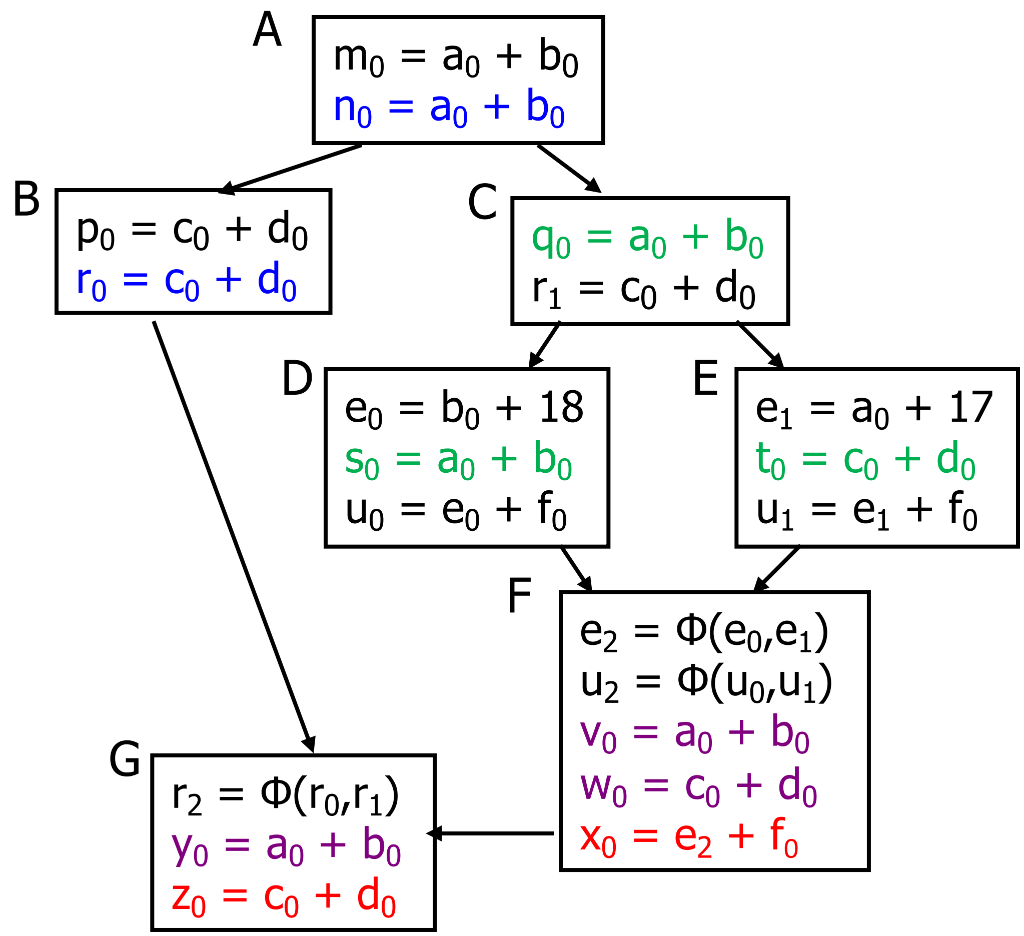
- Dataflow analysis: a framework and algorithm for many common compiler analyses
- Initial example: dataflow analysis for common subexpression elimination
- Other analysis problems that work in the same framework
- Some of these are optimizations we've seen, but now more formally and with details

The Story So Far…

- Redundant expression elimination
  - Local Value Numbering
  - Superlocal Value Numbering
    - Extends VN to EBBs
    - SSA-like namespace
  - Dominator VN Technique (DVNT)
- All of these propagate along forward edges
- None are global
  - In particular, can’t handle back edges (loops)

Dominator Value Numbering

- Most sophisticated algorithm so far
- Still misses some opportunities
- Can't handle loops

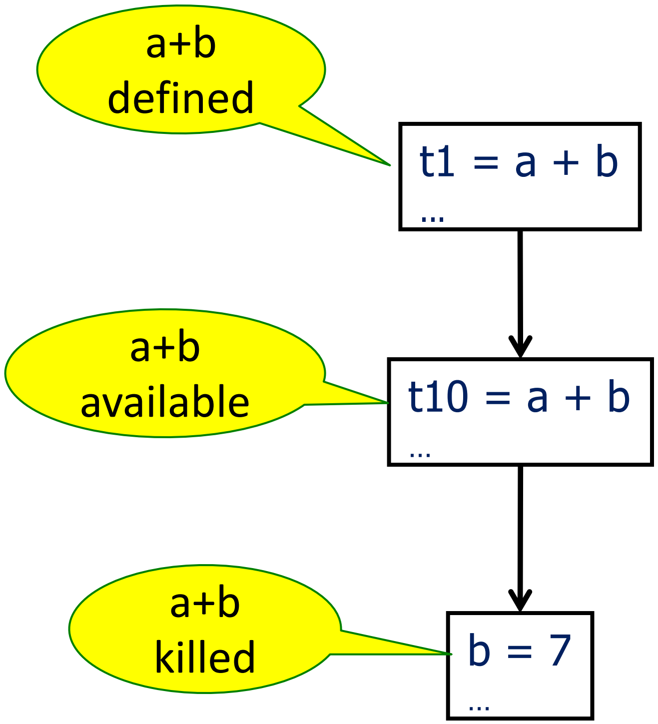


Available Expressions

- Goal: use dataflow analysis to find common subexpressions whose range spans basic blocks
- Idea: calculate *available expressions* at beginning of each basic block
- Avoid re-evaluation of an available expression – use a copy operation

"Available" and Other Terms

- An expression *e* is *defined* at point *p* in the CFG if its value is computed at *p*
  - Sometimes called *definition site*
- An expression *e* is *killed* at point *p* if one of its operands is defined at *p*
  - Sometimes called *kill site*
- An expression *e* is *available* at point *p* if every path leading to *p* contains a prior definition of *e* and *e* is not killed between that definition and *p*



Available Expression Sets

- To compute available expressions, for each block *b*, define
  - AVAIL(b) – the set of expressions available on entry to *b*
  - NKILL(b) – the set of expressions <u>not killed</u> in *b*
    - i.e., all expressions in the program *except* for those killed in *b*
  - DEF(b) – the set of expressions defined in *b* and not subsequently killed in *b*

Computing Available Expressions

- AVAIL(b) is the set

  $\text{AVAIL}(b) = \cap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))$

  - preds(b) is the set of b's predecessors in the CFG
  - The set of expressions available on entry to *b* is the set of expressions that were available at the end of *every* predecessor basic block *x*
  - The expressions available on exit from block *b* are those defined in *b* or available on entry to *b* and not killed in *b*
- This gives a system of simultaneous equations – a dataflow problem

Name Space Issues

- In previous value-numbering algorithms, we used a SSA-like renaming to keep track of versions
- In global dataflow problems, we use the original namespace
  - we require a+b have the same value along *all* paths to its use
  - If a or b is updated along *any* path to its use, then a+b has the “wrong” value
  - so original names are exactly what we want
- The KILL information captures when a value is no longer available

Computing Available Expressions

- Big Picture
  - Build control-flow graph
  - Calculate initial local data – DEF(*b*) and NKILL(*b*)
    - This only needs to be done once for each block *b* and depends only on the statements in *b*
  - Iteratively calculate AVAIL(*b*) by repeatedly evaluating equations until nothing changes
    - Another fixed-point algorithm

Computing DEF and NKILL (1)

- First, figure out which expressions are killed in each block (i.e., clobbered by some assignment later in that block)
- For each block *b* with operations o_1, o_2, …, o_k

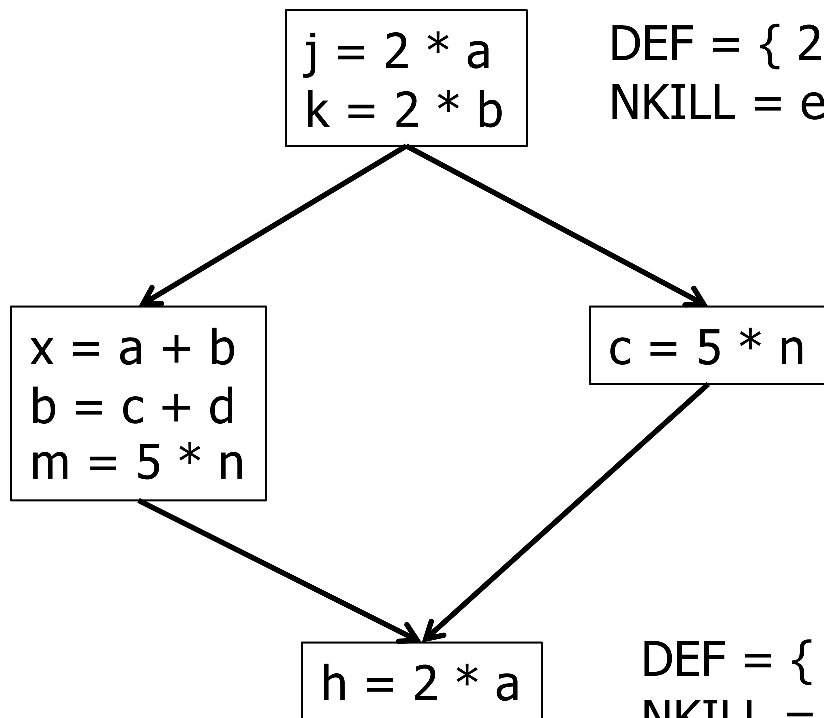
```
KILLED = ∅        // variables killed (later) in b, not expressions
DEF(b) = ∅
for i = k to 1    // note: working back to front
  assume oi is "x = y + z"
  add x to KILLED
  if (y ∉ KILLED and z ∉ KILLED)
    add "y + z" to DEF(b)        // i.e., neither y nor z killed
                                 // after this point in b
…
```

Computing DEF and NKILL (2)

- After computing DEF and KILLED for a block *b*, compute set of all expressions in the program not killed in *b*

```
NKILL(b) = { all expressions }
for each expression e
   for each variable v ∈ e
      if v ∈ KILLED then
         NKILL(b) = NKILL(b) - e
```

Example: Compute DEF and NKILL



DEF = { 2*a, 2*b }
NKILL = exprs w/o j or k

DEF = { 5*n, c+d }
NKILL = exprs w/o m, x, b

DEF = { 5*n }
NKILL = exprs w/o c

DEF = { 2*a }
NKILL = exprs w/o h

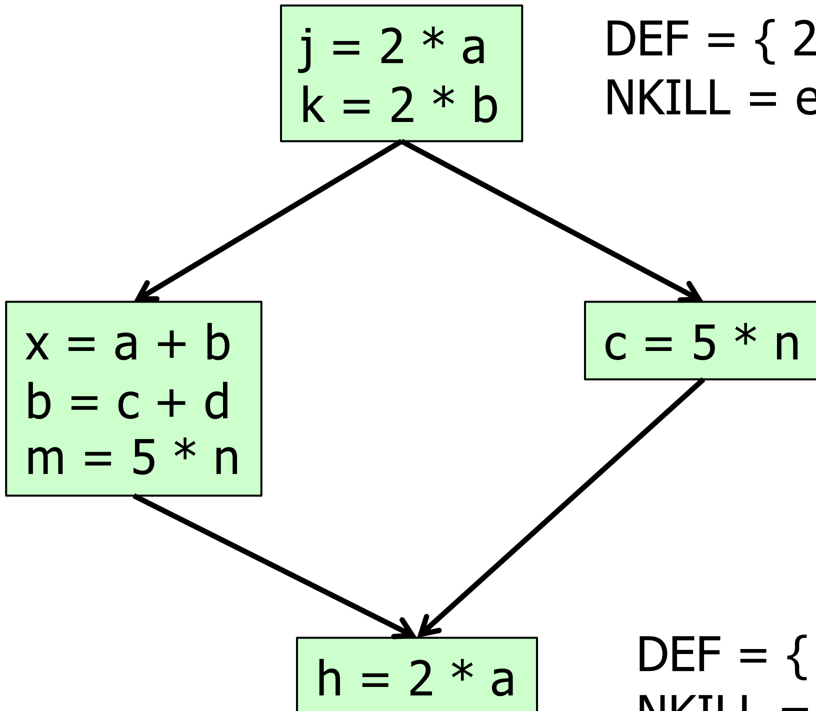
Computing Available Expressions

Once DEF(b) and NKILL(b) are computed for all blocks b

> Worklist = { all blocks b_i }
> while (Worklist $\neq \emptyset$)
>   remove a block *b* from Worklist
>   recompute AVAIL(*b*)
>   if AVAIL(*b*) changed
>   Worklist = Worklist $\cup$ successors(*b*)

Example: Find Available Expressions

$$AVAIL(b) = \bigcap_{x \in preds(b)} (DEF(x) \cup (AVAIL(x) \cap NKILL(x)))$$



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= in worklist

= processing

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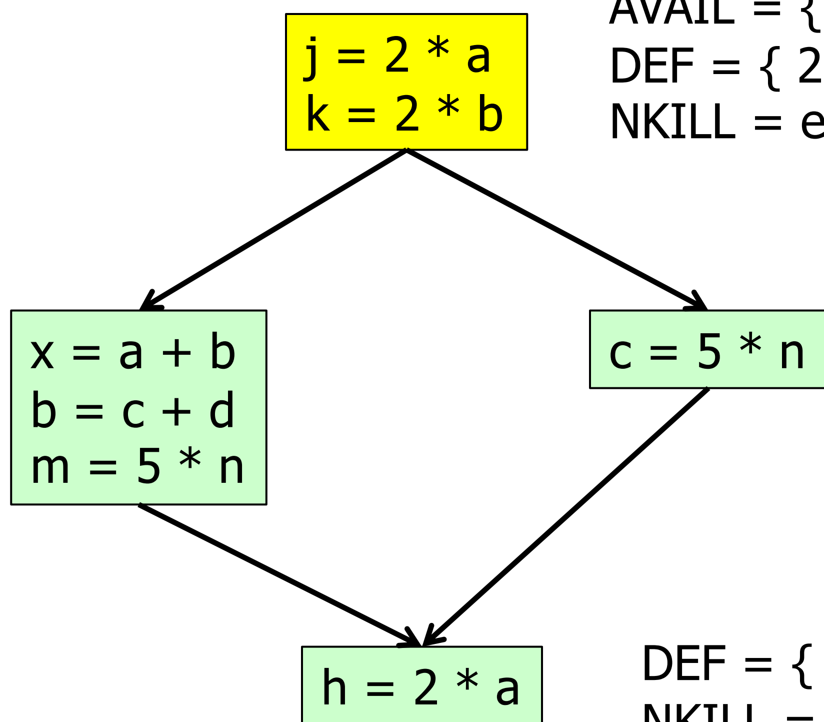

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(green) = in worklist

(yellow) = processing

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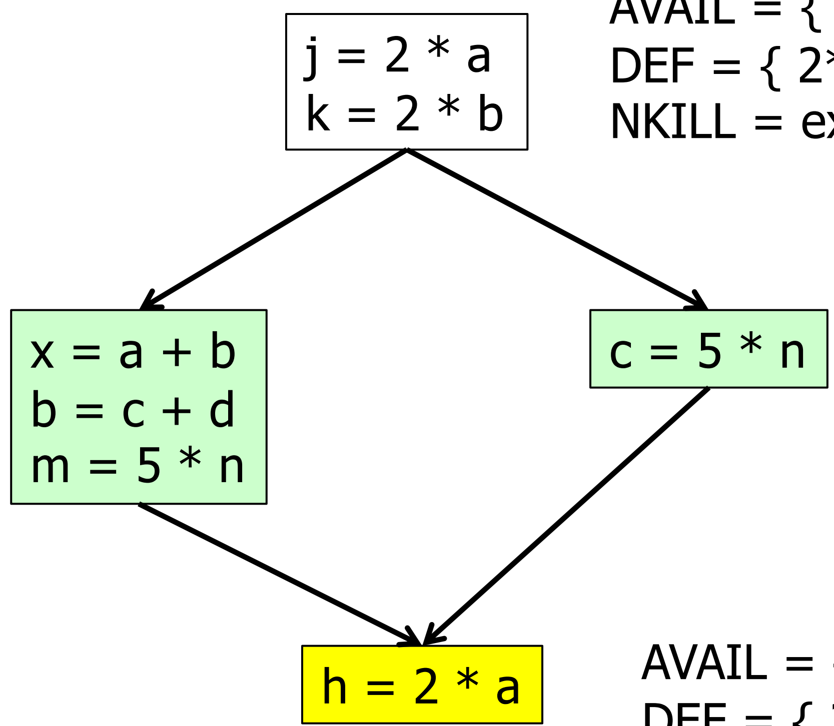

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AVAIL = { 5*n }
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[green box] = in worklist

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Example: Find Available Expressions

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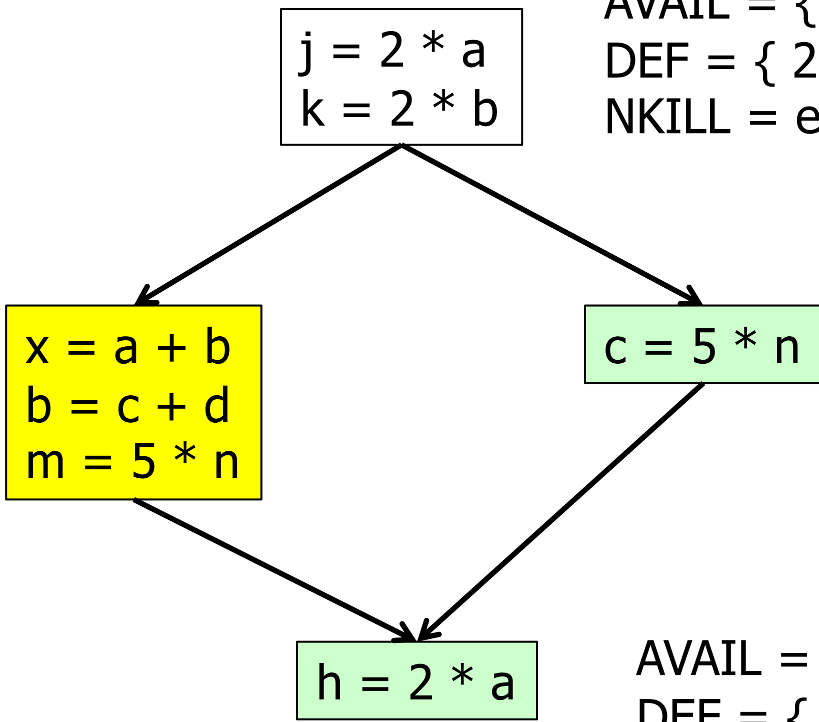

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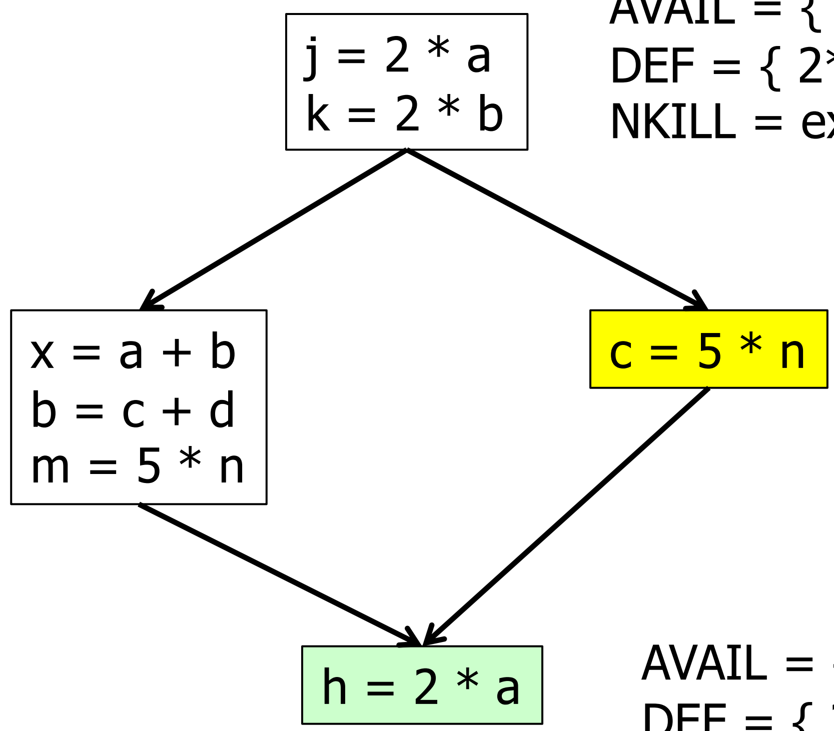

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= in worklist

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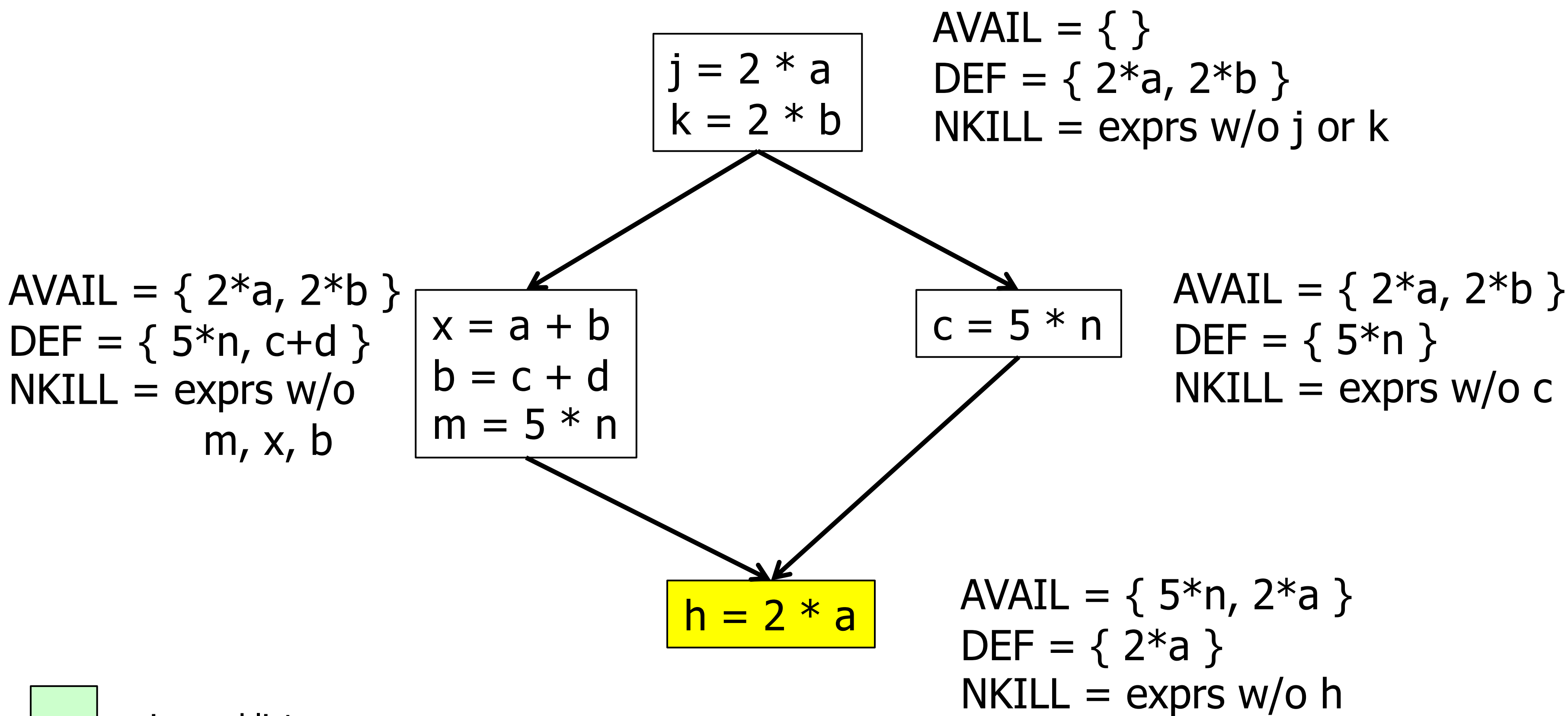
Example: Find Available Expressions

AVAIL(b) = $\cap_{x \in preds(b)}$ (DEF(x) $\cup$ (AVAIL(x) $\cap$ NKILL(x)))



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Example: Find Available Expressions

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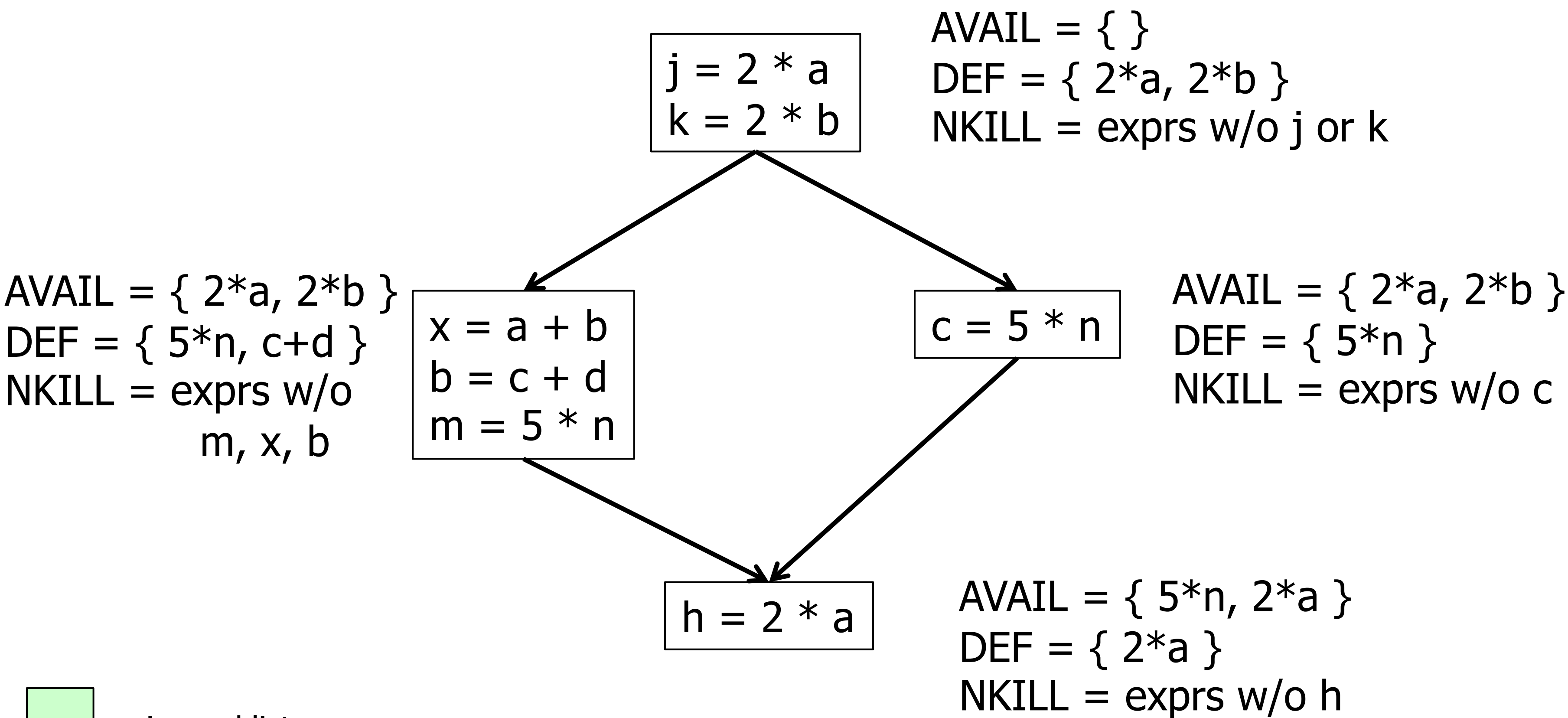

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And the common subexpression is???

Example: Find Available Expressions

$AVAIL(b) = \cap_{x \in preds(b)} (DEF(x) \cup (AVAIL(x) \cap NKILL(x)))$

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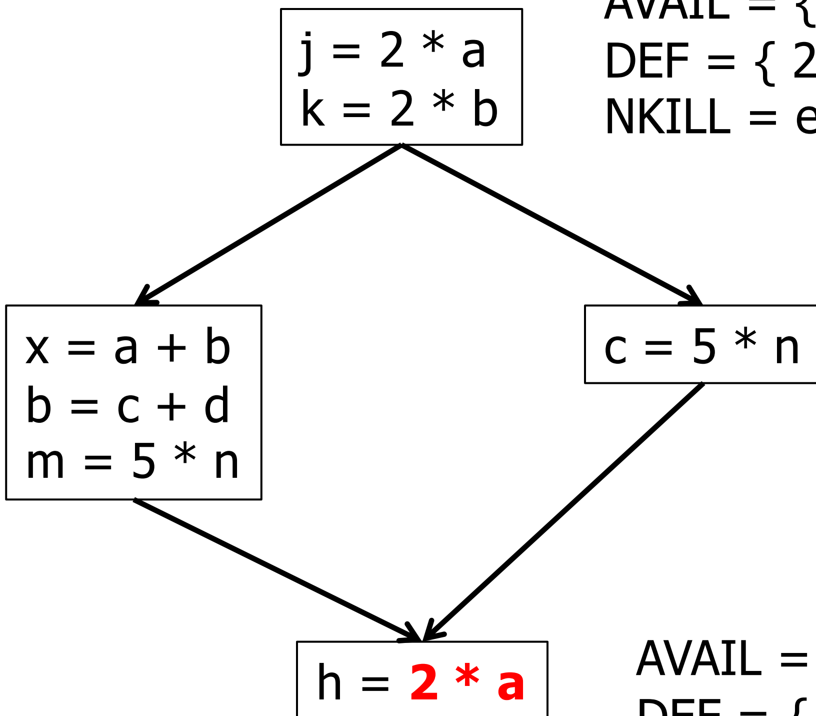


AVAIL = { 2*a, 2*b }
DEF = { 5*n, c+d }
NKILL = exprs w/o m, x, b

AVAIL = { 2*a, 2*b }
DEF = { 5*n }
NKILL = exprs w/o c

AVAIL = { 5*n, **2*a** }
DEF = { 2*a }
NKILL = exprs w/o h

= in worklist

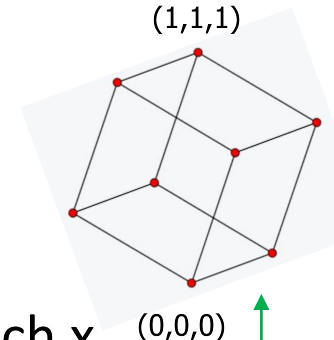
= processing

Example: Find Available Expressions

$\text{AVAIL}(b) = \cap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))$

- Termination?
  - Always
  - AVAIL(*b*) initially all empty
  - In equation above, DEF & NKILL are unchanging, and adding to AVAIL(*x*) can't shrink AVAIL(*b*)
  - Only a finite number of exprs in the program, so the alg is climbing a finite n-cube; can't climb forever
- Order of worklist removals?
  - Any will work
  - Some are faster than others; e.g., if CFG is a DAG, then go in topological order (which would have been faster on the previous example)

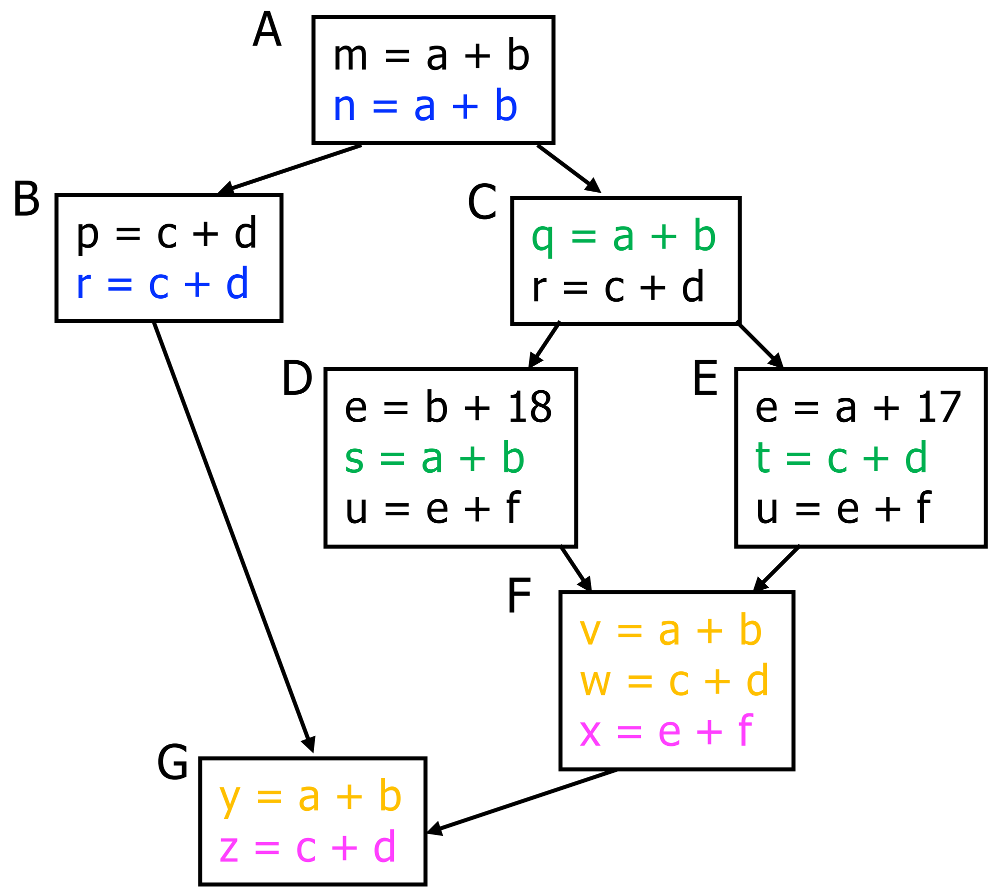
Termination – more generally



- Suppose algorithm has a "state" vector x = $(x_1,x_2,\ldots,x_n)$, each x_i from a *finite*, ordered set, say {0,1} or {1,2,3}
- If each state transition (iteration of an alg, such as prev few slides) allowed, say, x_i to go up while x_j goes down, then ∞ iteration is possible: (0,1) → (1,0) → (0,1) → …
- *BUT*, if alg ensures that, at each iteration, old-x_i ≤ new-x_i, then termination is certain: You can only increase x_i a *finite* number of times before you hit the top value
  - Available expressions: set is bounded by set of all exprs in code
- E.g., if $x_i \in \{0,1\}$, $x = (x_1,x_2,\ldots,x_n)$ are corners of an n-cube; at worst, alg walks from (0,0,…,0) to (1,1,…,1) in ≤ n steps
- Math Jargon: such a structure is typically called a "lattice".

Comparing Algorithms

- LVN – Local Value Numbering
- SVN – Superlocal Value Numbering
- DVN – Dominator-based Value Numbering
- GRE – Global Redundancy Elimination



Comparing Algorithms (2)

- LVN => SVN => DVN form a strict hierarchy – later algorithms find a superset of previous information
- Global RE finds a somewhat different set
  - Discovers e+f in F (computed in both D and E)
  - Misses identical values if they have different names (e.g., a+b and c+d when a=c and b=d)
    - Value Numbering catches this

Scope of Analysis

- Larger context (EBBs, regions, global, interprocedural) sometimes helps
  - More opportunities for optimizations
- But not always
  - Introduces uncertainties about flow of control
  - Usually only allows weaker analysis
  - Sometimes has unwanted side effects
    - Can create additional pressure on registers, for example

Dataflow analysis

- Available expressions is an example of a *dataflow analysis* problem
- Many similar problems can be expressed in a similar framework
- Only the first part of the story – once we've discovered facts, we then need to use them to improve code

Characterizing Dataflow Analysis

- All of these algorithms involve sets of facts about each basic block b

  IN(b) – facts true on entry to b

  OUT(b) – facts true on exit from b

  GEN(b) – facts created and not killed in b

  KILL(b) – facts killed in b
- These are related by the equation

  $$OUT(b) = GEN(b) \cup (IN(b) - KILL(b))$$

  – Solve this iteratively for all blocks
  – Sometimes information propagates forward; sometimes backward
    - But will reach correct solution (fixed point) regardless of order in which blocks are considered

Dataflow Analysis (1)

- A collection of techniques for compile-time reasoning about run-time values
- Almost always involves building a graph
  - Trivial for basic blocks
  - Control-flow graph or derivative for global problems
  - Call graph or derivative for whole-program problems

Dataflow Analysis (2)

- Usually formulated as a set of *simultaneous equations* (dataflow problem)
  - Sets attached to nodes and edges
  - Need a lattice (or semilattice) to describe values
    - In particular, has an appropriate operator to combine values and an appropriate “bottom” or minimal value

Dataflow Analysis (3)

- Desired solution is usually a *meet over all paths* (MOP) solution
  - "What is true on every path from entry"
  - "What can happen on any path from entry"
  - Usually relates to safety of optimization

Dataflow Analysis (4)

- Limitations
  - Precision – "up to symbolic execution"
    - Assumes all paths taken
  - Sometimes cannot afford to compute full solution
  - Arrays – classic analysis treats each array as a single fact
  - Pointers – difficult, expensive to analyze
    - Imprecision rapidly adds up
    - But gotta do it to effectively optimize things like C/C++
- For scalar values we can quickly solve simple problems

Example:Live Variable Analysis

- A variable *v* is *live* at point *p* iff there is *any* path from *p* to a use of *v* along which *v* is not redefined
- Some uses:
  - Register allocation – only live variables need a register
  - Eliminating useless stores – if variable not live at store, then stored variable will never be used
  - Detecting uses of uninitialized variables – if live at declaration (before initialization) then it might be used uninitialized
  - Improve SSA construction – only need Φ-function for variables that are live in a block (later)

Liveness Analysis Sets

- For each block b, define
  - use[*b*] = variable used in *b* before any def
  - def[*b*] = variable defined in *b* & not killed
  - in[*b*] = variables live on entry to *b*
  - out[*b*] = variables live on exit from *b*

Equations for Live Variables

- Given the preceding definitions, we have

  $\text{in}[b] = \text{use}[b] \cup (\text{out}[b] - \text{def}[b])$

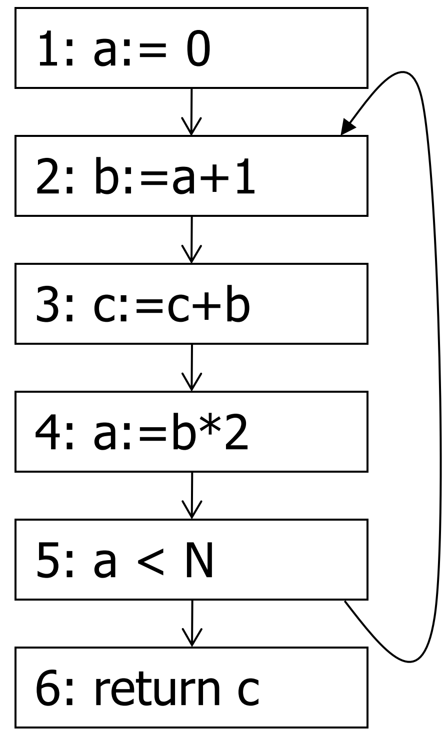
  $\text{out}[b] = \cup_{s \in \text{succ}[b]} \text{in}[s]$

- Algorithm
  - Set $\text{in}[b] = \text{out}[b] = \emptyset$
  - Update in, out until no change

Example (1 stmt per block)

- Code

```
    a := 0
L:  b := a+1
    c := c+b
    a := b*2
    if a < N goto L
    return c
```

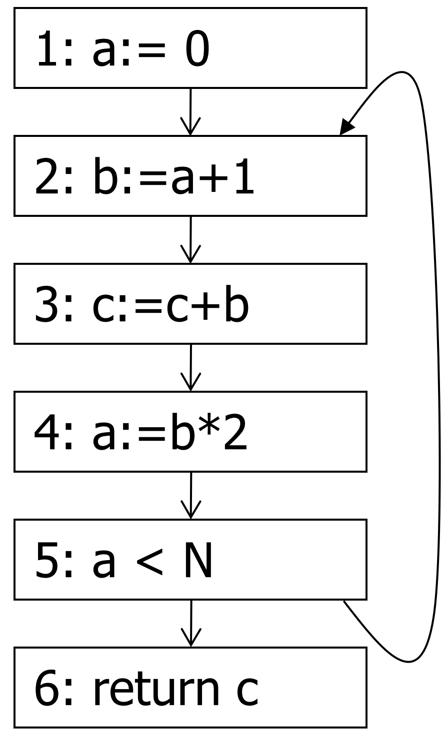


$in[b] = use[b] \cup (out[b] - def[b])$

$out[b] = \cup_{s \in succ[b]} in[s]$

Calculation

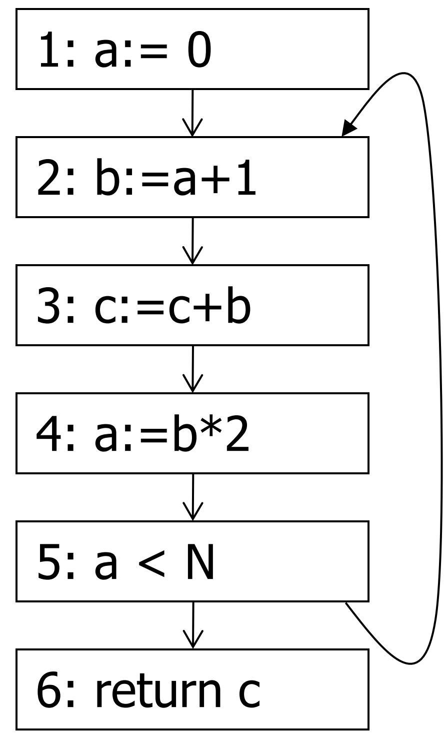
			I		II		III	
block	use	def	out	in	out	in	out	in
6								
5								
4								
3								
2								
1								



$$in[b] = use[b] \cup (out[b] - def[b])$$
$$out[b] = \cup_{s \in succ[b]} in[s]$$

Calculation

			I		II		III	
block	use	def	out	in	out	in	out	in
6	c	--	--	c	--	c		
5	a	--	c	a,c	a,c	a,c		
4	b	a	a,c	b,c	a,c	b,c		
3	b,c	c	b,c	b,c	b,c	b,c		
2	a	b	b,c	a,c	b,c	a,c		
1	--	a	a,c	c	a,c	c		



$$in[b] = use[b] \cup (out[b] - def[b])$$
$$out[b] = \cup_{s \in succ[b]} in[s]$$

Equations for Live Variables v2

- Many problems have more than one formulation. For example, Live Variables…
- Sets
  - USED(b) – variables used in b before being defined in b
  - NOTDEF(b) – variables not defined in b
  - LIVE(b) – variables live on *exit* from b
- Equation

$$\text{LIVE}(b) = \cup_{s \in \text{succ}(b)} \text{USED}(s) \cup (\text{LIVE}(s) \cap \text{NOTDEF}(s))$$

Efficiency of Dataflow Analysis

- The algorithms eventually terminate, but the expected time needed can be reduced by picking a good order to visit nodes in the CFG
  - Forward problems – reverse postorder
  - Backward problems – postorder

Example: Reaching Definitions

- A definition *d* of some variable *v* ***reaches*** operation *i* iff *i* reads the value of *v* and there is a path from *d* to *i* that does not define *v*
- Uses
  - Find all of the possible definition points for a variable in an expression

Equations for Reaching Definitions

- Sets
  - DEFOUT(b) – set of definitions in b that reach the end of b (i.e., not subsequently redefined in b)
  - SURVIVED(b) – set of all definitions not obscured by a definition in b
  - REACHES(b) – set of definitions that reach b
- Equation

$$REACHES(b) = \cup_{p \in preds(b)} DEFOUT(p) \cup (REACHES(p) \cap SURVIVED(p))$$

Example: Very Busy Expressions

- An expression *e* is considered *very busy* at some point *p* if *e* is evaluated and used along every path that leaves *p*, and evaluating *e* at *p* would produce the same result as evaluating it at the original locations
- Uses
  - Code hoisting – move *e* to *p* (reduces code size; no effect on execution time)

Equations for Very Busy Expressions

- Sets
  - USED(b) – expressions used in b before they are killed
  - KILLED(b) – expressions redefined in b before they are used
  - VERYBUSY(b) – expressions very busy on exit from b
- Equation

$$\text{VERYBUSY}(b) = \bigcap_{s \in succ(b)} \text{USED}(s) \cup (\text{VERYBUSY}(s) - \text{KILLED}(s))$$

Using Dataflow Information

- A few examples of possible transformations…

Classic Common-Subexpression Elimination (CSE)

- In a statement s: *z* := *x* op *y*, if *x* op *y* is *available* at s then it need not be recomputed
- Analysis: compute *reaching expressions* i.e., statements n: *v* := *x* op *y* such that the path from n to s does not compute *x* op *y* or define *x* or *y*

Classic CSE Transformation

- If x op y is defined at n and reaches s
  - Create new temporary t_i
  - Rewrite n: v := x op y as

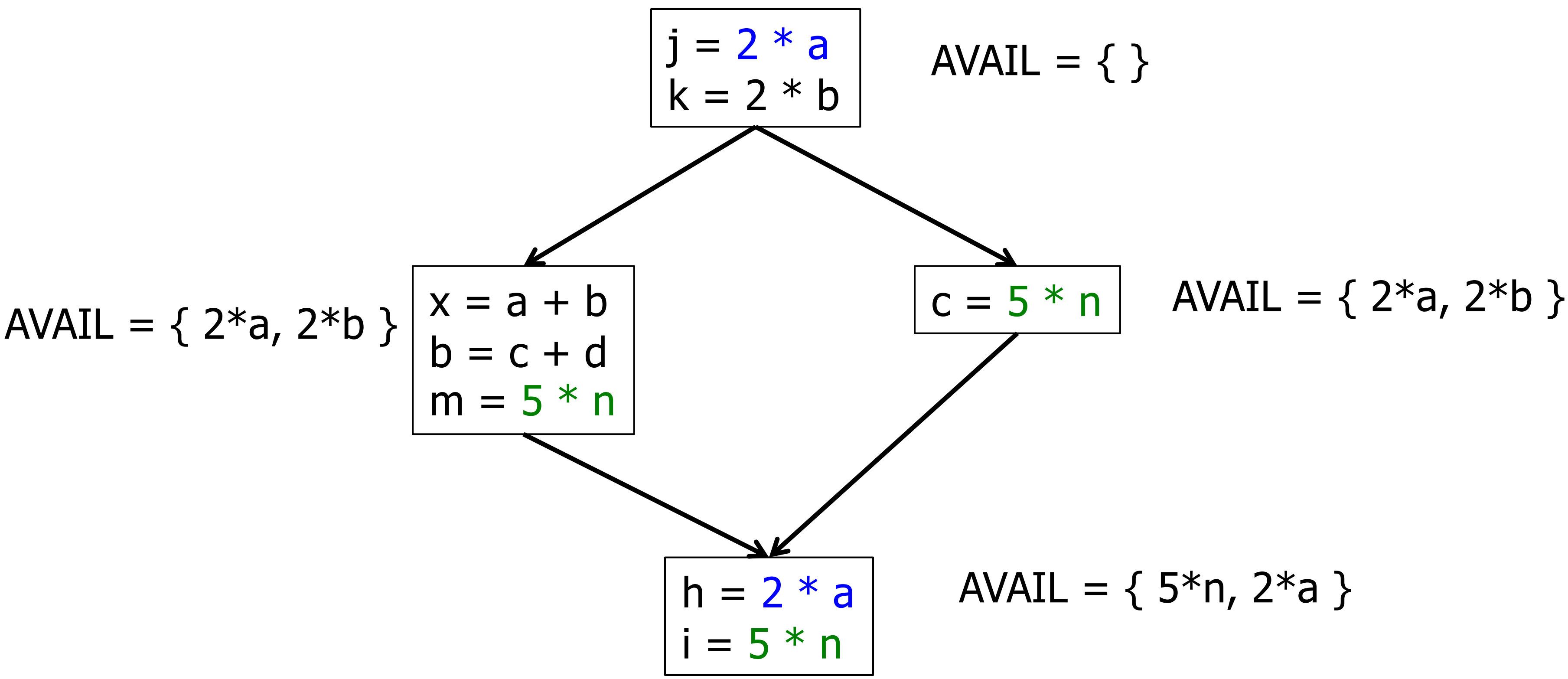
    n: t_i := x op y

    n': v := t_i
  - Rewrite statement s: z := x op y to be

    s: z := t_i

  - (Rely on copy propagation to remove extra assignments that are not really needed)

Revisiting Example (w/small change)



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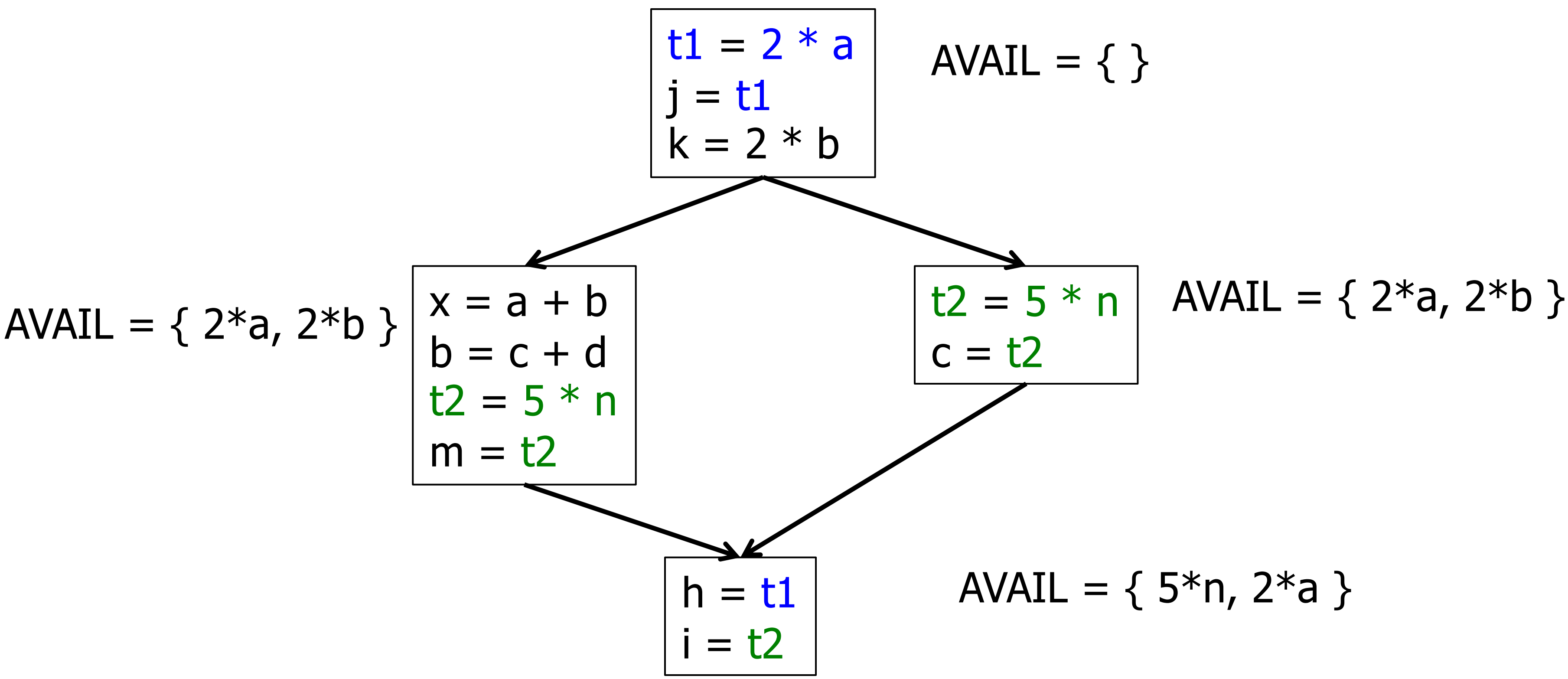
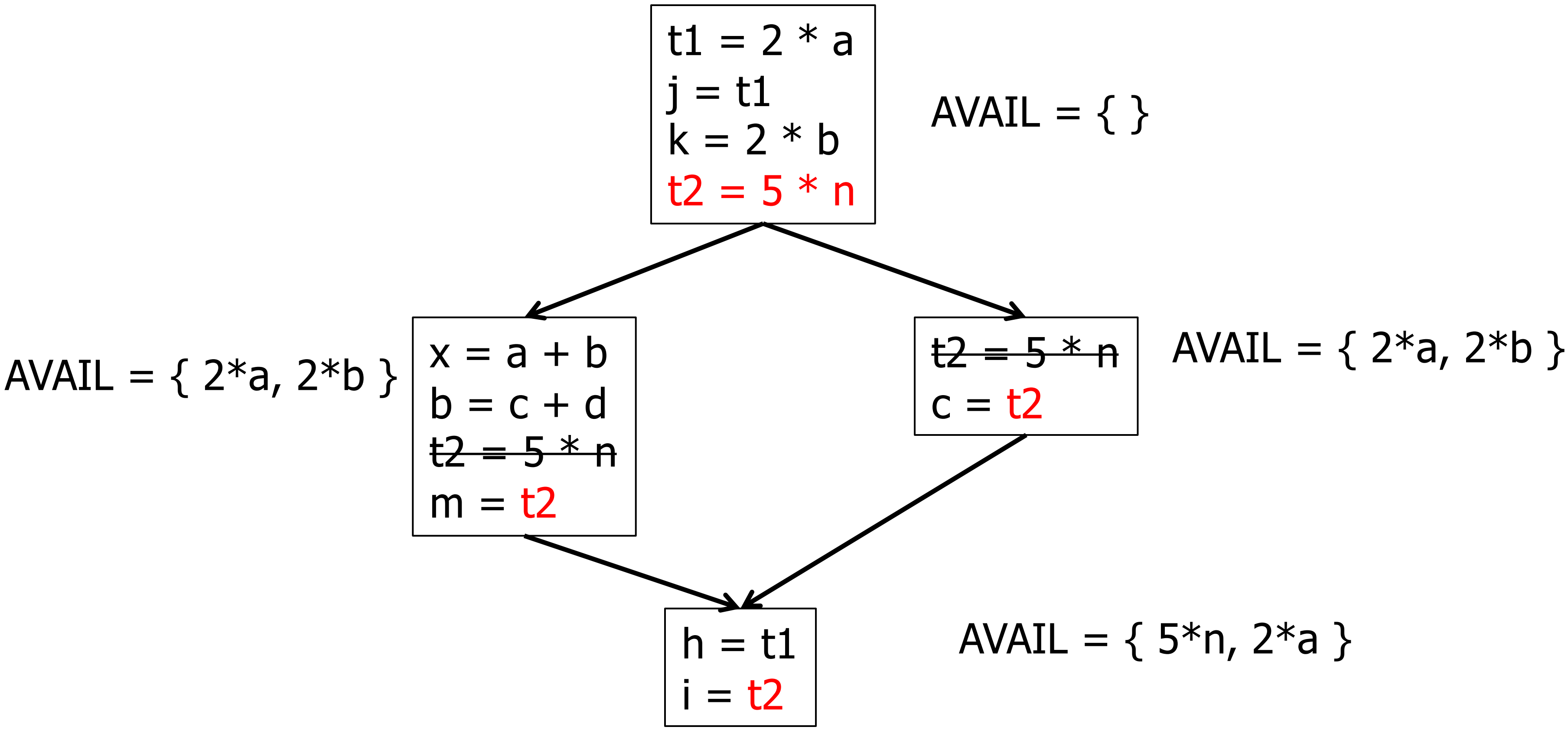


Then Apply Very Busy…



Constant Propagation

- Suppose we have
  - Statement d: *t* := *c*, where *c* is constant
  - Statement n that uses *t*
- If d reaches n and no other definitions of *t* reach n, then rewrite n to use *c* instead of *t*

Copy Propagation

- Similar to constant propagation
- Setup:
  - Statement d: *t* := *z*
  - Statement n uses *t*
- If d reaches n and no other definition of *t* reaches n, and there is no definition of *z* on any path from d to n, then rewrite n to use *z* instead of *t*
  - Recall that this can help remove dead assignments

Copy Propagation Tradeoffs

- Downside is that this can increase the lifetime of variable *z* and increase need for registers or memory traffic
- But it can expose other optimizations, e.g.,

```
a := y + z
u := y
c := u + z        // copy propagation makes this y + z
```

  – After copy propagation we can recognize the common subexpression

Dead Code (Assignment) Elimination

- If we have an instruction

  s: *a* := *b* op *c*

  and *a* is not live-out after s, then s can be eliminated
  - Provided it has no implicit side effects that are visible (output, exceptions, etc.)
    - If *b* or *c* are function calls, they have to be assumed to have unknown side effects unless the compiler can prove otherwise

Aliases

- A variable or memory location may have multiple names or *aliases*
  - Call-by-reference parameters
  - Variables whose address is taken (&x)
  - Expressions that dereference pointers (p.x, *p)
  - Expressions involving subscripts (a[i])
  - Variables in nested scopes

Aliases vs Optimizations

- Example:

  p.x := 5; q.x := 7; a := p.x;

  - Does reaching definition analysis show that the definition of p.x reaches a?
  - (Or: do p and q refer to the same variable/object?)
  - (Or: *can* p and q refer to the same thing?)

Aliases vs Optimizations

- Example

```
int f(int *p, int *q) {
  *p = 1; *q = 2;
  return *p;
}
```

  – How do we account for the possibility that p and q might refer to the same thing?
  – Safe approximation: since it's possible, assume it is true (but rules out a lot)
    - C programmers can use “restrict” to indicate no other pointer is an alias for this one

Types and Aliases (1)

- In Java, ML, MiniJava, and others, if two variables have incompatible types they cannot be names for the same location
  - Also helps that programmer cannot create arbitrary pointers to storage in these languages

Types and Aliases (2)

- Strategy: Divide memory locations into *alias classes* based on type information (every type, array, record field is a class)
- Implication: need to propagate type information from the semantics pass to optimizer
  - Not normally true of a minimally typed IR
- Items in different alias classes cannot refer to each other

Aliases and Flow Analysis

- Idea: Base alias classes on points where a value is created
  - Every new/malloc and each local or global variable whose address is taken is an alias class
  - Pointers can refer to values in multiple alias classes (so each memory reference is to a set of alias classes)
  - Use to calculate "may alias" information (e.g., *p* "may alias" *q* at program point s)

Using "may-alias" information

- Treat each alias class as a "variable" in dataflow analysis problems
- Example: framework for available expressions
  - Given statement s: M[a]:=b,

    gen[s] = { }

    kill[s] = { M[x] | a may alias x at s }

May-Alias Analysis

- Without alias analysis, #2 kills M[t] since x and t might be related
- If analysis determines that “x may-alias t” is false, M[t] is still available at #3; can eliminate the common subexpression and use copy propagation

- Code
  1: u := M[t]
  2: M[x] := r
  3: w := M[t]
  4: b := u+w

Where are we now?

- Dataflow analysis is the core of classical optimizations
  - Although not the only possible story
- Still to explore:
  - Discovering and optimizing loops
  - SSA – Static Single Assignment form