

CSE P 501 – Compilers

LR Parsing
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Agenda

- LR Parsing
- Table-driven Parsers
- Parser States
- Shift-Reduce and Reduce-Reduce conflicts

Bottom-Up Parsing

- Idea: Read the input left to right
- Whenever we've matched the right hand side of a production, reduce it to the appropriate non-terminal and add that non-terminal to the parse tree
- The upper edge of this partial parse tree is known as the *frontier*

Example

- Grammar

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$

- Bottom-up Parse

a b b c d e

Example

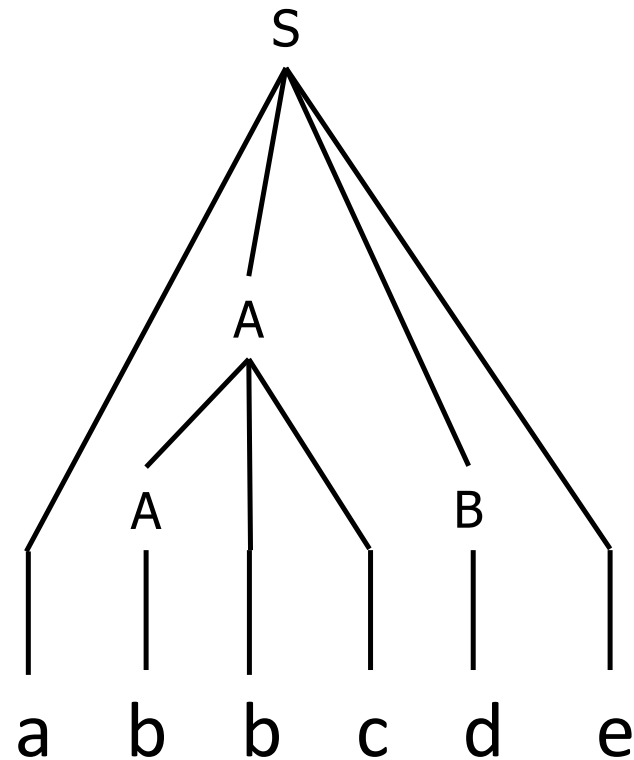
- Grammar

$S ::= aABe$

$A ::= A|bc \mid b$

$B ::= d$

- Bottom-up Parse



LR(1) Parsing

- We'll look at LR(1) parsers
 - Left to right scan, Rightmost derivation, 1 symbol lookahead
 - Almost all practical programming languages have a LR(1) grammar
 - LALR(1), SLR(1), etc. – subsets of LR(1)
 - LALR(1) can parse most real languages, tables are more compact, and is used by YACC/Bison/CUP/etc.

LR Parsing in Greek

- The bottom-up parser reconstructs a reverse rightmost derivation
- Given the rightmost derivation
$$S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = w$$
the parser will first discover $\beta_{n-1} \Rightarrow \beta_n$, then $\beta_{n-2} \Rightarrow \beta_{n-1}$, etc.
- Parsing terminates when
 - β_1 reduced to S (start symbol, success), or
 - No match can be found (syntax error)

How Do We Parse with This?

- Key: given what we've already seen and the next input symbol (the lookahead), decide what to do.
- Choices:
 - Perform a reduction
 - Look ahead further
- Can reduce $A \Rightarrow \beta$ if both of these hold:
 - $A \Rightarrow \beta$ is a valid production, *and*
 - $A \Rightarrow \beta$ is a step in *this* rightmost derivation
- This is known as a *shift-reduce* parser

Sentential Forms

- If $S \Rightarrow^* \alpha$, the string α is called a *sentential form* of the grammar
- In the derivation
$$S \Rightarrow \beta_1 \Rightarrow \beta_2 \Rightarrow \dots \Rightarrow \beta_{n-2} \Rightarrow \beta_{n-1} \Rightarrow \beta_n = w$$
each of the β_i are sentential forms
- A sentential form in a rightmost derivation is called a right-sentential form (similarly for leftmost and left-sentential)

Handles

- Informally, a production whose right hand side matches a substring of the tree frontier *that is part of the rightmost derivation of the current input string* (i.e., the “correct” production)
 - Even if $A ::= \beta$ is a production, it is a handle only if β matches the frontier at a point where $A ::= \beta$ was used in *this specific* derivation
 - β may appear in many other places in the frontier without being a part of a handle for that particular production
- Bottom-up parsing is all about finding handles

Handle Examples

- In the derivation

$S \Rightarrow aABe \Rightarrow aAde \Rightarrow aAbcde \Rightarrow abbcde$

– $a**b**cde$ is a right sentential form whose handle is

$A ::= b$ at position 2

– $a**Abc**de$ is a right sentential form whose handle is

$A ::= Abc$ at position 4

- Note: some books take the left end of the match as the position

Handles – The Dragon Book Defn.

- Formally, a *handle* of a right-sentential form γ is a production $A ::= \beta$ and a position in γ where β may be replaced by A to produce the previous right-sentential form in the rightmost derivation of γ

Implementing Shift-Reduce Parsers

- Key Data structures
 - A stack holding the frontier of the tree
 - A string with the remaining input (tokens)
- We also need something to encode the rules that tell us what action to take next, given the state of the stack and the lookahead symbol
 - Typically a table that encodes a finite automata

Shift-Reduce Parser Operations

- *Reduce* – if the top of the stack is the right side of a handle $A ::= \beta$, pop the right side β and push the left side A
- *Shift* – push the next input symbol onto the stack
- *Accept* – announce success
- *Error* – syntax error discovered

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

<u>Stack</u>	<u>Input</u>	<u>Action</u>
\$	abbcde\$	

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Stack	Input	Action
\$	abbcde\$	<i>shift</i>
\$a	bbcde\$	<i>shift</i>
\$ab	bcde\$	<i>reduce</i>
\$aA	bcde\$	<i>shift</i>
\$aAb	cde\$	<i>shift</i>
\$aAbc	de\$	<i>reduce</i>
\$aA	de\$	<i>shift</i>
\$aAd	e\$	<i>reduce</i>
\$aAB	e\$	<i>shift</i>
\$aABe	\$	<i>reduce</i>
\$S	\$	<i>accept</i>

How Do We Automate This?

- Cannot use clairvoyance in a real parser (alas...)
- Defn. *Viable prefix* – a prefix of a right-sentential form that can appear on the stack of the shift-reduce parser
 - Equivalent: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form
 - In Greek: γ is a *viable prefix* of G if there is some derivation $S \Rightarrow_{rm}^* \alpha A w \Rightarrow_{rm} \alpha \beta w$ and γ is a prefix of $\alpha \beta$.
 - The occurrence of β in $\alpha \beta w$ is the right side of a *handle* of $\alpha \beta w$

How Do We Automate This?

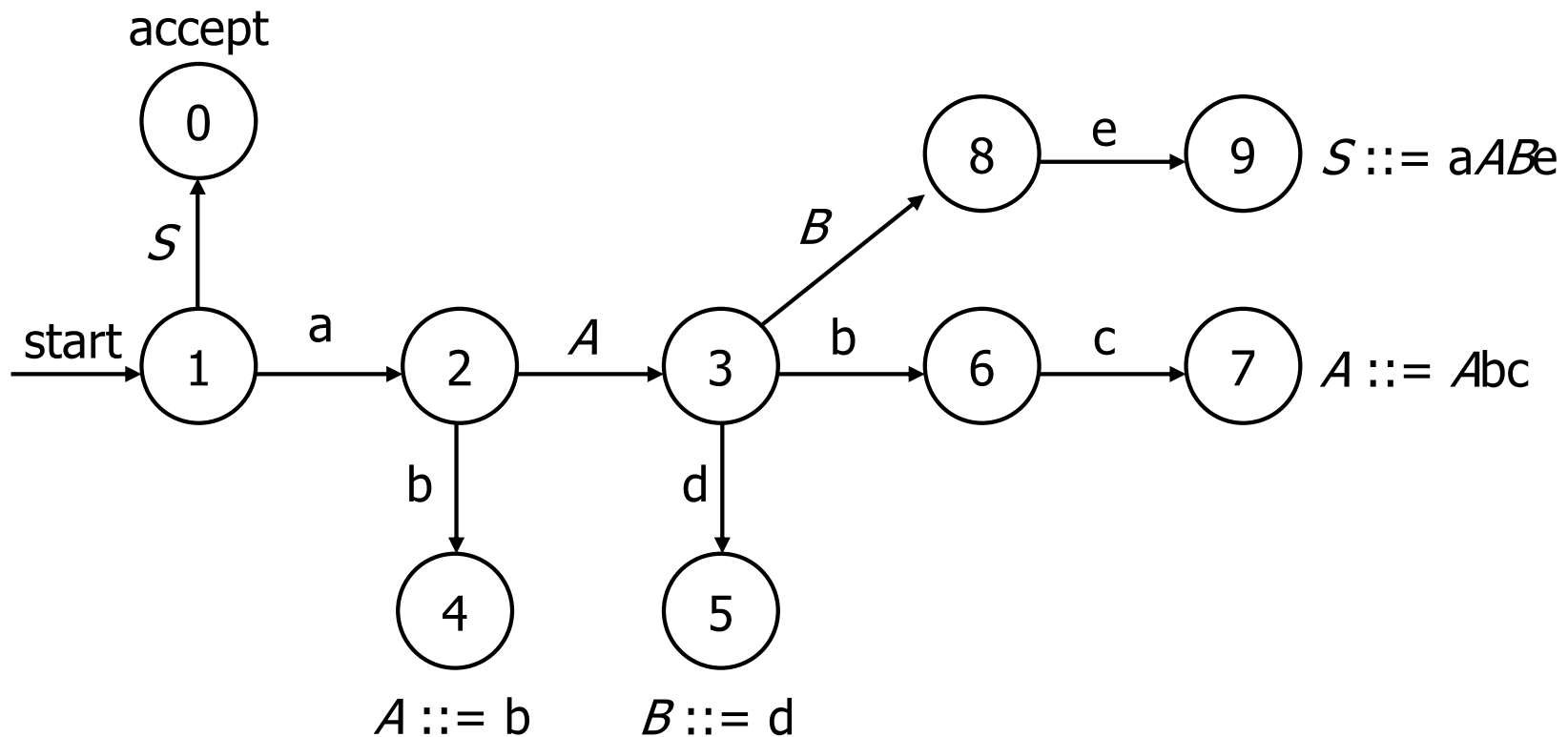
- Fact: the set of viable prefixes of a CFG is a regular language(!)
- Idea: Construct a DFA to recognize viable prefixes given the stack and remaining input
 - Perform reductions when we recognize the rhs of handles

DFA for prefixes of

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$

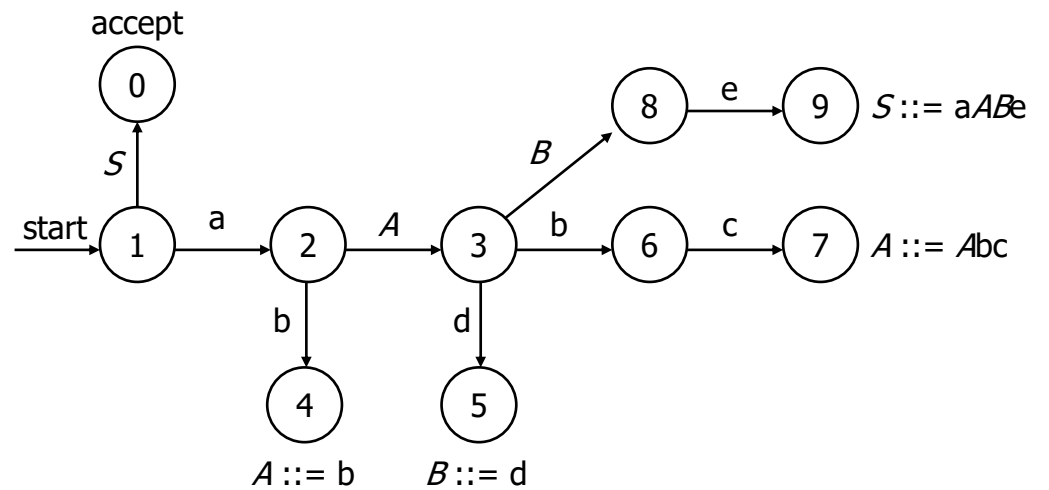


Trace

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Stack
\$

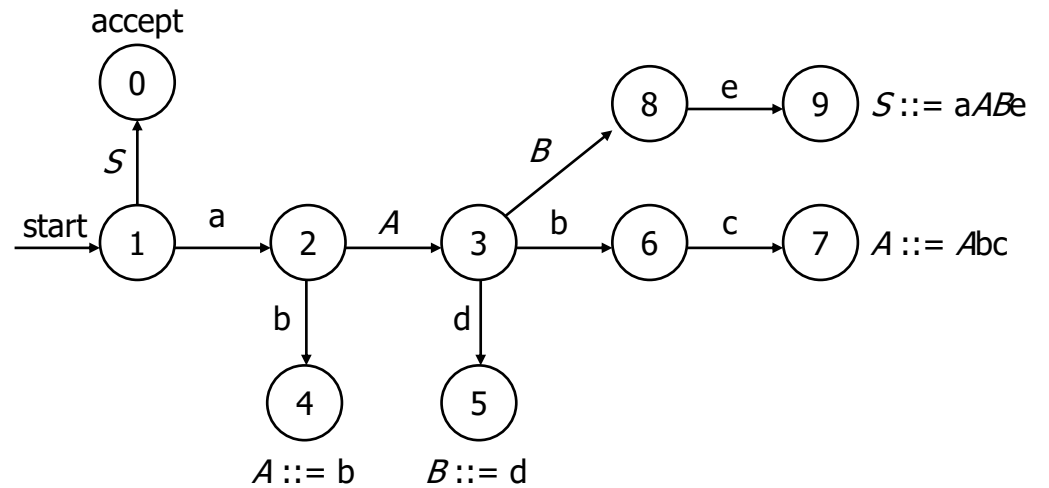
Input
abcde\$



Trace

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Stack	Input
\$	abbcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$
\$aAB	e\$
\$aABe	\$
\$S	\$



Observations

- Way too much backtracking
 - We want the parser to run in time proportional to the length of the input
- Where the heck did this DFA come from anyway?
 - From the underlying grammar
 - Defer construction details for now

Avoiding DFA Rescanning

- Observation: no need to restart DFA after a shift. Stay in the same state and process next token.
- Observation: after a reduction, the contents of the stack are the same as before except for the new non-terminal on top
 - ∴ Scanning the stack will take us through the same transitions as before until the last one
 - ∴ If we record state numbers on the stack, we can go directly to the appropriate state when we pop the right hand side of a production from the stack

Stack

- Change the stack to contain pairs of states and symbols from the grammar

$\$s_0 X_1 s_1 X_2 s_2 \dots X_n s_n$

- State s_0 is the start state
 - When we push a symbol on the stack, push the symbol plus the FA state
 - When we reduce, popping the handle will reveal the state of the FA just prior to reading the handle
- Observation: in an actual parser, only the state numbers are needed since they implicitly contain the symbol information, but for explanations / examples it can help to show both.

Encoding the DFA in a Table

- A shift-reduce parser's DFA can be encoded in two tables
 - One row for each state
 - *action* table encodes what to do given the current state and the next input symbol
 - *goto* table encodes the transitions to take after a reduction

Actions (1)

- Given the current state and input symbol, the main possible actions are
 - si – shift the input symbol and **state i** onto the stack (i.e., shift and move to state i)
 - rj – reduce using grammar **production j**
 - The production number tells us how many $\langle \text{symbol}, \text{state} \rangle$ pairs to pop off the stack (= number of symbols on rhs of production)
 - Each production needs a unique number, i.e., $A ::= \alpha \mid \beta$ needs to be split into $A ::= \alpha$ and $A ::= \beta$

Actions (2)

- Other possible *action* table entries
 - *accept*
 - **blank** – no transition – syntax error
 - A LR parser will detect an error as soon as possible on a left-to-right scan
 - A real compiler needs to produce an error message, recover, and continue parsing when this happens
 - (Often involves encoding error handling/recovery info in the action table)

Goto

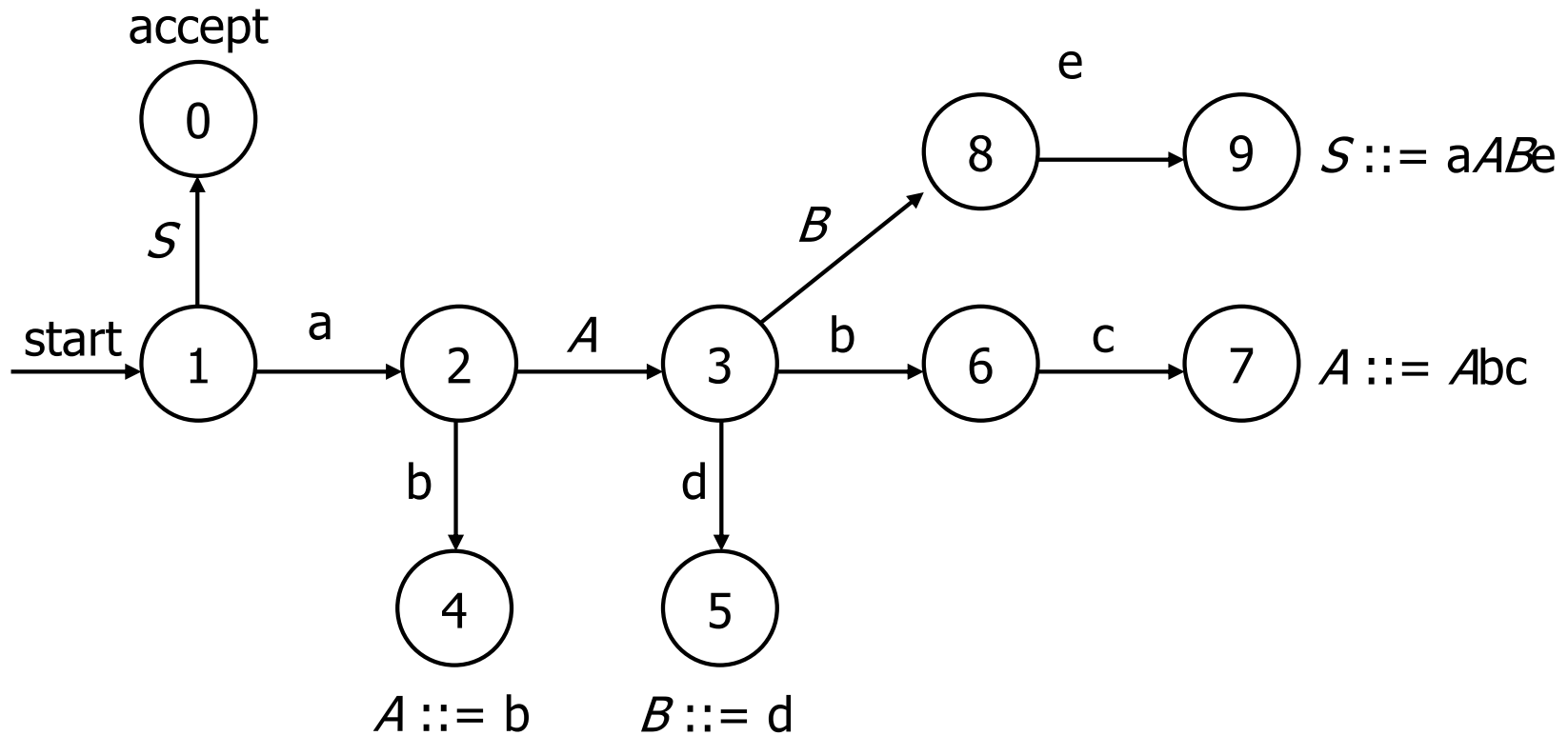
- When a reduction is performed using $A ::= \beta$, we pop $|\beta|$ $\langle \text{symbol}, \text{state} \rangle$ pairs from the stack revealing a state *uncovered_s* on the top of the stack
- $\text{goto}[\textit{uncovered_s}, A]$ is the new state to push on the stack when reducing production $A ::= \beta$ (after popping handle β and pushing A)

Aside: Extra Initial Production

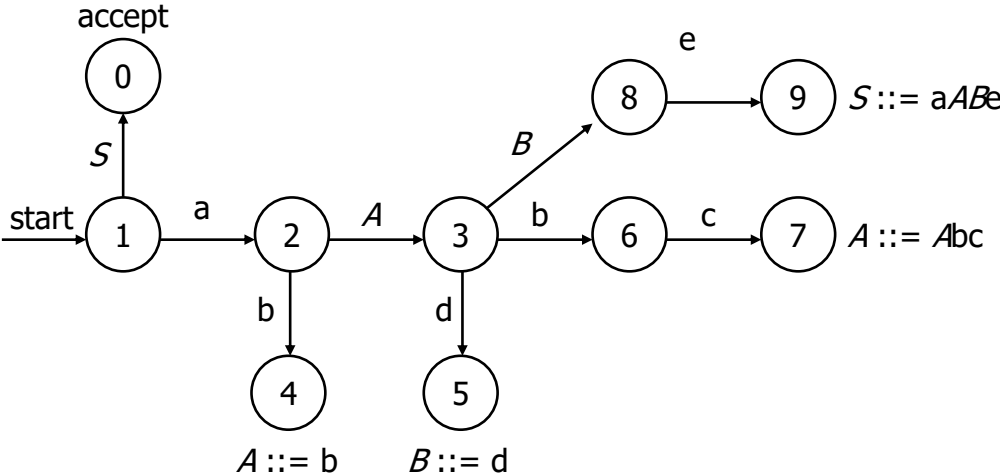
- When we construct the DFA we'll need to add a new production to handle end-of-file (i.e., end-of-input) correctly
- If S is the start state of the original grammar, add an initial production $S' ::= S \$$
 - $\$$ represents end-of-file (input)
 - Accept when we've reduced the input to S and there is no more input (i.e., lookahead is $\$$)

Reminder: DFA for

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$



LR Parse Table



State	action						goto		
	a	b	c	d	e	\$	A	B	S
0						acc			
1	s2								g0
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

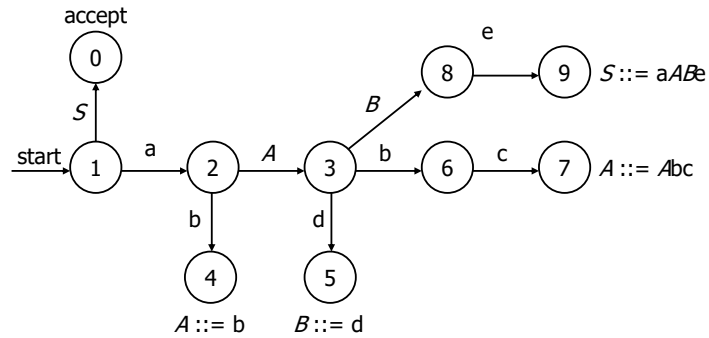
- 0. $S' ::= S\$$
- 1. $S ::= aABe$
- 2. $A ::= Abc$
- 3. $A ::= b$
- 4. $B ::= d$

LR Parsing Algorithm

```
tok = scanner.getToken();
while (true) {
    s = top of stack;
    if (action[s, tok] =  $si$ ) {
        push tok; push  $i$  (state);
        tok = scanner.getToken();
    } else if (action[s, tok] =  $rj$ ) {
        pop  $2 * \text{length of right side of}$ 
        production  $j$  ( $2 * |\beta|$ );
        uncovered_s = top of stack;
        push left side  $A$  of production  $j$ ;
        push state goto[uncovered_s,  $A$ ];
    }
}
```

```
} else if (action[s, tok] = accept ) {
    return;
} else {
    // no entry in action table
    report syntax error;
    halt or attempt recovery;
}
```


Example



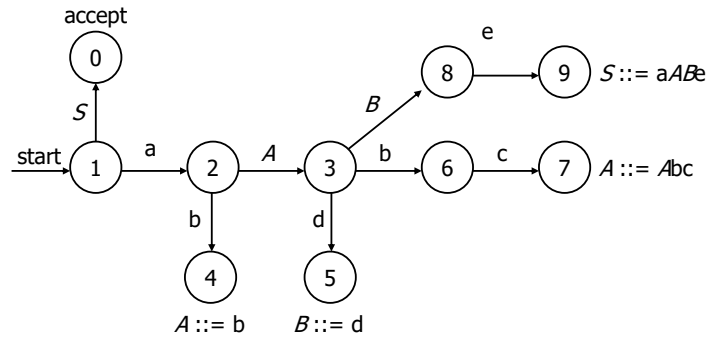
0. $S' ::= S\$$
1. $S ::= aABe$
2. $A ::= Abc$
3. $A ::= b$
4. $B ::= d$

Stack
\$1

Input
abbcde\$

S	action						goto		
	a	b	c	d	e	\$	A	B	S
0						ac			
1	s2								g0
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

Example



0. $S' ::= S\$$
1. $S ::= aABe$
2. $A ::= Abc$
3. $A ::= b$
4. $B ::= d$

Stack	Input
\$1	abbcde\$
\$1a2	bbcde\$
\$1a2b4	bcde\$
\$1a2A3	bcde\$
\$1a2A3b6	cde\$
\$1a2A3b6c7	de\$
\$1a2A3	de\$
\$1a2A3d5	e\$
\$1a2A3B8	e\$
\$1a2A3B8e9	\$
\$1S0	\$

S	action						goto		
	a	b	c	d	e	\$	A	B	S
0						ac			
1	s2								g0
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

LR States

- Idea is that each state encodes
 - The set of all possible productions that we could be looking at, given the current state of the parse, and
 - *Where* we are in the right hand side of each of those productions

Items

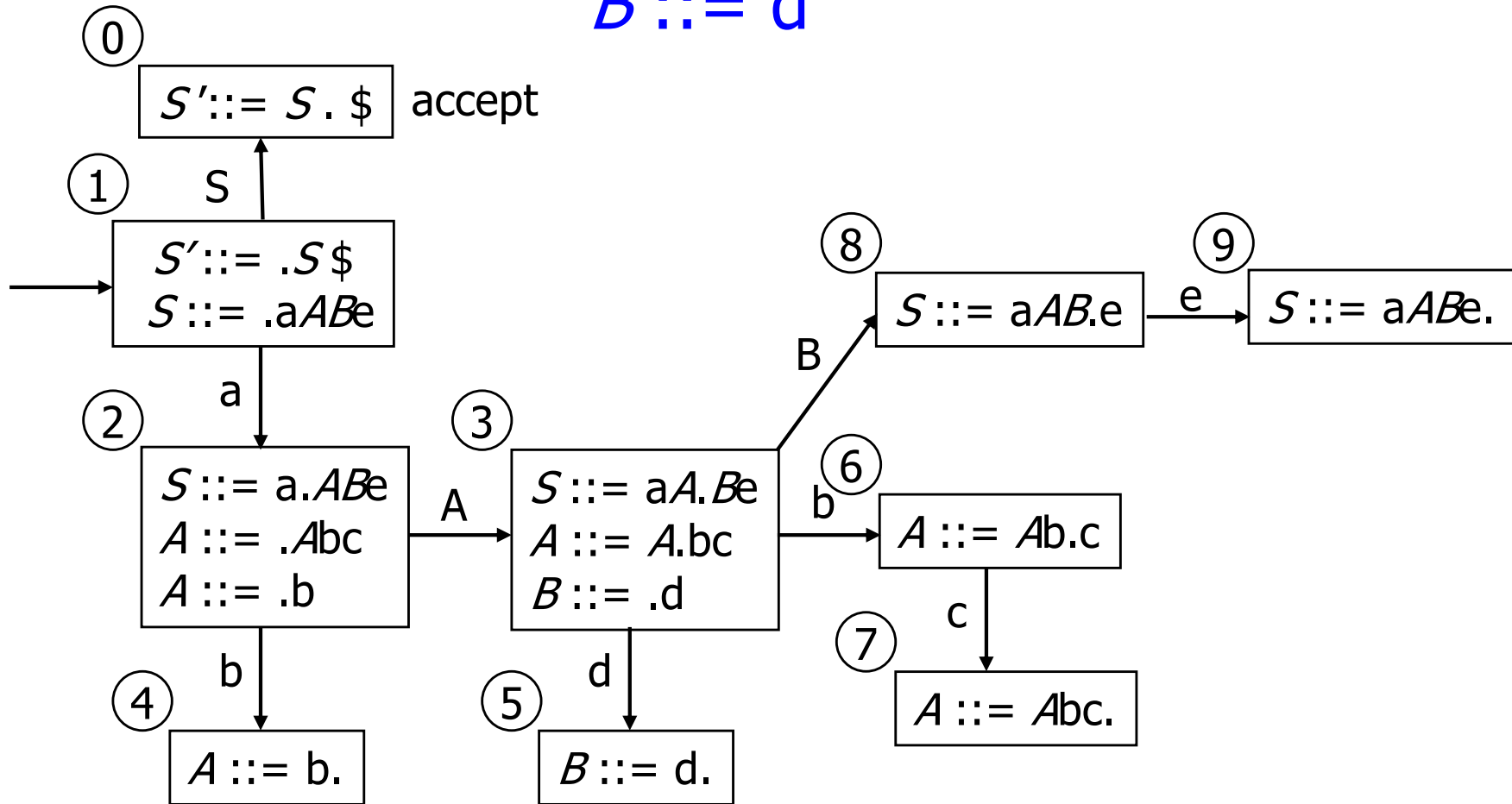
- An *item* is a production with a dot in the right hand side
- Example: Items for production $A ::= X Y$
 - $A ::= . X Y$
 - $A ::= X . Y$
 - $A ::= X Y .$
- Idea: The dot represents a position in the production

DFA for

$S ::= aABe$

$A ::= Abc \mid b$

$B ::= d$



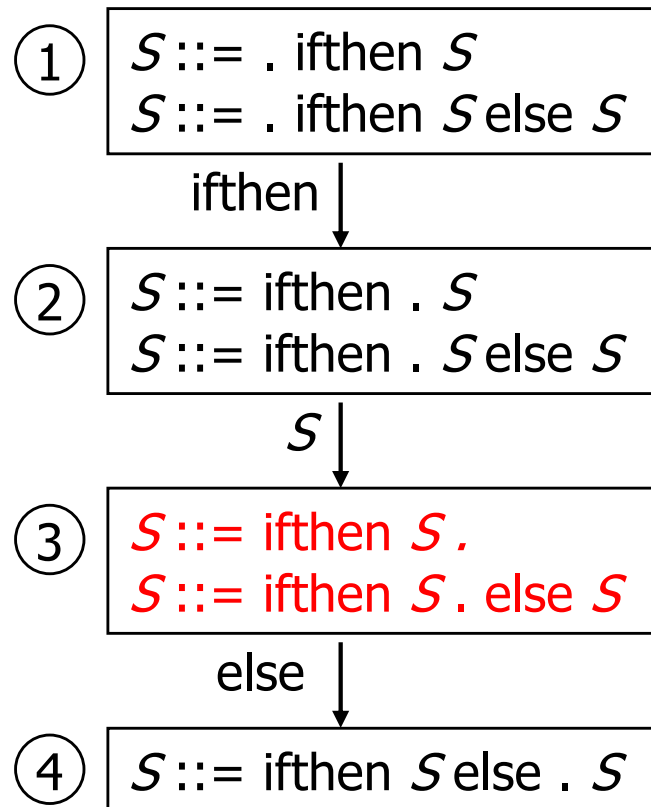
Problems with Grammars

- Grammars can cause problems when constructing a LR parser
 - Shift-reduce conflicts
 - Reduce-reduce conflicts

Shift-Reduce Conflicts

- Situation: both a shift and a reduce are possible at a given point in the parse (equivalently: in a particular state of the DFA)
- Classic example: if-else statement
 $S ::= \text{ifthen } S \mid \text{ifthen } S \text{ else } S$

Parser States for



1. $S ::= \text{ifthen } S$
2. $S ::= \text{ifthen } S \text{ else } S$

- State 3 has a shift-reduce conflict
 - Can shift past else into state 4 (s4)
 - Can reduce (r1)
 $S ::= \text{ifthen } S$

(Note: other $S ::= . \text{ifthen}$ items not included in states 2-4 to save space)

Solving Shift-Reduce Conflicts

- Fix the grammar
 - Done in Java reference grammar, others
- Use a parse tool with a “longest match” rule – i.e., if there is a conflict, choose to shift instead of reduce
 - Does exactly what we want for if-else case
 - Guideline: a few shift-reduce conflicts are fine, but be sure they do what you want (and that this behavior is guaranteed by the tool specification)

Reduce-Reduce Conflicts

- Situation: two different reductions are possible in a given state
- Contrived example

$S ::= A$

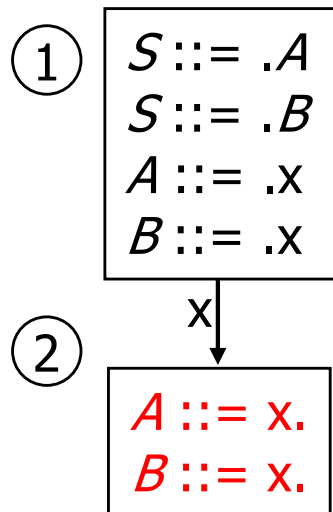
$S ::= B$

$A ::= x$

$B ::= x$

Parser States for

1. $S ::= A$
2. $S ::= B$
3. $A ::= x$
4. $B ::= x$



- State 2 has a reduce-reduce conflict (r3, r4)

Handling Reduce-Reduce Conflicts

- These normally indicate a serious problem with the grammar.
- Fixes
 - Use a different kind of parser generator that takes lookahead information into account when constructing the states
 - Most practical tools (Yacc, Bison, CUP, et al) do this
 - Fix the grammar

Another Reduce-Reduce Conflict

- Suppose the grammar tries to separate arithmetic and boolean expressions

$expr ::= aexp \mid bexp$

$aexp ::= aexp * aident \mid aident$

$bexp ::= bexp \&\& bident \mid bident$

$aident ::= id$

$bident ::= id$

- This will create a reduce-reduce conflict after recognizing *id*

Covering Grammars

- A solution is to merge *aident* and *bident* into a single non-terminal like *ident* (or just use *id* in place of *aident* and *bident* everywhere they appear)
- This is a *covering grammar*
 - Will generate some programs (sentences) that are not generated by the original grammar
 - Use the type checker or other static semantic analysis to weed out illegal programs later

Coming Attractions

- Constructing LR tables
 - We'll present a simple version (SLR(0)) in lecture, then talk about adding lookahead and then a little bit about how this relates to LALR(1) used in most parser generators
- LL parsers and recursive descent
- Continue reading ch. 3