CSE P 501 – Compilers

Value Numbering & Optimizations

Hal Perkins

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Agenda

- Optimization (Review)
  - Goals
  - Scope: local, superlocal, regional, global (intraprocedural), interprocedural
- Control flow graphs (reminder)
- Value numbering
- Dominators
- Ref.: Cooper/Torczon ch. 8

Code Improvement (1)

- Pick a better algorithm(!)
- Use machine resources efficiently
  - Instructions, registers
  - More later…

Code Improvement (2)

- Local optimizations – basic blocks
  - Algebraic simplifications
  - Constant folding
  - Common subexpression elimination (i.e., redundancy elimination)
  - Dead code elimination
  - Specialize computation based on context
  - etc., etc., …

Code Improvement (3)

- Global optimizations
  - Code motion
  - Moving invariant computations out of loops
  - Strength reduction (replace multiplications by repeated additions, for example)
  - Global common subexpression elimination
  - Global register allocation
  - Many others…

“Optimization”

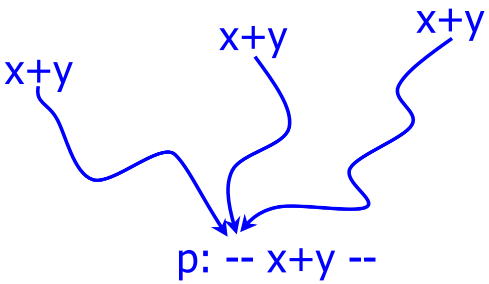
- None of these improvements are truly “optimal”
  - Hard problems (in theory-of-computation sense)
  - Proofs of optimality assume artificial restrictions
- Best we can do is to improve things
      - Most (much?) (some?) of the time
  - Realistically: try to do better for common idioms both in the code and on the machine

Optimization Phase

- Goal
  - Discover, at compile time, information about the runtime behavior of the program, and use that information to improve the generated code

A First Running Example: Redundancy Elimination

- An expression x+y is *redundant* at a program point iff, along every path from the procedure's entry, it has been evaluated and its constituent subexpressions (x and y) have not been redefined
- If the compiler can prove the expression is redundant:
  - Can store the result of the earlier evaluation
  - Can replace the redundant computation with a reference to the earlier (stored) result



Common Pattern for Code Improvement

- Typical for most compiler optimizations
- First, discover opportunities through program analysis
- Then, modify the IR to take advantage of the opportunities
  - Historically, goal usually was to decrease execution time
  - Other possibilities: reduce space, power, …

Issues (1)

- Safety – transformation must not change program meaning
  - Must generate correct results
  - Can't generate spurious errors
  - Optimizations must be conservative
  - Large part of analysis goes towards proving safety
  - Can pay off to speculate (be optimistic) but then need to recover if reality is different

Issues (2)

- Profitibility
  - If a transformation is possible, is it profitable?
  - Example: loop unrolling
    - Can increase amount of work done on each iteration, i.e., reduce loop overhead
    - Can eliminate duplicate operations done on separate iterations

Issues (3)

- Downside risks
  - Even if a transformation is generally worthwhile, need to think about potential problems
  - Example:
    - Transformation might need more temporaries, putting additional pressure on registers
    - Increased code size could cause cache misses, or, in bad cases, increase page working set

Example: Value Numbering

- Technique for eliminating redundant expressions: assign an identifying number VN(n) to each expression
  - VN(x+y)=VN(j) if x+y and j have the same value
  - Use hashing over value numbers for effeciency
- Old idea (Balke 1968, Ershov 1954)
  - Invented for low-level, linear IRs
  - Equivalent methods exist for tree IRs, e.g., build a DAG

Uses of Value Numbers

- Improve the code
  - Replace redundant expressions
  - Simplify algebraic identities
  - Discover, fold, and propagate constant valued expressions

Local Value Numbering

- Algorithm
  - For each operation o = <op, o1,o2> in a basic block
    - 1. Get value numbers for operands from hash lookup
    - 2. Hash <op, VN(o1), VN(o2)> to get a value number for o (If op is commutative, sort VN(o1), VN(o2) first)
    - 3. If o already has a value number, replace o with a reference to the value
    - 4. If o1 and o2 are constant, evaluate o at compile time and replace with an immediate load
- If hashing behaves well, this runs in linear time

Example

Original	With VNs	Rewritten
a = x + y	$a^3 = x^1 + y^2$	$a^3 = x^1 + y^2$
b = x + y	$b^3 = x^1 + y^2$	$b^3 = a^3$
a = 17	$a^4 = 17^4$	$a^4 = 17^4$
c = x + y	$c^3 = x^1 + y^2$	$c^3 = a^3$
		WHOOPS!

VN table

expr	vn
x	1
y	2
<+ 1 2>	3
a	3
b	3
17	4
a	4
c	3

Bug in Simple Example

- If we use the original names, we get in trouble when a name is reused
- Solutions
  - Be clever about which copy of the value to use (e.g., use c=b in last statement)
  - Create an extra temporary
  - Rename around it (best!)

Renaming

- Idea: give each value a unique name

  a_i^j means i^{th} definition of a with VN = j
- Somewhat complex notation, but meaning is reasonably clear
- This is the idea behind SSA (Static Single Assignment)
  - Popular modern IR – exposes many opportunities for optimizations
  - Key is keeping track of values separately from memory locations (variable names)

Example

Original	With VNs	Rewritten
a = x + y	$a_0^3 = x_0^1 + y_0^2$	$a_0^3 = x_0^1 + y_0^2$
b = x + y	$b_0^3 = x_0^1 + y_0^2$	$b_0^3 = a_0^3$
a = 17	$a_1^4 = 17^4$	$a_1^4 = 17^4$
c = x + y	$c_0^3 = x_0^1 + y_0^2$	$c_0^3 = a_0^3$
		AHHHH…

VN table

expr	vn
x_0	1
y_0	2
<+ 1 2>	3
a_0	3
b_0	3
17	4
a_1	4
c_0	3

Notation: x_i^j means
x version i with VN j

Simple Extensions to Value Numbering

- Constant folding
  - Add a bit that records when a value is constant
  - Evaluate constant values at compile time
  - Replace op with load immediate
- Algebraic identities: x+0, x*1, x-x, …
  - Many special cases
    - Switch on op to narrow down checks needed
    - Replace result with input VN

Larger Scopes

- This algorithm works on straight-line blocks of code (basic blocks)
  - Best possible results for single basic blocks
  - Loses all information when control flows to another block
- To go further we need to represent multiple blocks of code and the control flow between them

Control Flow Graph (CFG) reminder

- Nodes: basic blocks
  - Key property: all statements executed sequentially if any are
- Edges: include a directed edge from n1 to n2 if there is *any* possible way for control to transfer from block n1 to n2 during execution

Optimization Categories (1)

- *Local methods*
  - Usually confined to basic blocks
  - Simplest to analyze and understand
  - Most precise information

Optimization Categories (2)

- *Superlocal methods*
  - Operate over *Extended Basic Blocks* (EBBs)
    - An EBB is a set of blocks b_1, b_2, …, b_n where b_1 has multiple predecessors and each of the remaining blocks b_i ($2 \leq i \leq n$) have only b_{i-1} as its unique predecessor
    - The EBB is entered only at b_1, but may have multiple exits
    - A single block b_i can be the head of multiple EBBs (these EBBs form a tree rooted at b_i)
  - Use information discovered in earlier blocks to improve code in successors

Optimization Categories (3)

- *Regional methods*
  - Operate over scopes larger than an EBB but smaller than an entire procedure/function/method
  - Typical example: loop body
  - Difference from superlocal methods is that there may be merge points in the graph (i.e., a block with two or more predecessors)
    - Facts true at merge point are facts known to be true on all possible paths to that point

Optimization Categories (4)

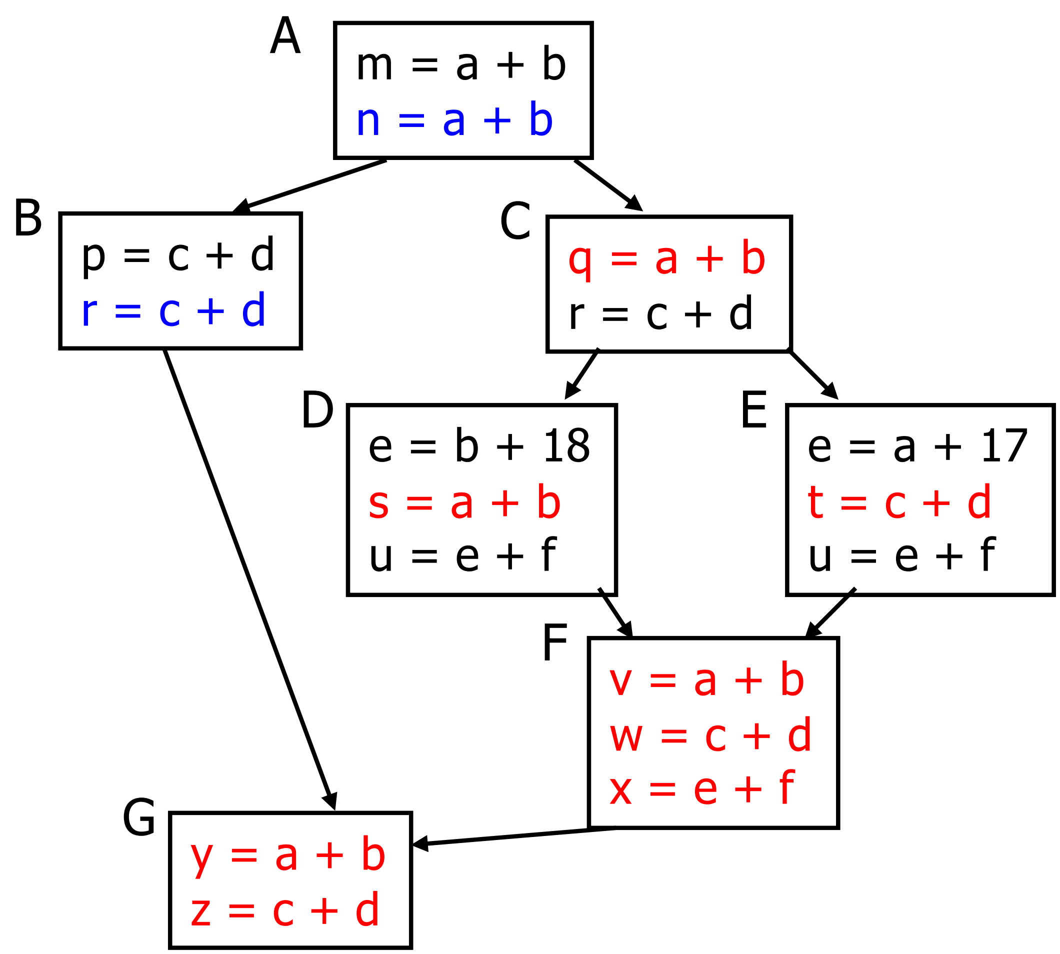
- *Global methods*
  - Operate over entire procedures
  - Sometimes called *intraprocedural* methods
  - Motivation is that local optimizations sometimes have bad consequences in larger context
  - Procedure/method/function is a natural unit for analysis, separate compilation, etc.
  - Almost always need global *data-flow* analysis information for these

Optimization Categories (5)

- *Whole-program methods*
  - Operate over more than one procedure
  - Sometimes called *interprocedural* methods
  - Challenges: name scoping and parameter binding issues at procedure boundaries
  - Classic examples: inline method substitution, interprocedural constant propagation
  - Common in aggressive JIT compilers and optimizing compilers for object-oriented languages

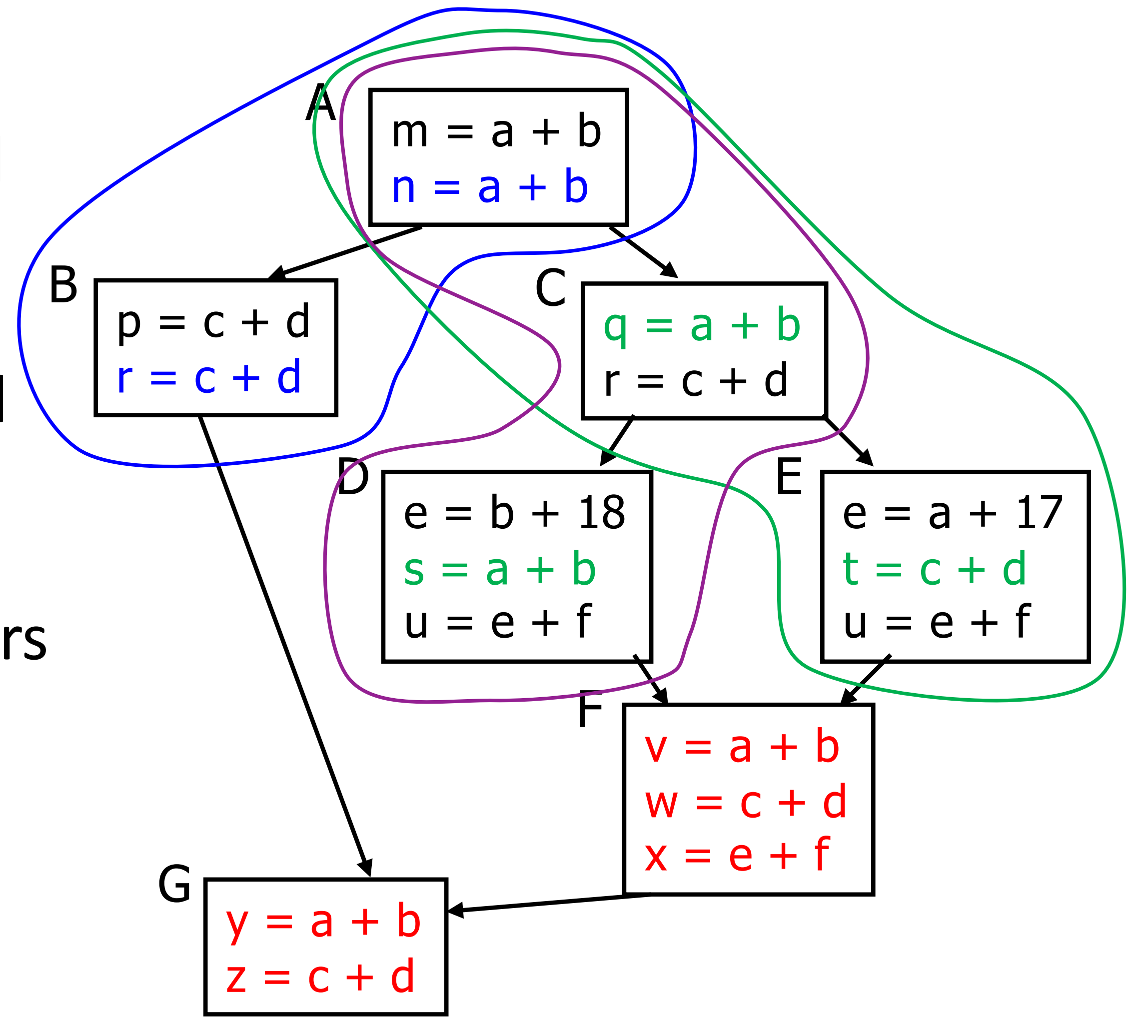
Value Numbering Revisited

- Local Value Numbering
  - 1 block at a time
  - Strong local results
  - No cross-block effects
- Missed opportunities



Superlocal Value Numbering

- Idea: apply local method to EBBs
  - {A,B}, {A,C,D}, {A,C,E}
- Final info from A is initial info for B, C; final info from C is initial for D, E
- Gets reuse from ancestors
- Avoid reanalyzing A, C
- Doesn't help with F, G



SSA Name Space (from before)

Code	Rewritten
$a_0^3 = x_0^1 + y_0^2$	$a_0^3 = x_0^1 + y_0^2$
$b_0^3 = x_0^1 + y_0^2$	$b_0^3 = a_0^3$
$a_1^4 = 17^4$	$a_1^4 = 17^4$
$c_0^3 = x_0^1 + y_0^2$	$c_0^3 = a_0^3$

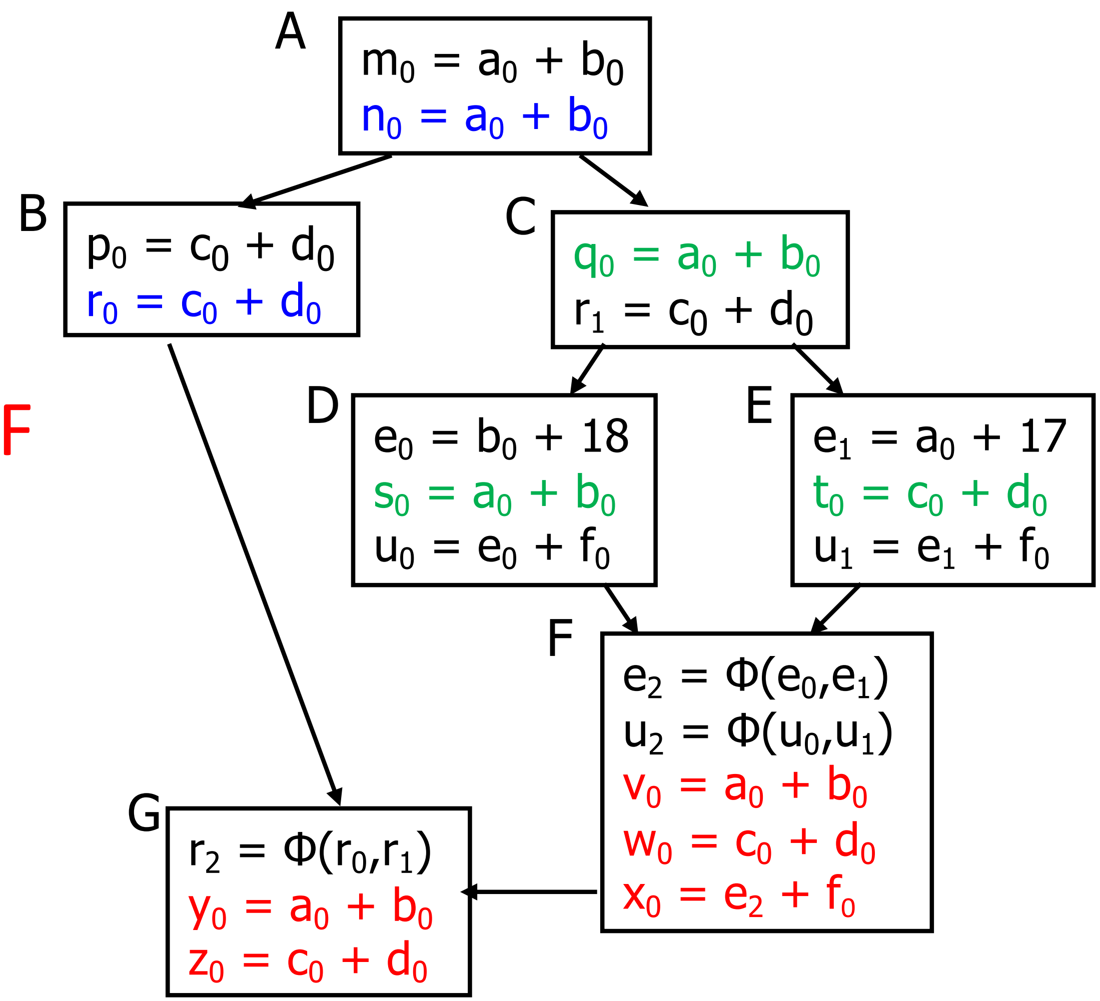
- Unique name for each definition
- Name ⇔ VN
- a_0^3 is available to assign to c_0^3

SSA Name Space

- Two Principles
  - Each name is defined by exactly one operation
  - Each operand refers to exactly one definition
- Need to deal with merge points
  - Add Φ functions at merge points to reconcile names
  - Use subscripts on variable names for uniqueness

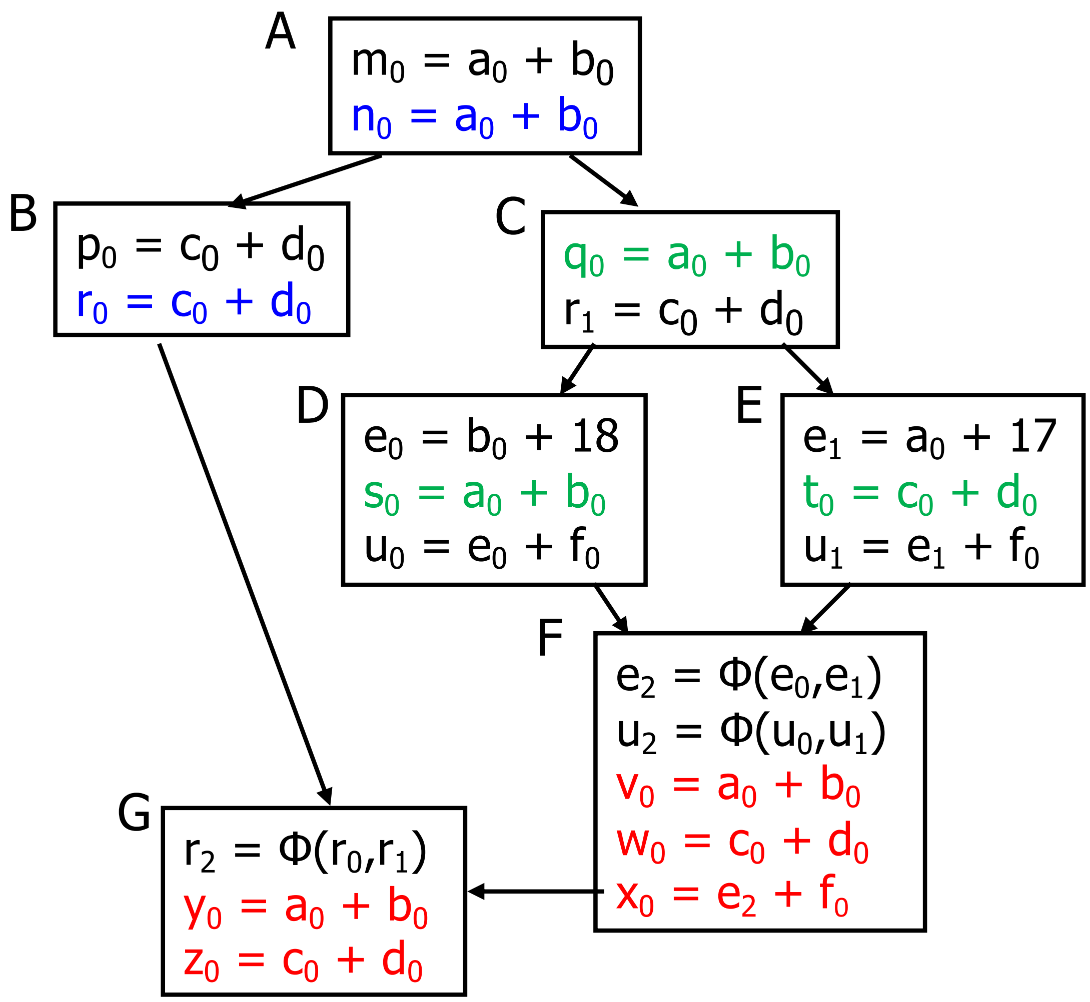
Superlocal Value Numbering with All Bells & Whistles

- Finds more redundancies
- Little extra cost
- Still does nothing for F and G

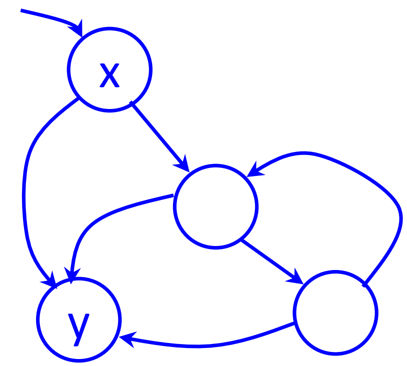


Larger Scopes

- Still have not helped F and G
- Problem: multiple predecessors
- Must decide what facts hold in F and in G
  - For G, combine B & F?
  - Merging states is expensive
  - Fall back on what we know



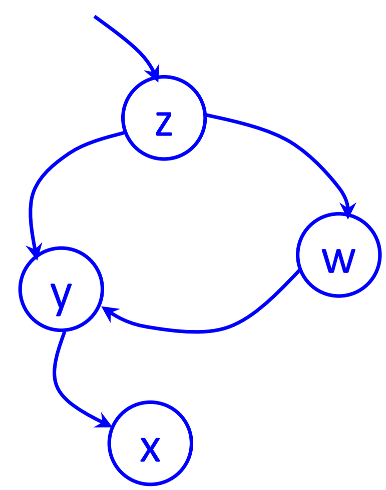
Dominators



- Definition
  - x *dominates* y iff every path from the entry of the control-flow graph to y includes x
- By definition, x dominates x
- Associate a Dom set with each node
  - | Dom(x) | ≥ 1
- Many uses in analysis and transformation
  - Finding loops, building SSA form, code motion

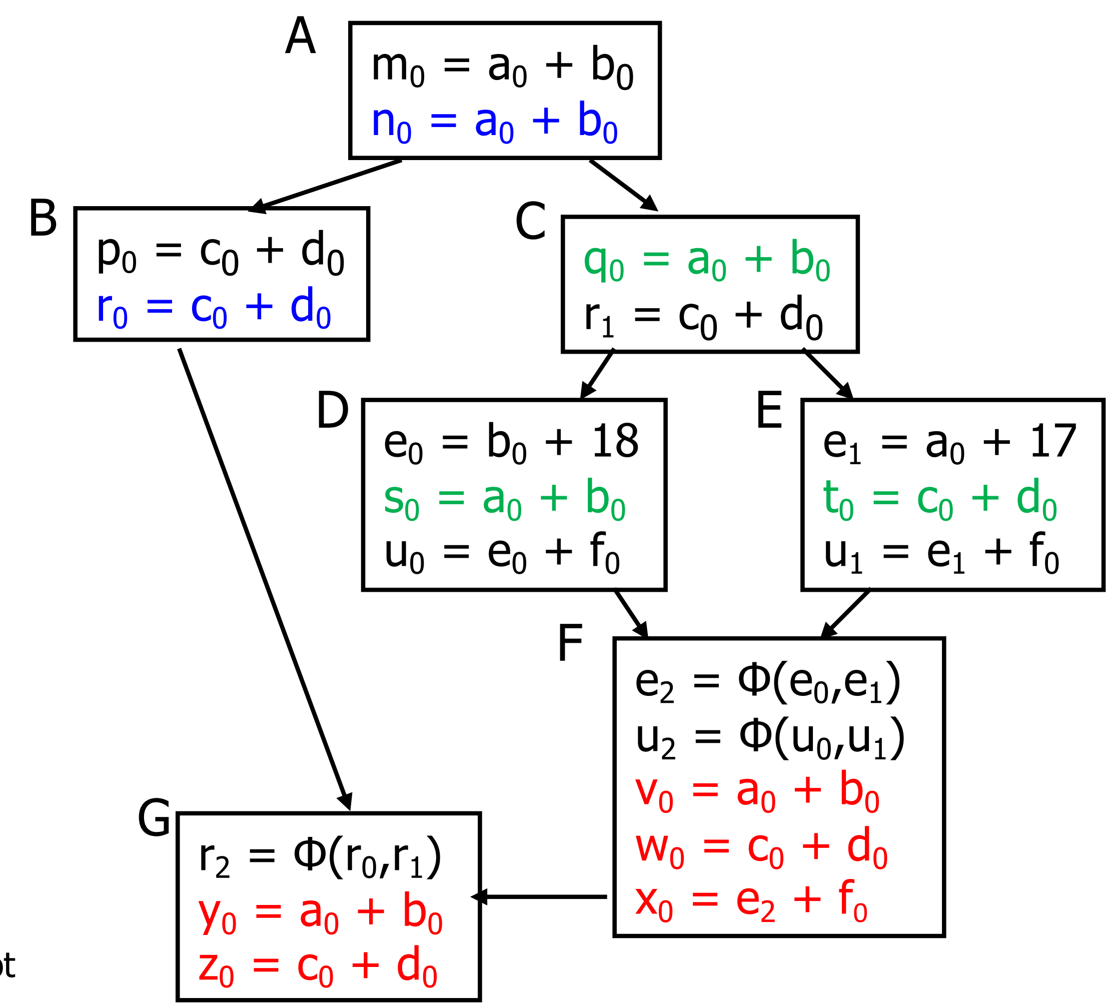
Immediate Dominators

- For any node x, there is a y in Dom(x) closest to x
- This is the *immediate dominator* of x
  - Notation: IDom(x)



Dominator Sets

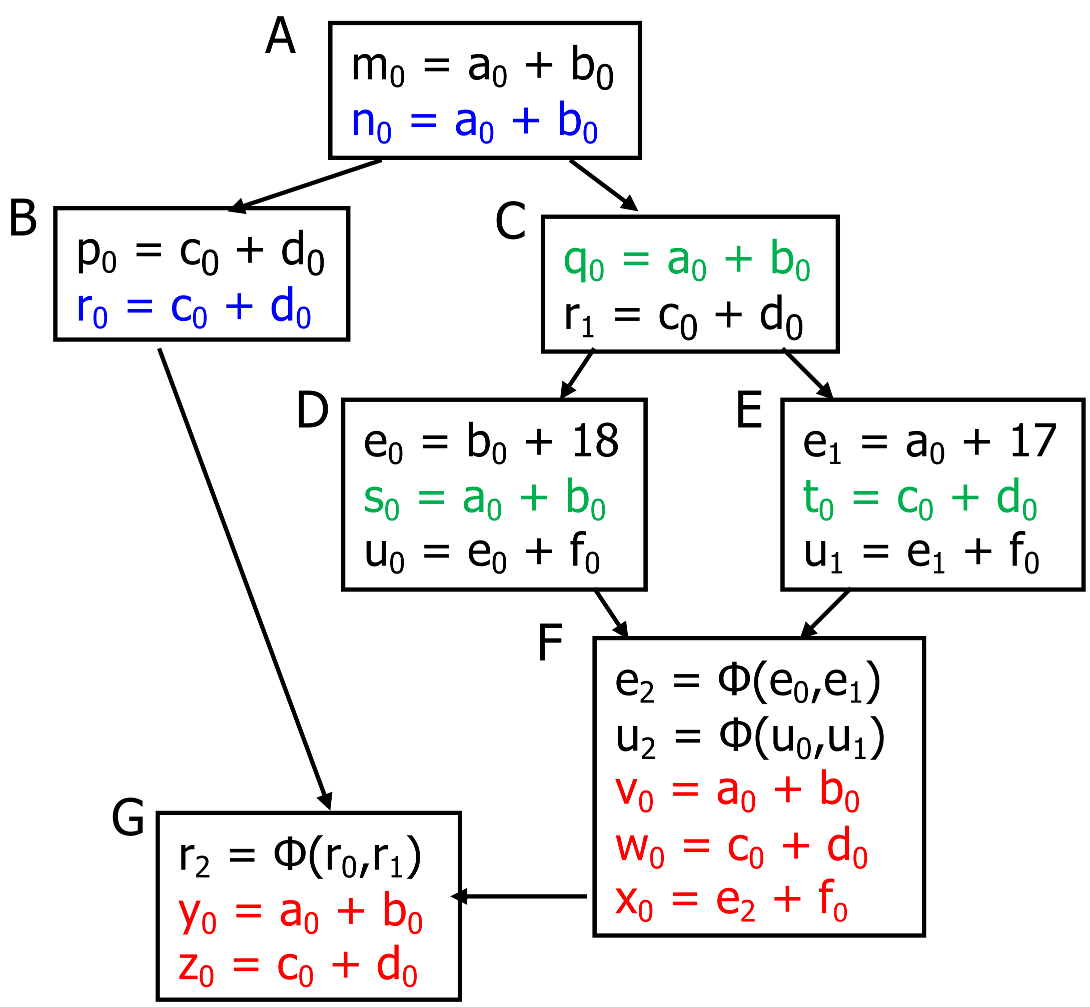
Block	Dom	IDom
A	A	—
B	A, B	A
C	A, C	A
D	A, C, D	C
E	A, C, E	C
F	A, C, F	C
G	A, G	A



Note: the IDOM relation defines a tree with A at the root

Dominator Value Numbering

- Still looking for a way to handle F and G
- Idea: Use info from IDom(x) to start analysis of x
  - Use C for F and A for G
- Dominator VN Technique (DVNT)

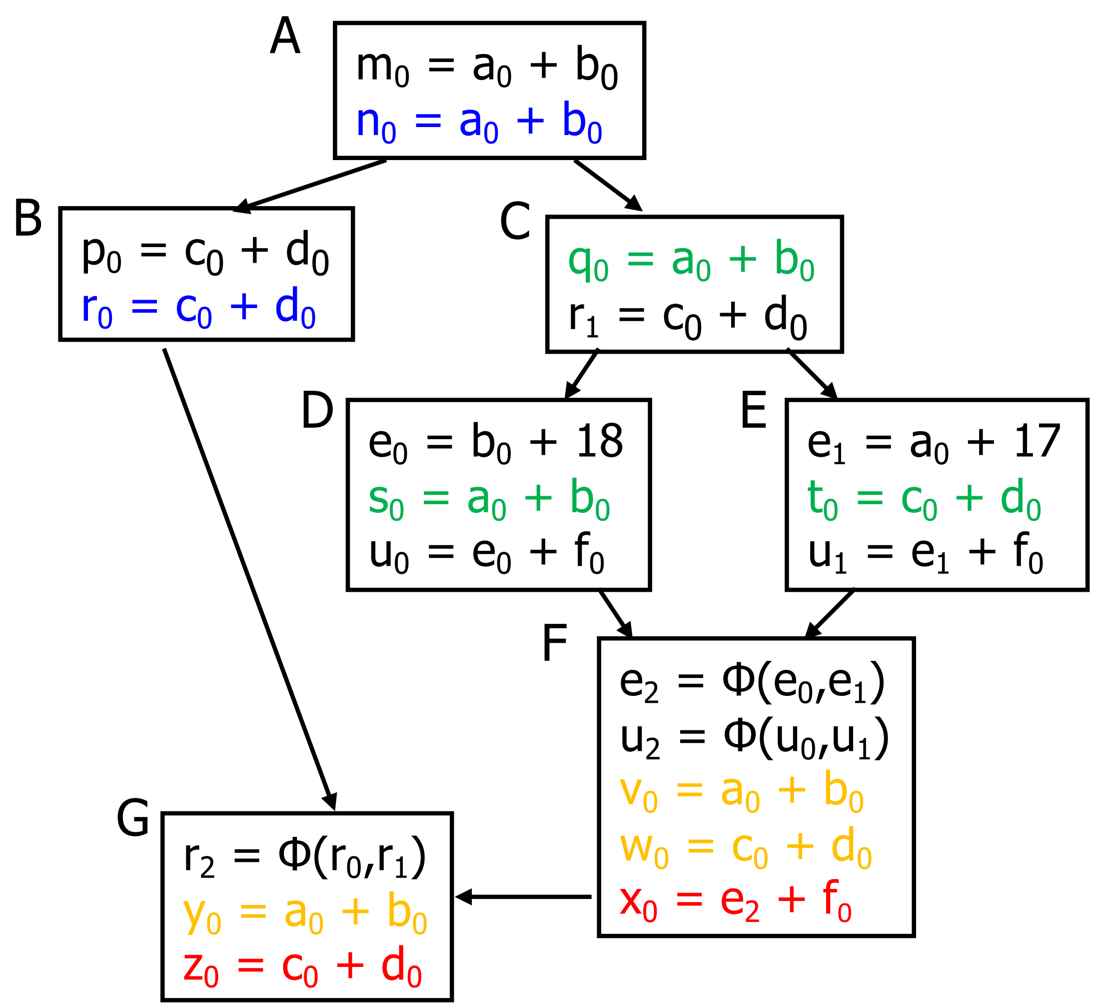


DVNT algorithm

- Use superlocal algorithm on extended basic blocks
  - Use scoped hash tables & SSA name space as before
- Start each node with table from its IDOM
- No values flow along back edges (i.e., loops)
- Constant folding, algebraic identities as before

Dominator Value Numbering

- Advantages
  - Finds more redundancy
  - Little extra cost
- Shortcomings
  - Misses some opportunities (common calculations in ancestors that are not IDOMs)
  - Doesn't handle loops or other back edges



The Story So Far…

- Local algorithm
- Superlocal extension
  - Some local methods extend cleanly to superlocal scopes
- Dominator VN Technique (DVNT)
- All of these propagate along forward edges
- None are global

Coming Attractions

- Data-flow analysis
  - Provides global solution to redundant expression analysis
    - Catches some things missed by DVNT, but misses some others
  - Generalizes to many other analysis problems, both forward and backward
- Loops
- SSA for general transformations