CSE P 501 – Compilers

LR Parser Construction

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Agenda

- LR(0) state construction
- FIRST, FOLLOW, and nullable
- Variations: SLR, LR(1), LALR

LR State Machine

- Idea: Build a DFA that recognizes handles
  - Language generated by a CFG is generally not regular, but
  - Language of viable prefixes for a CFG is regular
    - So a DFA can be used to recognize handles
  - LR Parser reduces when DFA accepts a handle

Prefixes, Handles, &c (review)

- If *S* is the start symbol of a grammar *G*,
  - If $S \Rightarrow^* \alpha$ then α is a *sentential form* of *G*
  - γ is a *viable prefix* of *G* if there is some derivation $S \Rightarrow^*_{rm} \alpha A w \Rightarrow_{rm} \alpha\beta w$ and γ is a prefix of $\alpha\beta$.
    - These are the strings that can appear on the LR parser stack
  - The occurrence of β in $\alpha\beta w$ is the right side of a *handle* of $\alpha\beta w$
- An *item* is a marked production (a . at some position in the right hand side)
  - [*A* ::= . *X Y*]   [*A* ::= *X* . *Y*]   [*A* ::= *X Y* .]

Building the LR(0) States

- Example grammar

  S' ::= *S* \$

  S ::= (*L*)

  S ::= x

  L ::= *S*

  L ::= *L* , *S*

  – We add a production S' with the original start symbol followed by end of file (\$)
    - We accept if we reach the end of this production

  – Question: What language does this grammar generate?

Start of LR Parse

0. *S'* ::= *S* $
1. *S* ::= (*L*)
2. *S* ::= x
3. *L* ::= *S*
4. *L* ::= *L* , *S*

- Initially
  - Stack is empty
    - (except for start state number usually)
  - Input is the right hand side of *S'*, i.e., *S* $
  - Initial configuration is [*S'* ::= . *S* $]
  - But, since position is just before *S*, we are also just before anything that can be derived from *S*

Initial state

0. $S' ::= S \,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

$S' ::= .\ S\ \$$ ← start

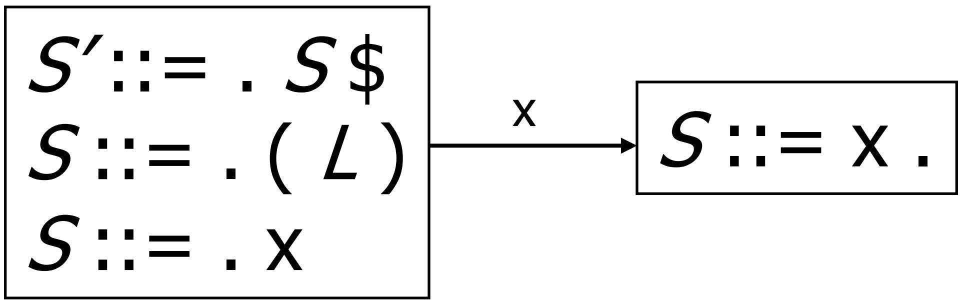
$S ::= .\ (\ L\ )$ ← completion

$S ::= .\ \mathrm{x}$ ← completion

- A state is just a set of items
  - Start: an initial set of items
  - Completion (or closure): additional productions whose left-hand side nonterminal appears immediately following a dot in some item already in the state

Shift Actions (1)

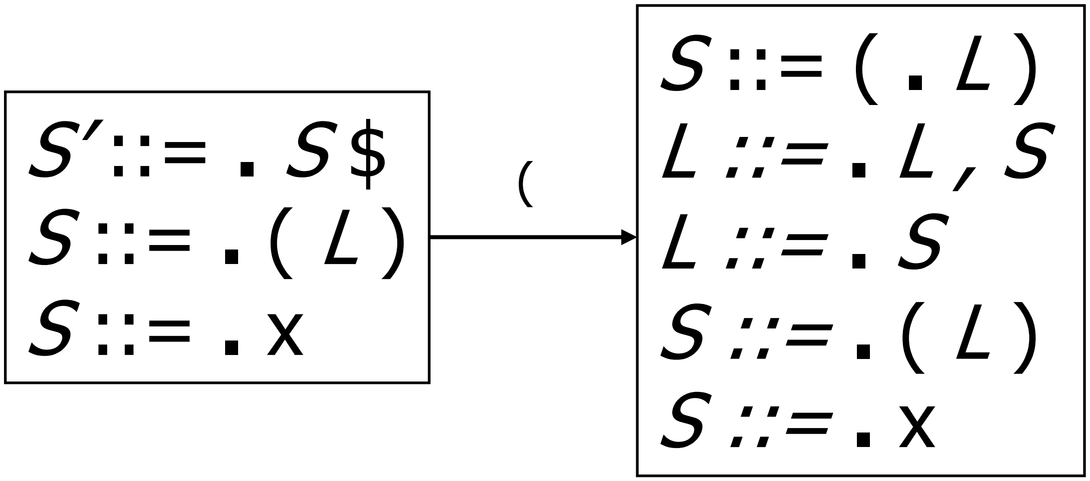
0. *S′* ::= *S* $
1. *S* ::= (*L*)
2. *S* ::= x
3. *L* ::= *S*
4. *L* ::= *L* , *S*



- To shift past the x, add a new state with appropriate item(s), including their closure
  - In this case, a single item; the closure adds nothing
  - This state will lead to a reduction since no further shift is possible

Shift Actions (2)

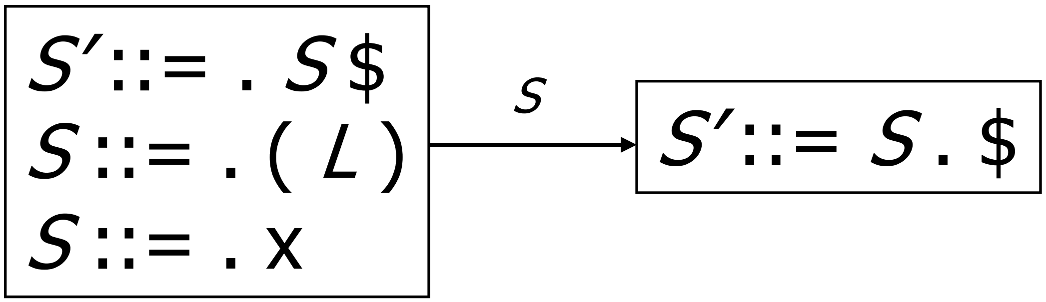
0. $S' ::= S \$$
1. $S ::= (L)$
2. $S ::= x$
3. $L ::= S$
4. $L ::= L , S$



- If we shift past the (, we are at the beginning of *L*
- The closure adds all productions that start with *L*
  - and that requires adding all productions starting with *S*

Goto Actions

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$



- Once we reduce *S*, we’ll pop the rhs from the stack exposing a previous state. Add a *goto* transition on *S* for this (i.e., if we back up into this state having reduced a rhs to *S*, then we need a goto transition on *S* to another state).

Basic Operations

- *Closure* (*S*)
  - Adds all items implied by items already in *S*
- *Goto* (*I, X*)
  - *I* is a set of items
  - *X* is a grammar symbol (terminal or non-terminal)
  - *Goto* moves the dot past the symbol *X* in all appropriate items in set *I*

Closure Algorithm

- *Closure* (*S*) =

  repeat

  for any item [A ::= α **.** B β] in *S*

  for all productions B ::= γ

  add [B ::= **.** γ] to *S*

  until *S* does not change

  return *S*

- Classic example of a fixed-point algorithm

Goto Algorithm

- *Goto* (*I, X*) =

  set *new* to the empty set

  for each item [A ::= α **.** *X* β] in *I*

  add [A ::= α *X* **.** β] to *new*

  return *Closure* (*new*)

  - This may create a new state, or may return an existing one

LR(0) Construction

- First, augment the grammar with an extra start production *S'* ::= *S* $
- Let *T* be the set of states
- Let *E* be the set of edges
- Initialize *T* to *Closure* ([*S'* ::= . *S* $])
- Initialize *E* to empty

LR(0) Construction Algorithm

repeat
  for each state *I* in *T*
    for each item [*A* ::= α **.** *X* β] in *I*
      Let *new* be *Goto*(*I*, *X*)
      Add *new* to *T* if not present
      Add *I* $\xrightarrow{X}$ *new* to *E* if not present
until *E* and *T* do not change in this iteration

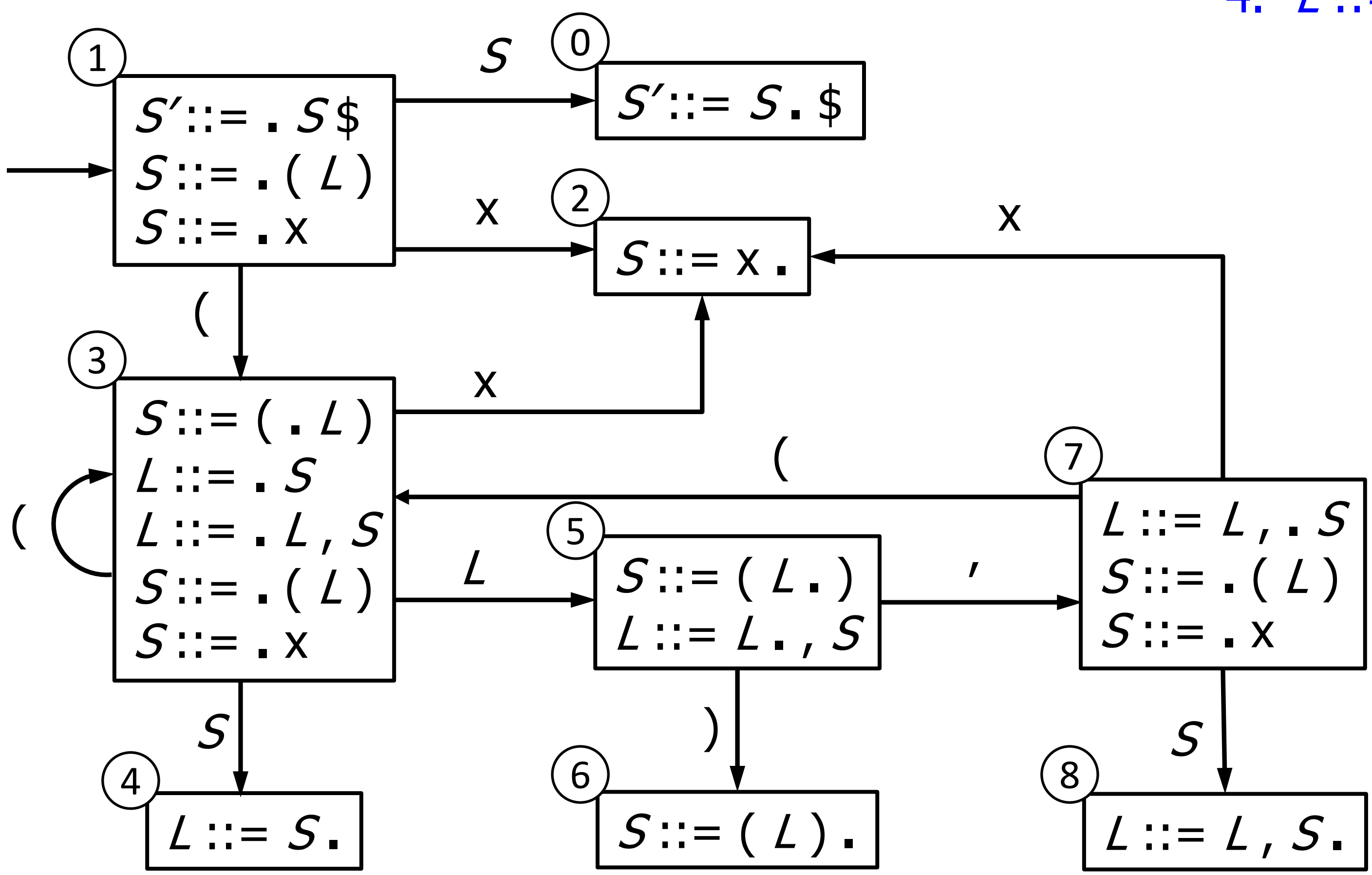
- Footnote: For symbol $, we don't compute *goto*(*I*, $); instead, we make this an *accept* action.

Example: States for

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Example: States for

0. $S' ::= S\,\$$
1. $S ::= (\,L\,)$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\,,\,S$



Building the Parse Tables (1)

- For each edge $I \xrightarrow{X} J$
  - if X is a terminal, put s*j* in column X, row *I* of the action table (shift to state *j*)
  - If X is a non-terminal, put g*j* in column X, row *I* of the goto table (go to state *j*)

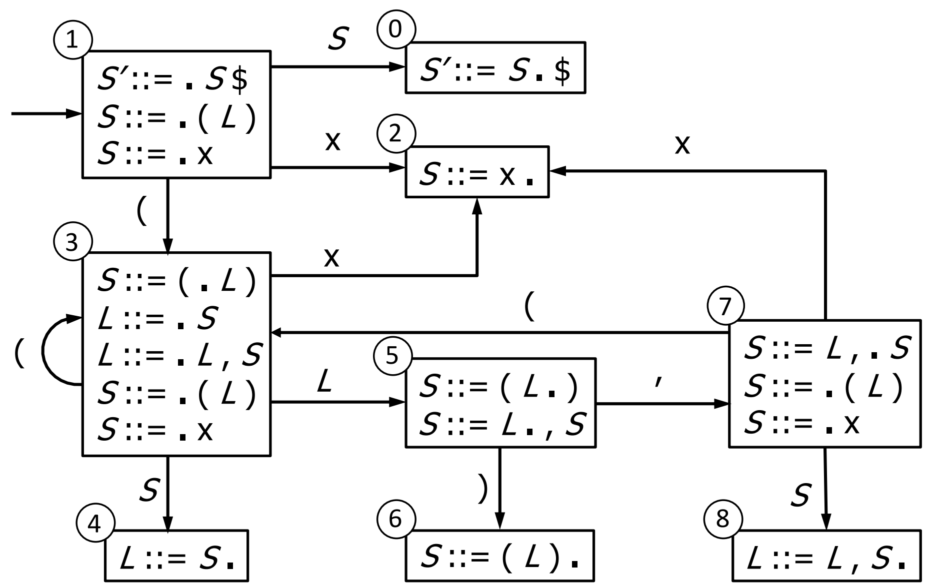
Building the Parse Tables (2)

- For each state *I* containing an item [*S'* ::= *S* . $], put *accept* in column $ of row *I*
- Finally, for any state containing [*A* ::= γ .] put action r*n* (reduce) in every column of row *I* in the table, where *n* is the *production* number (*not* a state number)
  – i.e., when it reaches this state, the DFA has discovered that *A* ::= γ is a *handle*, so the parser should reduce γ to *A*

Example: Tables for

0. $S' ::= S\ \$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

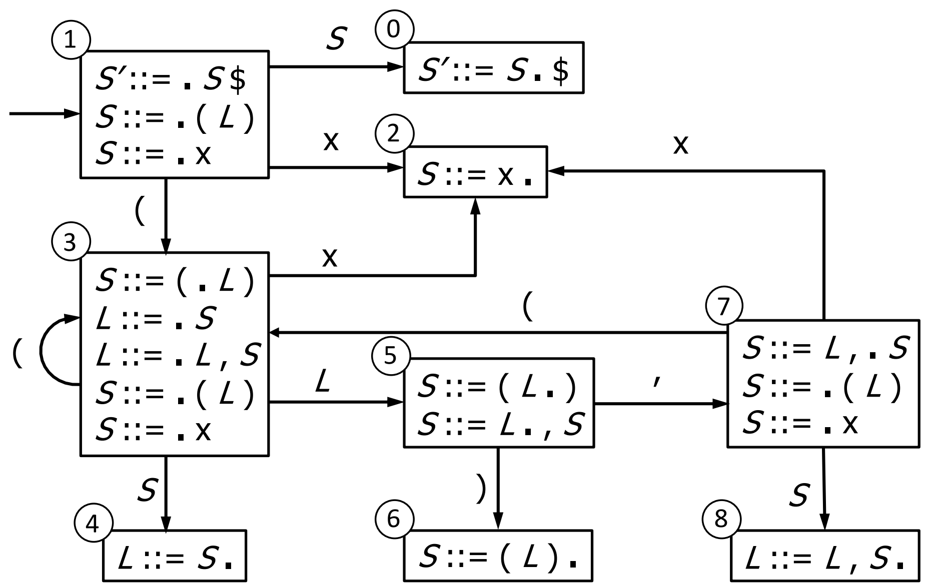
	(	)	x	,	$	S	L
0							
1							
2							
3							
4							
5							
6							
7							
8							



Example: Tables for

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \text{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

	(	)	x	,	$	S	L
0					acc		
1	s3		s2			g0	
2	r2	r2	r2	r2	r2		
3	s3		s2			g4	g5
4	r3	r3	r3	r3	r3		
5		s6		s7			
6	r1	r1	r1	r1	r1		
7	s3		s2			g8	
8	r4	r4	r4	r4	r4		



Where Do We Stand?

- We have built the LR(0) state machine and parser tables
  - No lookahead yet
  - Different variations of LR parsers add lookahead information, but basic idea of states, closures, and edges remains the same
- A grammar is LR(0) if its LR(0) state machine (equiv. parser tables) has no shift-reduce or reduce-reduce conflicts.

A Grammar that is not LR(0)

- Build the state machine and parse tables for a simple expression grammar

  E' ::= *E* $

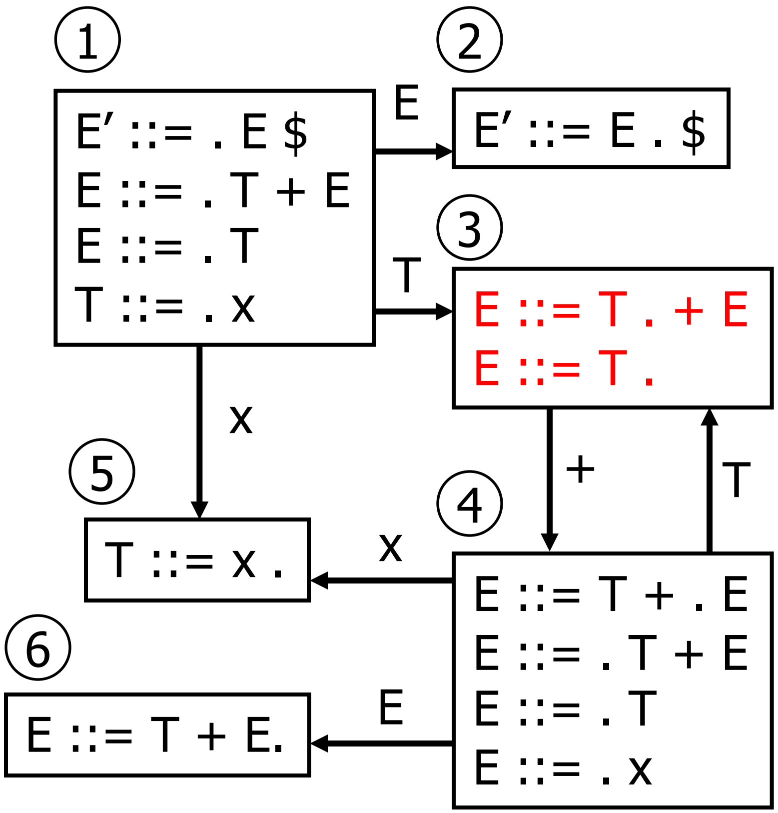
  E ::= *T* + *E*

  E ::= *T*

  T ::= x

LR(0) Parser for

0. $E' ::= E\ \$$
1. $E ::= T + E$
2. $E ::= T$
3. $T ::= x$



	x	+	$	E	T
1	s5			g2	G3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	G3
5	r3	r3	r3		
6	r1	r1	r1		

- State 3 is has two possible actions on +
  - shift 4, or reduce 2
- ∴ Grammar is not LR(0)

How can we solve conflicts like this?

- Idea: look at the next symbol after the handle before deciding whether to reduce
- Easiest: SLR – Simple LR. Reduce only if next input terminal symbol could follow resulting nonterminal
  - Suppose we've reached [*A* ::= β **.**] and the next input is x
  - Don't reduce unless *A*x can appear in some sentential form
    - This is the 🔑 🗝️ idea!
- More complex: LR and LALR. Store lookahead symbols in items to keep track of what can follow a *particular instance* of a reduction
  - LALR used by YACC/Bison/CUP; we won't examine in detail

SLR Parsers

- Idea: Use information about what can follow a non-terminal to decide if we should perform a reduction; don't reduce if the next input symbol can't follow the resulting non-terminal
- To decide, for each non-terminal *A,* we need to know FOLLOW(*A*) – the set of terminal symbols that can follow *A* in some possible derivation
  - i.e., t is in FOLLOW(*A*) if there is some possible derivation that contains *A*t
  - To compute this, we need to compute FIRST(γ) for strings γ that can follow *A*

Calculating FIRST(γ)

- Sounds easy… If $\gamma = X\ Y\ Z$, then FIRST(γ) is FIRST(X), right?

  – But what if we have the rule $X ::= \varepsilon$?
  – In that case, FIRST(γ) includes anything that can follow X, i.e. FOLLOW(X), which includes FIRST(Y) and, if Y can derive ε, FIRST(Z), and if Z can derive ε, …
  – So computing FIRST and FOLLOW involves knowing FIRST and FOLLOW for other symbols, as well as which ones can derive ε

FIRST, FOLLOW, and nullable

- nullable(*X*) is true if *X* can derive the empty string
- Given a string γ of terminals and non-terminals, FIRST(γ) is the set of terminals that can begin any strings derived from γ
  - For SLR we only need this for single terminal or non-terminal symbols, not arbitrary strings γ
- FOLLOW(*X*) is the set of terminals that can immediately follow *X* in some derivation
- All three of these are computed together

- Footnote: Textbook doesn't use a separate nullable(X) attribute, instead it indicates nullable by including ε in FIRST(X). Both will wind up with same results, but one or the other might be easier to follow, so to speak.

Computing FIRST, FOLLOW, and nullable (1)

- Initialization

  set FIRST and FOLLOW to be empty sets

  set nullable to false for all non-terminals

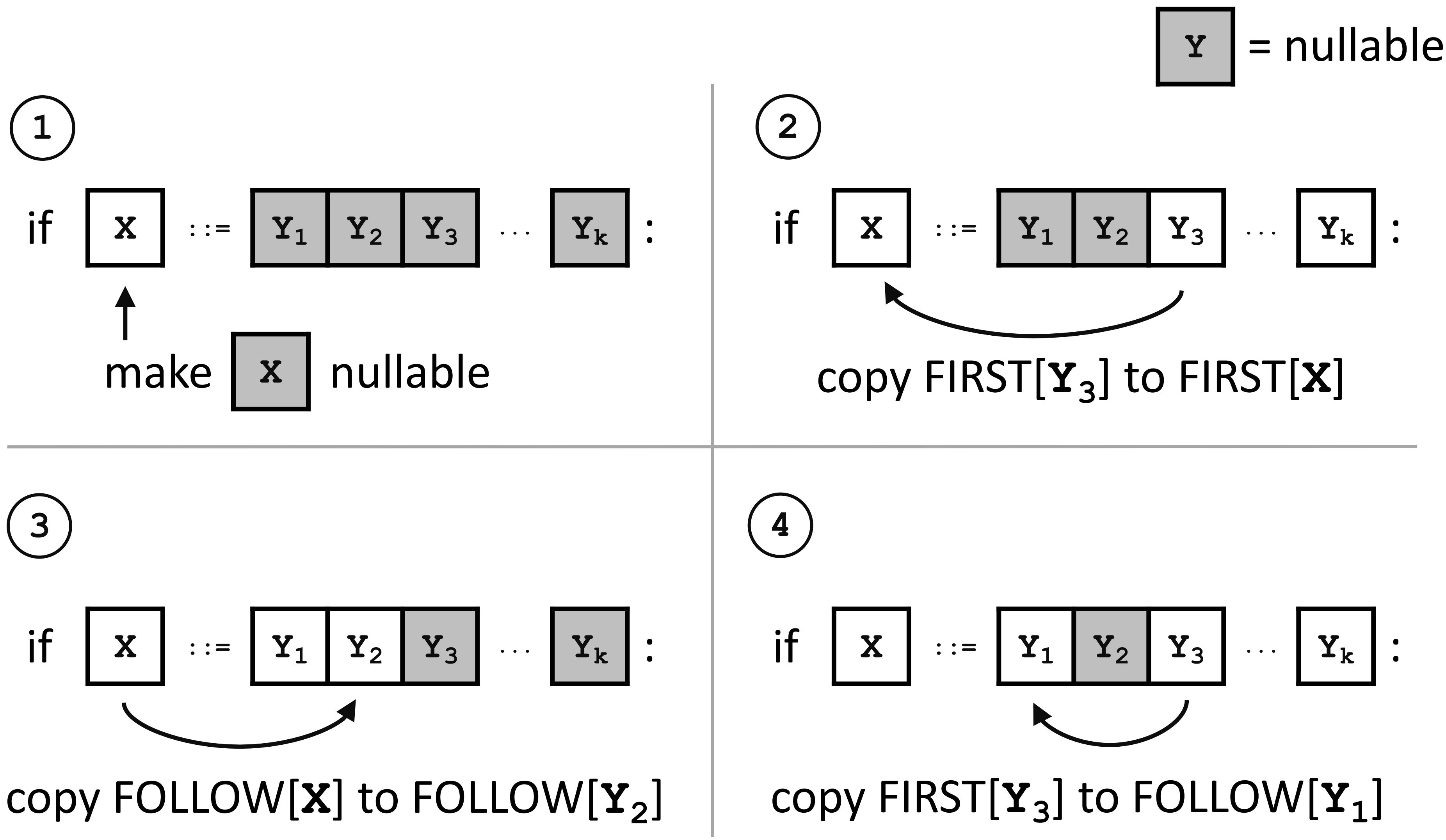
  set FIRST[a] to a for all terminal symbols a

- Repeatedly apply four simple observations to update these sets
  - Stop when there are no further changes
  - Another fixed-point algorithm

Computing FIRST, FOLLOW, and nullable (2)

repeat
  for each production $X := Y_1\ Y_2\ Y_3 \ldots Y_{k-2}\ Y_{k-1}\ Y_k$
   if $Y_1 \ldots Y_k$ are all nullable (or if $k = 0$)
    set nullable[X] = true 1
   for each i from 1 to k and each j from i +1 to k
    if $Y_1 \ldots Y_{i-1}$ are all nullable (or if $i = 1$)
     add FIRST[Y_i] to FIRST[X] 2
    if $Y_{i+1} \ldots Y_k$ are all nullable (or if $i = k$)
     add FOLLOW[X] to FOLLOW[Y_i] 3
    if $Y_{i+1} \ldots Y_{j-1}$ are all nullable (or if i+1=j)
     add FIRST[Y_j] to FOLLOW[Y_i] 4
Until FIRST, FOLLOW, and nullable do not change

Computing FIRST, FOLLOW, & nullable (3)



Example (initial)

- Grammar

  Z ::= d

  Z ::= *X Y Z*

  Y ::= ε

  Y ::= c

  X ::= *Y*

  X ::= a

	nullable	FIRST	FOLLOW
X	no		
Y	no		
Z	no		

Example (final)

- Grammar

  Z ::= d

  Z ::= X Y Z

  Y ::= ε

  Y ::= c

  X ::= Y

  X ::= a

	nullable	FIRST	FOLLOW
X	~~no~~ yes	a, c	a, c, d
Y	~~no~~ yes	c	a, c, d
Z	no	a, c, d	

LR(0) Reduce Actions (review)

- In a LR(0) parser, if a state contains a reduction, it is unconditional regardless of the next input symbol
- Algorithm:

  Initialize *R* to empty

  for each state *I* in *T*

    for each item [A ::= α .] in *I*

      add (*I*, A ::= α) to *R*

SLR Construction

- This is identical to LR(0) – states, etc., except for the calculation of reduce actions
- Algorithm:

  Initialize *R* to empty

  for each state *I* in *T*

    for each item [*A* ::= α .] in *I*

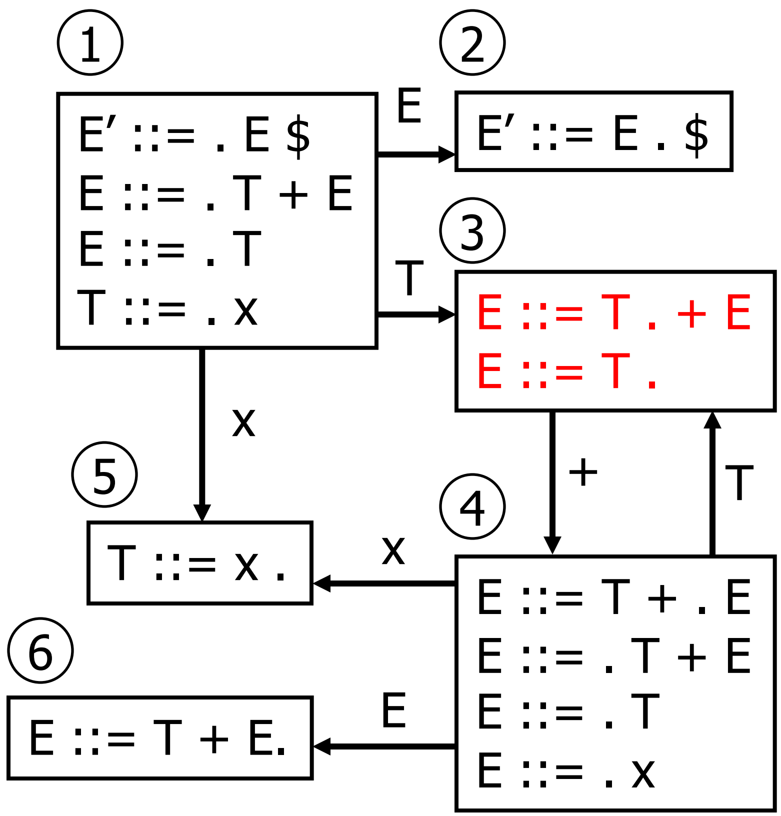
      for each terminal a in FOLLOW(*A*)   ☜ new!

        add (*I*, a, *A* ::= α) to *R*

  – i.e., reduce α to *A* in state *I* only on lookahead a

SLR Parser for

0. E′ ::= E $
1. E ::= T + E
2. E ::= T
3. T ::= x



	x	+	$	E	T
1	s5			g2	g3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	g3
5	r3	r3	r3		
6	r1	r1	r1		

Ghost yellow = reductions omitted in SLR parser because next terminal is not in FOLLOW(non-terminal)

On To LR(1)

- Many practical grammars are SLR
- LR(1) is more powerful yet
- Similar construction, but notion of an item is more complex, incorporating lookahead information
  - So lookahead information is associated with specific items rather than FOLLOW for a non-terminal regardless of specific context where that non-terminal appears in the derivation

LR(1) Items

- An LR(1) item [A ::= α . β, a] is
  - A grammar production (A ::= $\alpha\beta$)
  - A right hand side position (the dot)
  - A lookahead symbol (a)
- Idea: This item indicates that α is the top of the stack and the next input is derivable from βa.
- Full construction: see the book

LR(1) Tradeoffs

- LR(1)
  - Pro: extremely precise; largest set of grammars
  - Con: potentially ***very*** large parse tables with many states

LALR(1)

- Variation of LR(1), but merge any two states that differ only in lookahead
  - Example: these two would be merged

    [*A* ::= x . , a]

    [*A* ::= x . , b]

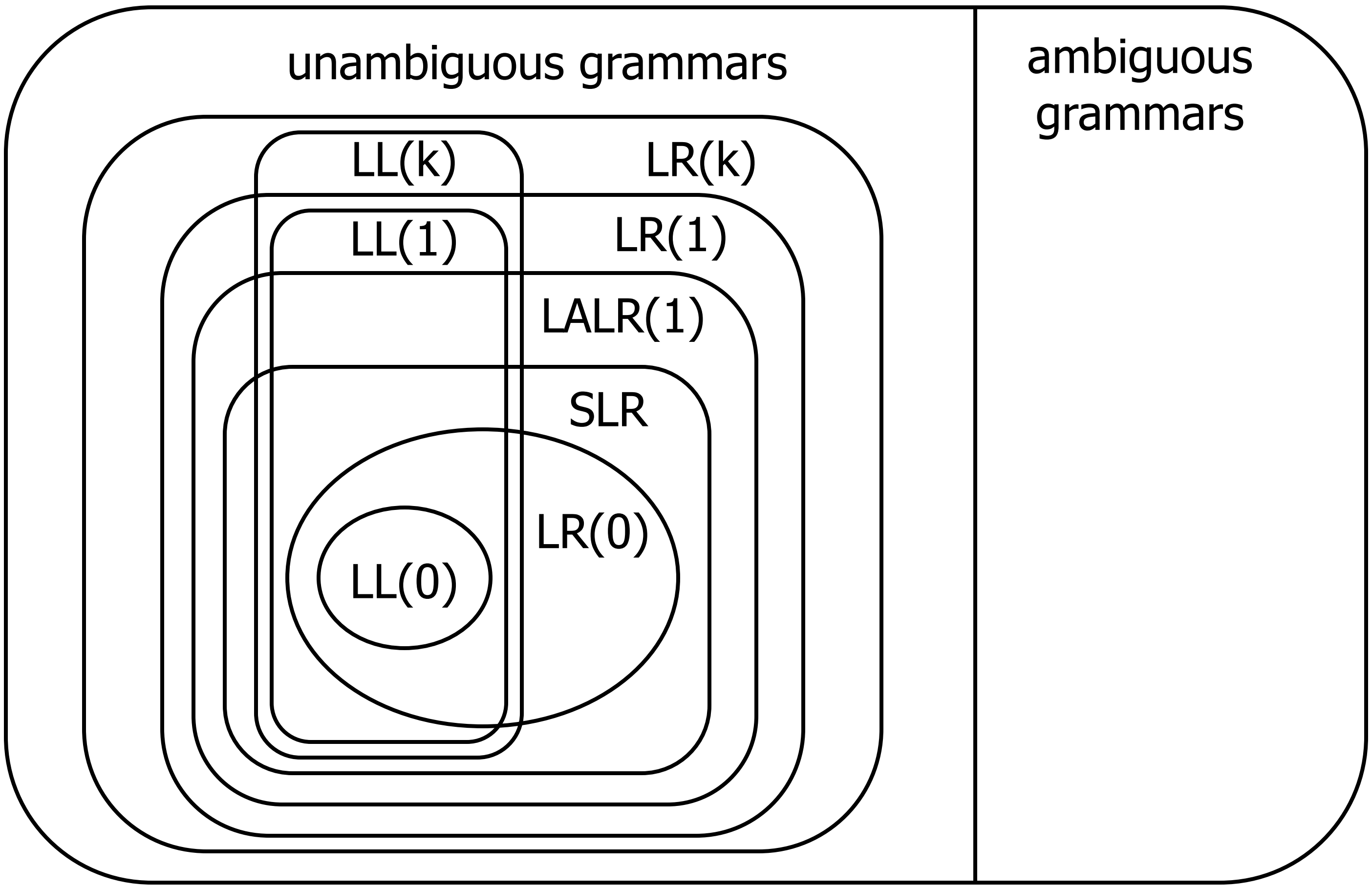
    to produce

    [A ::= x . y , ab]

LALR(1) vs LR(1)

- LALR(1) tables can have many fewer states than LR(1)
  - Somewhat surprising result: will actually have same number of states as SLR parsers, even though LALR(1) is more powerful
  - After the merge step, acts like SLR parser with "smarter" FOLLOW sets (can be specific to particular handles)
- LALR(1) may have reduce conflicts where LR(1) would not (but in practice this doesn't happen often)
- Most practical bottom-up parser tools are LALR(1) (e.g., yacc, bison, CUP, …)

Language Hierarchies



Coming Attractions

Rest of Parsing…

- LL(k) Parsing – Top-Down
- Recursive Descent Parsers
  - What you can do if you want a parser in a hurry

Then…

- AST construction – what do do while you parse!
- Visitor Pattern – how to traverse ASTs for further processing (type checking, code generation, …)