Question 1. (12 points) Regular expressions. Our new summer intern is designing a programming language and has decided that the traditional syntax for comments is old and boring. The comments in this new programming language will be strings that start with the three characters <## and end with the two characters #>. Examples of comments: <## comment ##> <### xyzzy ###> <####>. Examples that are not comments: <##> (need at least <## at the beginning and #> at the end), <# # #> (first three characters <## can’t include spaces or other characters), etc.

(a) (6 points) Give a regular expression that generates strings representing comments as described above. (Hint: you may want to work on parts (a) and (b) at the same time.)

Ground rules (the fine print): You may only use the basic regular expression operations of concatenation, choice (|), and repetition (*) plus the derived operators ? and +, and simple character classes like [abc0-9] and[^a-z]. You may use abbreviations like vowels = [aeiou]. You may not use more complex operators found in various software tools that handle extended regular expressions and you should not use ‘\’ or other escape characters. If you need to differentiate between terminal characters and regular expression operators, underline the terminal characters to distinguish them or do something equally simple and easy to read.

< # # ([^#]* | (#*[^#>]))* #+ >

(b) (6 points) Draw a DFA that accepts comments as defined above.
Question 2. (8 points) Ambiguity. The syntax used to specify regular expressions can itself be defined by a context-free grammar. Here is one possible grammar for regular expressions with the operators concatenation, choice ( | ), Kleene star ( * ), and parenthesized subexpressions over the alphabet \{ a, b \}.

\[
\begin{align*}
R &::= R R & \text{(concatenation)} \\
R &::= R | R & \text{(the $|$ here is the literal regular expression choice operator)} \\
R &::= R * & \text{(Kleene star)} \\
R &::= ( R ) \\
R &::= a \\
R &::= b
\end{align*}
\]

Show that this grammar for specifying the syntax of regular expressions is ambiguous.

There are many possible examples, and it suffices to show either two distinct leftmost or rightmost derivations for string, or two distinct parse trees that generate the same string. Here are two leftmost derivations for \( aa | a \):

\[
\begin{align*}
R &\Rightarrow RR \Rightarrow aR \Rightarrow aR | R \Rightarrow aa | R \Rightarrow aa | a \\
R &\Rightarrow R | R \Rightarrow RR | R \Rightarrow aR | R \Rightarrow aa | R \Rightarrow aa | a
\end{align*}
\]

The corresponding parse trees also show the ambiguity.
Question 3. (20 points) The you’re-probably-not-surprised-to-see-it LR parsing question. Consider the following grammar.

0. $S' ::= S \ $  ($ is end-of-file)
1. $S ::= (S) \ T$
2. $S ::= x$
3. $T ::= x$

(a) (12 points) Draw the LR(0) state machine for this grammar. (You do not need to include the table with shift/reduce and goto actions, although you can write that out later if you find it useful to answer other parts of the question.)

(continued next page)
Question 3. (cont.) Grammar repeated for reference

0. \( S' ::= S \$
1. \( S ::= ( S ) T \)
2. \( S ::= \times \)
3. \( T ::= \times \)

(b) (4 points) Compute FIRST, FOLLOW, and Nullable for each of the non-terminals in this grammar.

<table>
<thead>
<tr>
<th>Non-terminal</th>
<th>FIRST</th>
<th>FOLLOW</th>
<th>nullable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S' )</td>
<td>( \times )</td>
<td></td>
<td>No</td>
</tr>
<tr>
<td>( S )</td>
<td>( \times )</td>
<td>) $</td>
<td>No</td>
</tr>
<tr>
<td>( T )</td>
<td>\times</td>
<td>) $</td>
<td>No</td>
</tr>
</tbody>
</table>

(c) (2 points) Is this grammar LR(0)? Why or why not?

Yes. No shift-reduce or reduce-reduce conflicts in the LR(0) state machine or tables.

(d) (2 points) Is this grammar SLR? Why or why not?

Yes. Same reason as (c), or any LR(0) grammar is also SLR.
Question 4. (8 points) (LL parsing/grammars) Here is the grammar from the previous question again:

0. $S' ::= S \$ 
1. $S ::= ( S ) T$
2. $S ::= x$
3. $T ::= x$

Is this a LL(1) grammar suitable for top-down predictive parsing? If yes, give a specific technical justification for your answer. If not, give a grammar that generates the same language and is LL(1) if that is possible. If no LL(1) grammar can generate the same language produced by the original grammar, give an explanation of why this is not possible.

Hint: You might want compute the FIRST/FOLLOW/nullable information for this grammar by answering that part of the previous question before you answer this question.

Yes, this grammar is LL(1). None of the non-terminals are nullable, so we don’t need to consider FOLLOW sets. The only non-terminal with more than one production is $S$, and the first sets of the right-hand sides of those productions are disjoint (\{ ( \} and \{ x \}), so we can write a predictive top-down parser for the grammar as given.
Question 5. (26 points) Compiler hacking. For this question we would like to add a new counting loop to MiniJava. (A copy of the MiniJava grammar is included at the end of the test for reference as needed.) For our new loop, we’ll add the following rule to the MiniJava grammar:

Statement ::= “for” Identifier “from” Expression “to” Expression “do” Statement

The idea is that the Statement in the loop body is executed repeatedly with the Identifier assigned successive integer values starting with the value of the first Expression and increasing by 1 each time the loop repeats until the final iteration where the Identifier has the value of the second Expression. For example, the following code stores the value 1 + 2 + … + 10 in variable `sum`:

```java
sum = 0;
for i from 1 to 10 do sum = sum + i;
```

The Identifier in the `for` statement must have been declared previously and must have type `int`. The two Expressions are only evaluated once, before the Identifier is assigned its initial value and before the loop body executes. The Expressions are not re-evaluated again as the loop executes. So, for example, the following code has exactly the same effect as the previous example:

```java
sum = 0; i = 0;
for i from i+1 to i+10 do sum = sum + i;
```

In other words, the loop bounds `i+1` and `i+10` are evaluated before the loop begins execution and before the initial assignment to `i`, and are not reevaluated again. The Expressions are evaluated in order from left to right. If the value of the first Expression is greater than the value of the second Expression, then the body of the loop is not executed. The value in the Identifier is not defined after the loop terminates – it might be equal to the value of one of the Expressions, or it could have any other value depending on what the implementation does.

(a) (3 points) What new tokens and/or keywords would need to be added to the scanner and parser of our MiniJava compiler to add this new `for` statement to the original MiniJava grammar? Just list the tokens; you don’t need to give JFlex or CUP specifications for them.

**We need tokens for the four new keywords: FOR, FROM, TO, and DO**
Question 5. (cont.) (b) (5 points) Complete the following new AST class to define an AST node type for the new for statement. You only need to define instance variables and the constructor. Assume that all appropriate package and import declarations are supplied, and don’t worry about visitor code.

(Hint: recall that the AST package in MiniJava contains the following key classes: ASTNode, Exp extends ASTNode, Statement extends ASTNode, and Identifier extends ASTNode. Also remember that each AST node constructor has a Location parameter.)

```java
public class For extends Statement {
    // add instance variables below

    public Identifier id;
    public Exp e1, e2;
    public Statement s;

    // constructor - add parameters and method body below

    public For( Identifier id, Exp e1, Exp e2, Statement s, Location pos ){
        super(pos);
        this.id = id;
        this.e1 = e1;
        this.e2 = e2;
        this.s = s;
    }
}
```

Note: Constructor parameters could be in a different order as long as that order matched the AST node creation code in the CUP semantic action for the new comparison operator, below.

(continued on next page)
Question 5. (cont.) (c) (5 points) Complete the CUP specification below to define a production for the new for statement with the associated semantic action(s) needed to parse a for statement and create an appropriate For node (as defined in part (b) above) into the AST. We have added the necessary additional code to the parser rule for Statement to get started.

Hint: recall that the Location of an item foo in a CUP grammar production can be referenced as foo::left.

Statement ::= ... 
| ForStatement::s { : RESULT = s; :} 
... 
;

ForStatement ::= FOR Identifier::i FROM Exp::e1 TO Exp::e2 DO 
Statement::s 
{: RESULT = new For(i,e1,e2,s,ix::left) :}

(d) (5 points) Describe the checks that would be needed in the semantics/type-checking part of the compiler to verify that a for statement is legal. You do not need to give code for a visitor method or anything like that – just describe what language rules (if any) need to be checked.

- Verify that the for variable has been declared and has type int
- Verify that expressions e1 and e2 have type int

(continued on next page)
Question 5. (cont.) (e) (8 points) Show the code shape for this new for statement, i.e., what code should be generated in the assembly language program to properly execute a for loop as specified on the previous pages. You should show instructions, labels, and other assembly language-level details that are needed for the new for loop statement itself, and also show where the generated code for the two Expressions and Statement that makes up the loop body would appear. Your code does not need to be precisely correct x86-64 assembly code (i.e., you were not expected to memorize instruction details), but it should be close enough so that your intent is clear and it basically equivalent to real x86-64 code.

Hint: you probably won’t need all the space on this page for your answer. But be careful that your code matches the described operation of the for statement given previously.

This answer uses a strategy similar to the code shape outlined for our MiniJava compilers. Values are pushed on the stack to save them if they will be needed later. The code arranges for the value of the second expression (the limit) to be stored on the top of the stack when the statement that makes up the loop body is executed. We are also careful to evaluate both loop expressions before assigning to the loop variable. We also assume the loop variable is a local variable in the stack frame. The code could be generalized to allow the loop variable to be elsewhere but this shows the key ideas. Other reasonable pseudo-code was fine if done correctly.

```assembly
<code to evaluate first expression and leave value in %rax>
pushq  %rax          # save initial expression value
<code to evaluate second expression and leave value in %rax>
popq   %rdx          # reload initial expression value
movq   %rdx,offsetvar(%rbp) # store initial expression in loop variable
pushq  %rax          # save second expression on top of stack

test:
    movq  offsetvar(%rbp),%rax  # load variable into %rax
    popq  %rdx                   # load limit into %rdx
    cmpq  %rdx,%rax              # set cond codes with var-limit
    jg    done                  # exit if var-limit > 0, i.e., var > limit
    pushq %rdx                  # push limit back on stack
<code for loop body statement>
movq  offsetvar(%rbp),%rax  # increment variable by 1
addq  $1,%rax
movq  %rax,offsetvar(%rbp)  # repeat loop
jmp    test                  # end of loop; second (limit) expression

done:                             # already popped from stack here
```

Page 9 of 13
The remaining questions concern the following control flow graph.

The rest of this page contains reference material and definitions that might be useful when answering some of the remaining questions.

You should remove this page from the exam and use it while answering the remaining questions. Do not write on this page – it will not be scanned for grading.

Reference Material

Every control flow graph has a unique start node $s_0$.

Node $x$ dominates node $y$ if every path from $s_0$ to $y$ must go through $x$.
   - A node $x$ dominates itself.

A node $x$ strictly dominates node $y$ if $x$ dominates $y$ and $x \neq y$.

The dominator set of a node $y$ is the set of all nodes $x$ that dominate $y$.

An immediate dominator of a node $y$, idom($y$), has the following properties:
   - idom($y$) strictly dominates $y$ (i.e., dominates $y$ but is different from $y$)
   - idom($y$) does not dominate any other strict dominator of $y$

A node might not have an immediate dominator. A node has at most one immediate dominator.

The dominator tree of a control flow graph is a tree where there is an edge from every node $x$ to its immediate dominator idom($x$).

The dominance frontier of a node $x$ is the set of all nodes $y$ such that
   - $x$ dominates a predecessor of $y$, but
   - $x$ does not strictly dominate $y$
**Question 6.** (18 points) Dataflow analysis – available expressions.

Recall from lecture that an expression \( e \) is **available** at a program point \( p \) if every path leading to point \( p \) contains a prior definition of expression \( e \) and \( e \) is not killed along a path from a prior definition by having one of its operands re-defined on that path.

We would like to compute the set of available expressions at the beginning of each basic block in the flowgraph shown on the previous page.

For each basic block \( b \) we define the following sets:

- \( \text{AVAIL}(b) \) = the set of expressions available on entry to block \( b \)
- \( \text{NKILL}(b) \) = the set of expressions **not killed** in \( b \) (i.e., all expressions defined somewhere in the flowgraph except for those killed in \( b \))
- \( \text{DEF}(b) \) = the set of all expressions defined in \( b \) and not subsequently killed in \( b \)

The dataflow equation relating these sets is:

\[
\text{AVAIL}(b) = \bigcap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))
\]

i.e., the expressions available on entry to block \( b \) are the intersection of the sets of expressions available on exit from all of its predecessor blocks \( x \) in the flow graph.

On the next page, calculate the \( \text{DEF} \) and \( \text{NKILL} \) sets for each block, then use that information to calculate the \( \text{AVAIL} \) sets for each block. You will only need to calculate the \( \text{DEF} \) and \( \text{NKILL} \) sets once for each block. You may need to re-calculate some of the \( \text{AVAIL} \) sets more than once as information about predecessor blocks change.

Hint: notice that there are only three expressions calculated in this flowgraph: \( a+b \), \( b+c \), and \( c+d \). So all of the \( \text{AVAIL} \), \( \text{NKILL} \), and \( \text{DEF} \) sets for the different blocks will contain some, none, or all of those three expressions.

You should **remove this page from the exam** and use it while answering this question. **Do not write on this page** – it will not be scanned for grading.
Question 6. (cont.) (a) (8 points) For each of the blocks B0, B1, B2, and B3, write their DEF and NKILL sets in the table below.

<table>
<thead>
<tr>
<th>Block</th>
<th>DEF</th>
<th>NKILL</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>{ a + b, c + d }</td>
<td>{ a + b, b + c, c + d }</td>
</tr>
<tr>
<td>B1</td>
<td>{ b + c, c + d }</td>
<td>{ b + c, c + d }</td>
</tr>
<tr>
<td>B2</td>
<td>{ b + c }</td>
<td>{ b + c, c + d }</td>
</tr>
<tr>
<td>B3</td>
<td>{ b + c }</td>
<td>{ a + b, b + c, c + d }</td>
</tr>
</tbody>
</table>

(b) (10 points) Now, give the AVAIL sets showing the expressions available on entry to each block in the table below. If you need to update this information as you calculate the sets, be sure to cross out previous information so it is clear what your final answer is.

<table>
<thead>
<tr>
<th>Block</th>
<th>AVAIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>{ }</td>
</tr>
<tr>
<td>B1</td>
<td>{ c + d }</td>
</tr>
<tr>
<td>B2</td>
<td>{ a + b, c + d }</td>
</tr>
<tr>
<td>B3</td>
<td>{ b + c, c + d }</td>
</tr>
</tbody>
</table>
**Question 7.** (18 points) Dominators and SSA. (a) (8 points) Using the same control flow graph from the previous problem, complete the following table. List for each node: the nodes that dominate it, the node that is its immediate dominator (if any), and the nodes that are in its dominance frontier (if any):

<table>
<thead>
<tr>
<th>Node</th>
<th>Dominators</th>
<th>IDOM</th>
<th>Dominance Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>B0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>B1</td>
<td>B0, B1</td>
<td>B0</td>
<td>B3</td>
</tr>
<tr>
<td>B2</td>
<td>B0, B2</td>
<td>B0</td>
<td>B3</td>
</tr>
<tr>
<td>B3</td>
<td>B0, B3</td>
<td>B0</td>
<td>B1</td>
</tr>
</tbody>
</table>

(b) (10 points) Now redraw the flowgraph in SSA (static single-assignment) form. You need to insert appropriate Φ-functions where they are required and, once that is done, add appropriate version numbers to all variables that are assigned in the flowgraph. You should not insert extra Φ-functions at the beginning of a block if they clearly would not be appropriate there, but we will not penalize a few extraneous Φ-functions if they are correct, but possibly not needed. You do not need to trace the steps of any particular algorithm to place the Φ-functions as long as you add them to the flowgraph in appropriate places.

**Note:** this solution includes all of the Φ functions placed by the dominance frontier algorithm (which is the same set of functions place by the path-convergence criteria).