CSE P 501 – Compilers

Dataflow Analysis

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Agenda

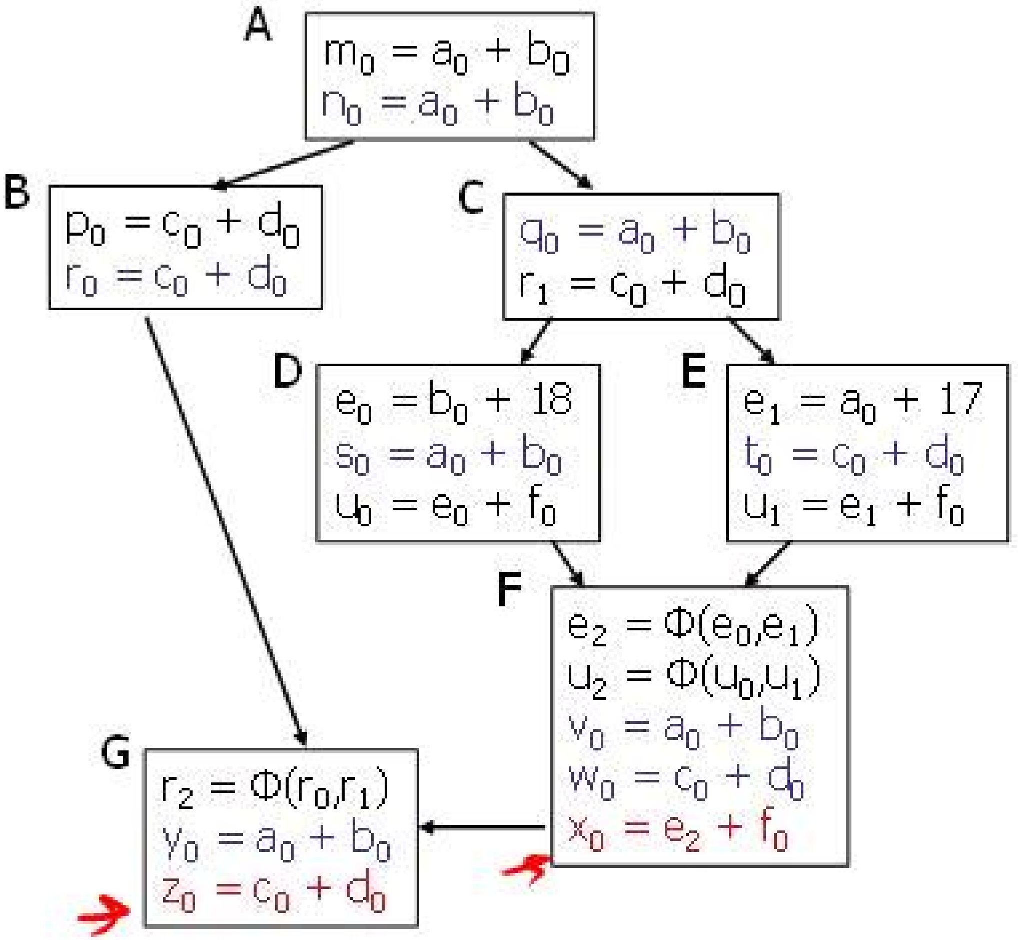
- Initial example: dataflow analysis for common subexpression elimination
- Other analysis problems that work in the same framework

The Story So Far…

- Redundant expression elimination
  - Local Value Numbering
  - Superlocal Value Numbering
    - Extends VN to EBBs
    - SSA-like namespace
  - Dominator VN Technique (DVNT)
- All of these propagate along forward edges
- None are global
  - In particular, can't handle back edges (loops)

Dominator Value Numbering

- Most sophisticated algorithm so far
- Still misses some opportunities
- Can't handle loops



Available Expressions

- Goal: use dataflow analysis to find common subexpressions whose range spans basic blocks
- Idea: calculate *available expressions* at beginning of each basic block
- Avoid re-evaluation of an available expression – use a copy operation

"Available" and Other Terms

a+b

- An expression *e* is *defined* at point *p* in the CFG if its value is computed at *p*
  - Sometimes called *definition site*

p: a:= ~e~

- An expression *e* is *killed* at point *p* if one of its operands is defined at *p*
  - Sometimes called *kill site*

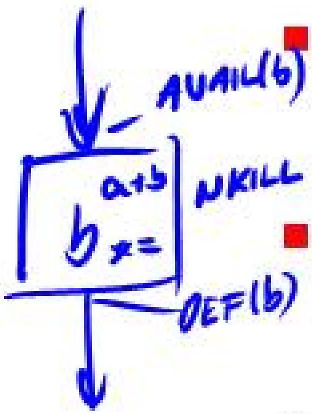
a+b a+b

p: ~a+b~

- An expression *e* is *available* at point *p* if every path leading to *p* contains a prior definition of *e* and *e* is not killed between that definition and *p*

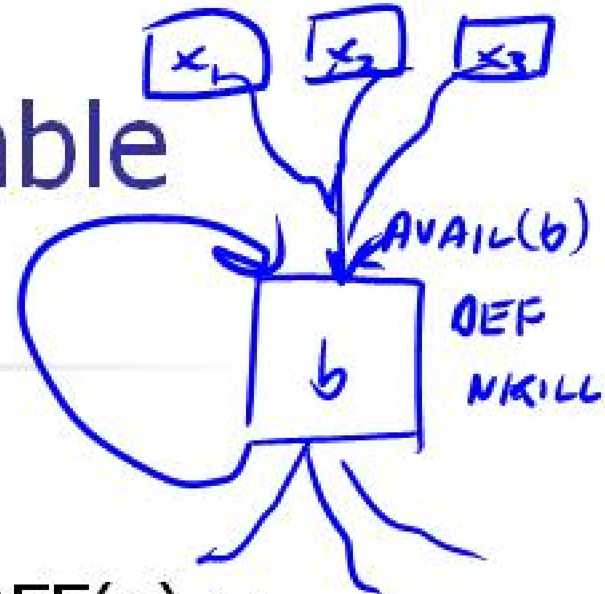
Available Expression Sets

- For each block *b*, define



  - AVAIL(b) – the set of expressions available on entry to *b*
  - NKILL(b) – the set of expressions <u>not killed</u> in *b*
  - DEF(b) – the set of expressions defined in *b* and not subsequently killed in *b*

Computing Available Expressions



- AVAIL(b) is the set

  $$\mathrm{AVAIL}(b) = \bigcap_{x \in \mathrm{preds}(b)} (\mathrm{DEF}(x) \cup (\mathrm{AVAIL}(x) \cap \mathrm{NKILL}(x)))$$

  - preds(b) is the set of b's predecessors in the control flow graph
- This gives a system of simultaneous equations – a dataflow problem !

Name Space Issues

- In previous value-numbering algorithms, we used a SSA-like renaming to keep track of versions
- In global dataflow problems, we use the original namespace
  - The KILL information captures when a value is no longer available

GCSE with Available Expressions

once

- For each block b, compute DEF(b) and NKILL(b)

iterate

- For each block b, compute AVAIL(b)
- For each block b, value number the block starting with AVAIL(b)
- Replace expressions in AVAIL(b) with references to the previously computed values

Global CSE Replacement

- After analysis and before transformation, assign a global name to each expression *e* by hashing on *e*
- During transformation step
  - At each evaluation of *e*, insert copy

    name(*e*) = *e*
  - At each reference to *e*, replace *e* with *name*(*e*)

Analysis

- Main problem – inserts extraneous copies at all definitions and uses of every *e* that appears in any AVAIL(b)
  - But the extra copies are dead and easy to remove
  - Useful copies often coalesce away when registers and temporaries are assigned
- Common strategy
  - Insert copies that might be useful
  - Let dead code elimination sort it out later

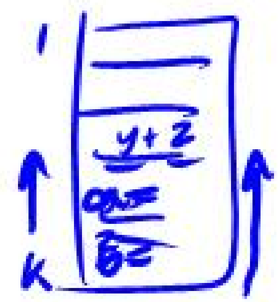
Computing Available Expressions

- Big Picture
  - Build control-flow graph
  - Calculate initial local data – DEF(b) and NKILL(b)
    - This only needs to be done once
  - Iteratively calculate AVAIL(b) by repeatedly evaluating equations until nothing changes
    - Another fixed-point algorithm

Computing DEF and NKILL (1)

- For each block *b* with operations $o_1, o_2, \ldots, o_k$

```
KILLED = ∅
DEF(b) = ∅
for i = k to 1
    assume o_i is "x = y + z"
    if (y ∉ KILLED and z ∉ KILLED)
        add "y + z" to DEF(b)
    add x to KILLED
…
```



Computing DEF and NKILL (2)

- After computing DEF and KILLED for a block b,

  a+b   x+y

  NKILL(b) = { all expressions }

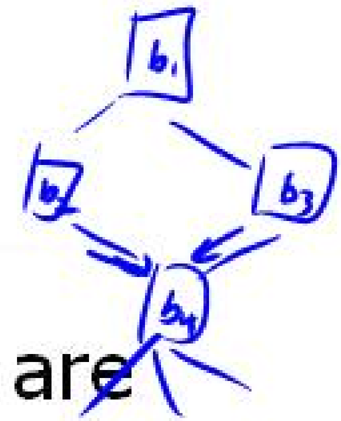
  for each expression *e*

    for each variable $v \in$ e

      if $v \in$ KILLED then

        NKILL(b) = NKILL(b) - *e*

Computing Available Expressions

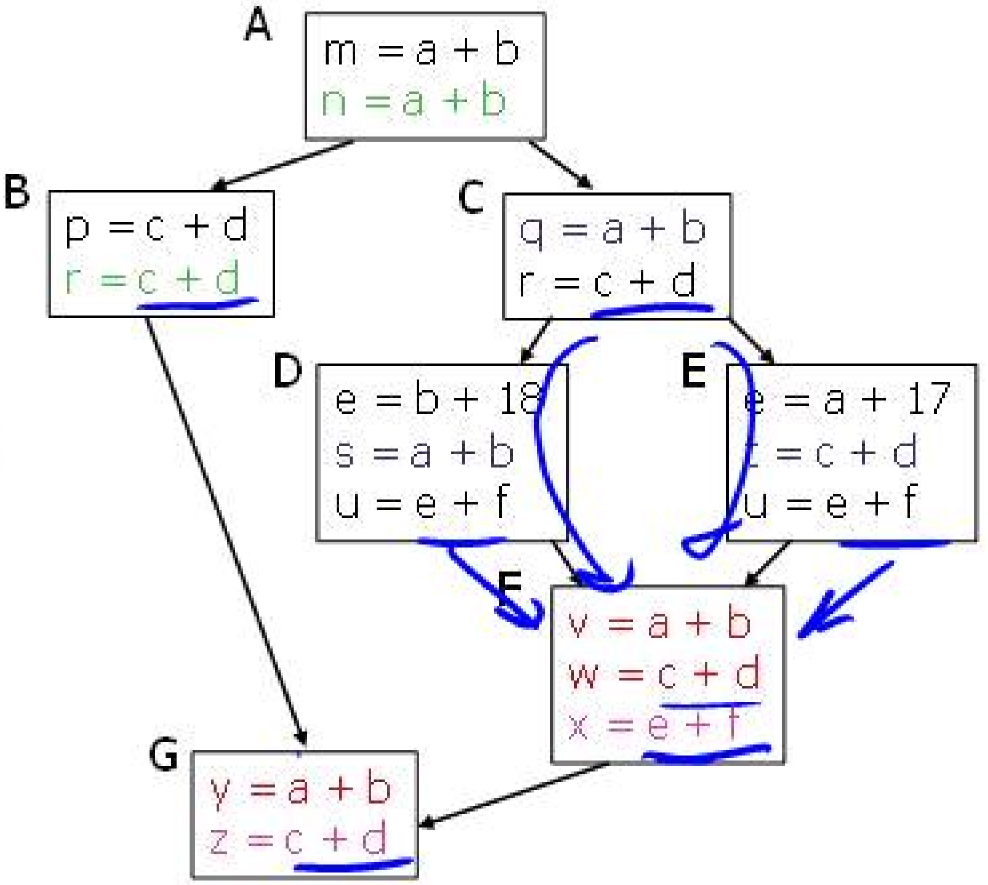


- Once DEF(b) and NKILL(b) are computed for all blocks b:

```
Worklist = { all blocks b }
while (Worklist ≠ ∅)
    remove a block b from Worklist
    recompute AVAIL(b)
    if AVAIL(b) changed
        Worklist = Worklist ∪ successors(b)
```

Comparing Algorithms

- LVN – Local Value Numbering
- SVN – Superlocal Value Numbering
- DVN – Dominator-based Value Numbering
- GRE – Global Redundancy Elimination



Comparing Algorithms (2)

- LVN => SVN => DVN form a strict hierarchy – later algorithms find a superset of previous information
- Global RE finds a somewhat different set
  - Discovers e+f in F (computed in both D and E)
  - Misses identical values if they have different names (e.g., a+b and c+d when a=c and b=d)
    - Value Numbering catches this

17 42

Dataflow analysis

- Global redundancy elimination is the first example of a *dataflow analysis* problem
- Many similar problems can be expressed in a similar framework
- Only the first part of the story – once we've discovered facts, we then need to use them to improve code

Dataflow Analysis (1)

- A collection of techniques for compile-time reasoning about run-time values
- Almost always involves building a graph
  - Trivial for basic blocks
  - Control-flow graph or derivative for global problems
  - Call graph or derivative for whole-program problems

Dataflow Analysis (2)

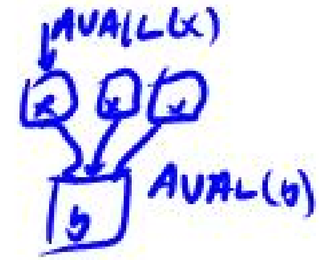
- Usually formulated as a set of *simultaneous equations* (dataflow problem)
  - Sets attached to nodes and edges
  - Need a lattice (or semilattice) to describe values
    - In particular, has an appropriate operator to combine values and an appropriate "bottom" or minimal value

Dataflow Analysis (3)

- Desired solution is usually a *meet over all paths* (MOP) solution
  - "What is true on every path from entry"
  - "What can happen on any path from entry"
  - Usually relates to safety of optimization

Dataflow Analysis (4)

- Limitations
  - Precision – "up to symbolic execution"
    - Assumes all paths taken
  - Sometimes cannot afford to compute full solution
  - Arrays – classic analysis treats each array as a single fact
  - Pointers – difficult, expensive to analyze
    - Imprecision rapidly adds up
- For scalar values we can quickly solve simple problems

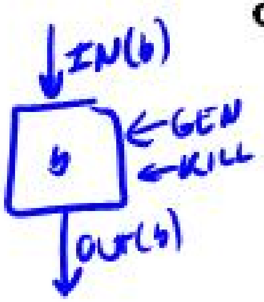


Example: Available Expressions

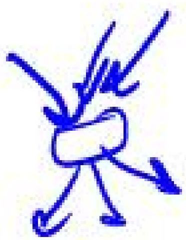
- This is the analysis we did earlier to eliminate redundant expression evaluations
- Equation:

$$\text{AVAIL}(b) = \bigcap_{x \in \text{preds}(b)} \left(\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x))\right)$$

Characterizing Dataflow Analysis



- All of these algorithms involve sets of facts about each basic block b
  - IN(b) – facts true on entry to b
  - OUT(b) – facts true on exit from b
  - GEN(b) – facts created and not killed in b
  - KILL(b) – facts killed in b



- These are related by the equation

$$OUT(b) = GEN(b) \cup (IN(b) - KILL(b))$$

  - Solve this iteratively for all blocks
  - Sometimes information propagates forward; sometimes backward

Example:Live Variable Analysis

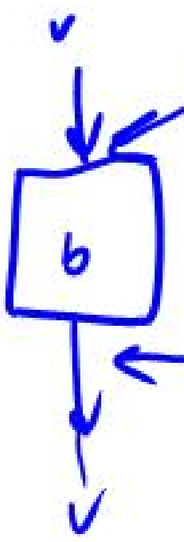
- A variable *v* is *live* at point *p* iff there is *any* path from *p* to a use of *v* along which *v* is not redefined
- Some uses:
  - Register allocation – only live variables need a register (or temporary)
  - Eliminating useless stores
  - Detecting uses of uninitialized variables
  - Improve SSA construction – only need Φ-function for variables that are live in a block (later)

p: ~v~

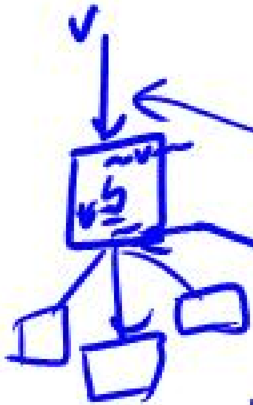
~v~

Liveness Analysis Sets

- For each block b, define
  - use[b] = variable used in b before any def
  - def[b] = variable defined in b & not killed
  - in[b] = variables live on entry to b
  - out[b] = variables live on exit from b



Equations for Live Variables

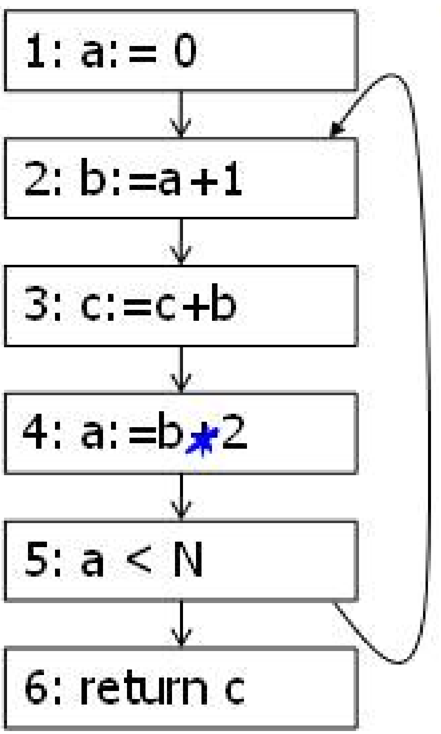


- Given the preceding definitions, we have
  - $in[b] = use[b] \cup (out[b] - def[b])$
  - $out[b] = \cup_{s \in succ[b]} in[s]$
- Algorithm
  - Set $in[b] = out[b] = \emptyset$
  - Update in, out until no change

Example (1 stmt per block)

- Code

```
    a := 0
L:  b := a+1
    c := c+b
    a := b*2
    if a < N goto L
    return c
```

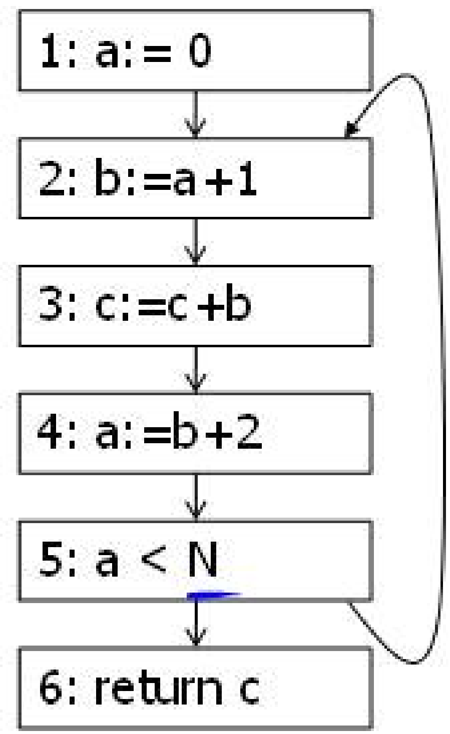


Calculation

$in[b] = use[b] \cup (out[b] - def[b])$

$out[b] = \cup_{s \in succ[b]} in[s]$

			I		II		III	
block	use	def	out	in	out	in	out	in
6	c	—	—	c	—	c		
5	a	—	c	a,c	a,c	a,c		same
4	b	a	a,c	b,c	a,c	b,c		
3	b,c	c	b,c	b,c	b,c	b,c		
2	a	b	b,c	a,c	b,c	a,c		
1	—	a	a,c	c	a,c	c		



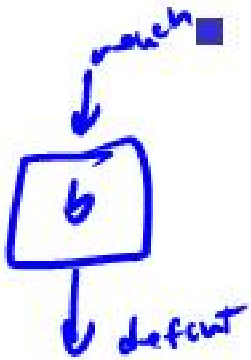
Equations for Live Variables v2

- Many problems have more than one formulation. For example, Live Variables…
- Sets
  - USED(b) – variables used in b before being defined in b
  - NOTDEF(b) – variables not defined in b
  - LIVE(b) – variables live on *exit* from b
- Equation
  - $\text{LIVE}(b) = \cup_{s \in succ(b)} \text{USED}(s) \cup (\text{LIVE}(s) \cap \text{NOTDEF}(s))$

Example: Reaching Definitions

- A definition *d* of some variable *v* *reaches* operation *i* iff *i* reads the value of *v* and there is a path from *d* to *i* that does not define *v*
- Use:
  - Find all of the possible definition points for a variable in an expression

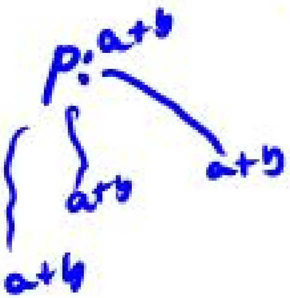
Equations for Reaching Definitions



- Sets
  - DEFOUT(b) – set of definitions in b that reach the end of b (i.e., not subsequently redefined in b)
  - SURVIVED(b) – set of all definitions not obscured by a definition in b
  - REACHES(b) – set of definitions that reach b
- Equation

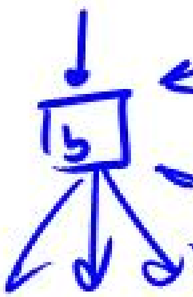
$$\mathrm{REACHES}(b) = \bigcup_{p \in preds(b)} \mathrm{DEFOUT}(p) \cup (\mathrm{REACHES}(p) \cap \mathrm{SURVIVED}(p))$$

Example: Very Busy Expressions



- An expression *e* is considered *very busy* at some point *p* if *e* is evaluated and used along every path that leaves *p*, and evaluating *e* at *p* would produce the same result as evaluating it at the original locations
- Use:
  - Code hoisting – move *e* to *p* (reduces code size; no effect on execution time)

Equations for Very Busy Expressions



- Sets
  - USED(b) – expressions used in b before they are killed
  - KILLED(b) – expressions redefined in b before they are used
  - VERYBUSY(b) – expressions very busy on exit from b
- Equation

$$\text{VERYBUSY}(b) = \bigcap_{s \in \text{succ}(b)} \text{USED}(s) \cup (\text{VERYBUSY}(s) - \text{KILLED}(s))$$

Efficiency of Dataflow Analysis

- The algorithms eventually terminate, but the expected time needed can be reduced by picking a good order to visit nodes in the CFG depending on how information flows
  - Forward problems – reverse postorder
  - Backward problems - postorder

Using Dataflow Information

- A few examples of possible tranformations…

Classic Common-Subexpression Elimination

- In a statement s: t := x op y, if x op y is *available* at s then it need not be recomputed
- Analysis: compute *reaching expressions* i.e., statements n: v := x op y such that the path from n to s does not compute x op y or define x or y

Classic CSE

- If x op y is defined at n and reaches s
  - Create new temporary w
  - Rewrite n as

    n: w := x op y

    n′: v := w
  - Modify statement s to be

    s: t := w

  - (Rely on copy propagation to remove extra assignments if not really needed)

Constant Propagation

- Suppose we have
  - Statement d: t := c, where c is constant
  - Statement n that uses t
- If d reaches n and no other definitions of t reach n, then rewrite n to use c instead of t

d

n $\cancel{t}$ c

Copy Propagation

- Similar to constant propagation
- Setup:
  - Statement d: t := z
  - Statement n uses t
- If d reaches n and no other definition of t reaches n, and there is no definition of z on any path from d to n, then rewrite n to use z instead of t

Copy Propagation Tradeoffs

- Downside is that this can increase the lifetime of variable z and increase need for registers or memory traffic
  - Not worth doing if only reason is to eliminate copies – let the register allocate deal with that
- But it can expose other optimizations, e.g.,

  a := y + z
  u := y
  c := ~~u~~ y + z

  - After copy propagation we can recognize the common subexpression

Dead Code Elimination

- If we have an instruction

  s: a := b op c
  b / c

  a:=

  and a is not live-out after s, then s can be eliminated
  - Provided it has no implicit side effects that are visible (output, exceptions, etc.)

Aliases

- A variable or memory location may have multiple names or *aliases*
  - Call-by-reference parameters
  - Variables whose address is taken (&x)
  - Expressions that dereference pointers (p.x, *p)
  - Expressions involving subscripts (a[i])
  - Variables in nested scopes