CSE P 501 – Compilers

LR Parser Construction

Hal Perkins

Autumn 2009

Agenda

- LR(0) state construction
- FIRST, FOLLOW, and nullable
- Variations: SLR, LR(1), LALR

LR State Machine

- Idea: Build a DFA that recognizes handles
  - Language generated by a CFG is generally not regular, but
  - Language of handles for a CFG is regular
    - So a DFA can be used to recognize handles
  - Parser reduces when DFA accepts

Prefixes, Handles, &c (review)

- If S is the start symbol of a grammar G,
  - If $S \Rightarrow^* \alpha$ then α is a *sentential form* of G
  - γ is a *viable prefix* of G if there is some derivation $S \Rightarrow^*_{rm} \alpha A w \Rightarrow^*_{rm} \alpha\beta w$ and γ is a prefix of $\alpha\beta$.
  - The occurrence of β in $\alpha\beta w$ is a *handle* of $\alpha\beta w$
- An *item* is a marked production (a . at some position in the right hand side)
  - [*A* ::= . *X Y*]   [*A* ::= *X* . *Y*]   [*A* ::= *X Y* .]

Building the LR(0) States

- Example grammar

  $S' ::= S\,\$$

  $S ::= (\ L\ )$

  $S ::= \mathrm{x}$

  $L ::= S$

  $L ::= L\ ,\ S$

  - We add a production S' with the original start symbol followed by end of file ($)
  - Question: What language does this grammar generate?

0. $S' ::= S \$$
1. $S ::= (L)$
2. $S ::= x$
3. $L ::= S$
4. $L ::= L , S$

Start of LR Parse

- Initially
  - Stack is empty
  - Input is the right hand side of S', i.e., $S \$$
  - Initial configuration is $[S' ::= \,.\, S \$]$
  - But, since position is just before S, we are also just before anything that can be derived from S

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Initial state

$$\boxed{\begin{array}{l} S' ::= \,.\, S\,\$ \\ S ::= \,.\, (\ L\ ) \\ S ::= \,.\, \mathrm{x} \end{array}}$$

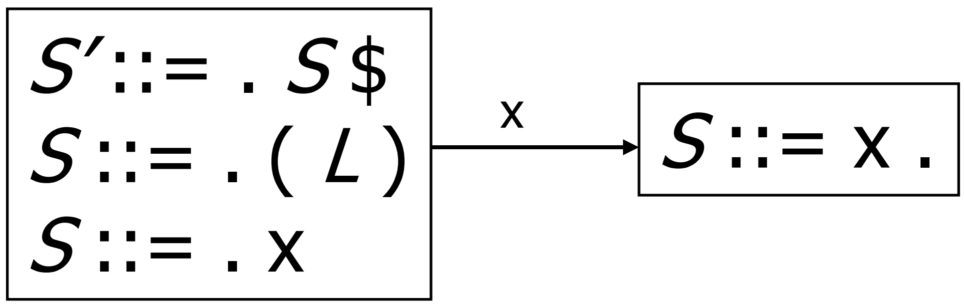
start

completion

- A state is just a set of items
  - Start: an initial set of items
  - Completion (or closure): additional productions whose left hand side appears to the right of the dot in some item already in the state

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Shift Actions (1)



- To shift past the x, add a new state with the appropriate item(s)
  - In this case, a single item; the closure adds nothing
  - This state will lead to a reduction since no further shift is possible

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Shift Actions (2)

$$\boxed{\begin{array}{l} S' ::= .\ S\ \$ \\ S ::= .\ (\ L\ ) \\ S ::= .\ \mathrm{x} \end{array}} \xrightarrow{\ (\ } \boxed{\begin{array}{l} S ::= (\ .\ L\ ) \\ L ::= .\ L\ ,\ S \\ L ::= .\ S \\ S ::= .\ (\ L\ ) \\ S ::= .\ \mathrm{x} \end{array}}$$

- If we shift past the (, we are at the beginning of *L*
- the closure adds all productions that start with *L*, which requires adding all productions starting with *S*

Goto Actions

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

$$\boxed{\begin{array}{l} S' ::= .\ S\,\$ \\ S ::= .\ (\ L\ ) \\ S ::= .\ \mathrm{x} \end{array}} \xrightarrow{S} \boxed{S' ::= S\ .\ \$}$$

- Once we reduce *S*, we'll pop the rhs from the stack exposing the first state. Add a *goto* transition on *S* for this.

Basic Operations

- *Closure* (*S*)
  - Adds all items implied by items already in *S*
- *Goto* (*I*, *X*)
  - *I* is a set of items
  - *X* is a grammar symbol (terminal or non-terminal)
  - *Goto* moves the dot past the symbol *X* in all appropriate items in set *I*

Closure Algorithm

- *Closure* (*S*) =

  repeat

    for any item [A ::= α . $X\,\beta$] in *S*

      for all productions X ::= γ

        add [X ::= . γ] to *S*

  until *S* does not change

  return *S*

Goto Algorithm

- *Goto* (*I*, *X*) =

  set *new* to the empty set

  for each item [A ::= α . *X* β] in *I*

  add [A ::= α *X* . β] to *new*

  return *Closure* (*new*)

  - This may create a new state, or may return an existing one

LR(0) Construction

- First, augment the grammar with an extra start production $S' ::= S\,\$$
- Let T be the set of states
- Let E be the set of edges
- Initialize T to $Closure$([$S' ::= .\ S\,\$$])
- Initialize E to empty

LR(0) Construction Algorithm

repeat
  for each state I in T
    for each item $[A ::= \alpha \, . \, X \; \beta]$ in I
      Let *new* be *Goto*(I, X)
      Add *new* to T if not present
      Add $I \xrightarrow{X}$ *new* to E if not present
until E and T do not change in this iteration

- Footnote: For symbol \$, we don't compute *goto* (I, \$); instead, we make this an *accept* action.

LR(0) Reduce Actions

- Algorithm:

  Initialize R to empty

  for each state I in T

  for each item $[A ::= \alpha\ .]$ in I

  add $(I, A ::= \alpha)$ to R

Building the Parse Tables (1)

- For each edge $I \xrightarrow{X} J$
  - if X is a terminal, put sj in column X, row I of the action table (shift to state j)
  - If X is a non-terminal, put gj in column X, row I of the goto table

Building the Parse Tables (2)

- For each state I containing an item $[S' ::= S\ .\ \$]$, put *accept* in column \$ of row I
- Finally, for any state containing $[A ::= \gamma\ .]$ put action r*n* in every column of row I in the table, where *n* is the production number

Example: States for

0. $S' ::= S \$$
1. $S ::= (L)$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L , S$

Example: Tables for

0. $S' ::= S\,\$$
1. $S ::= (\ L\ )$
2. $S ::= \mathrm{x}$
3. $L ::= S$
4. $L ::= L\ ,\ S$

Where Do We Stand?

- We have built the LR(0) state machine and parser tables
  - No lookahead yet
  - Different variations of LR parsers add lookahead information, but basic idea of states, closures, and edges remains the same

A Grammar that is not LR(0)

- Build the state machine and parse tables for a simple expression grammar

  $S ::= E\,\$$

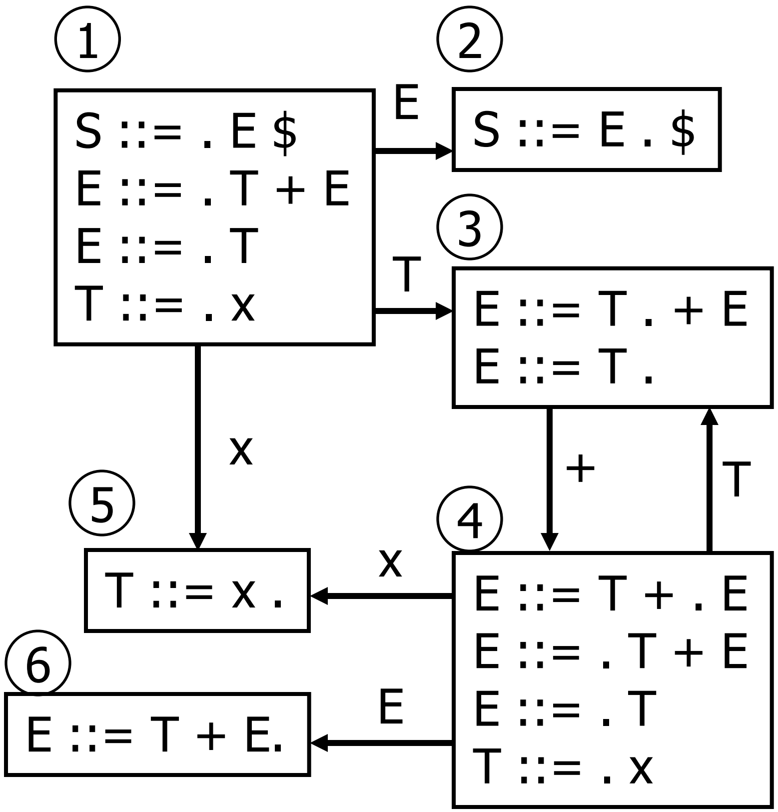
  $E ::= T + E$

  $E ::= T$

  $T ::= \mathrm{x}$

LR(0) Parser for

0. $S ::= E\,\$$
1. $E ::= T + E$
2. $E ::= T$
3. $T ::= x$



	x	+	$	E	T
1	s5			g2	G3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	G3
5	r3	r3	r3		
6	r1	r1	r1		

- State 3 is has two possible actions on +
  - shift 4, or reduce 2
- ∴ Grammar is not LR(0)

SLR Parsers

- Idea: Use information about what can follow a non-terminal to decide if we should perform a reduction
- Easiest form is SLR – Simple LR
- So we need to be able to compute FOLLOW(*A*) – the set of symbols that can follow *A* in any possible derivation
  - But to do this, we need to compute FIRST(γ) for strings γ that can follow *A*

Calculating FIRST(γ)

- Sounds easy… If $\gamma = X\ Y\ Z$, then FIRST(γ) is FIRST(X), right?
  - But what if we have the rule $X ::= \varepsilon$?
  - In that case, FIRST(γ) includes anything that can follow an X – i.e. FOLLOW(X)

FIRST, FOLLOW, and nullable

- nullable(X) is true if X can derive the empty string
- Given a string γ of terminals and non-terminals, FIRST(γ) is the set of terminals that can begin strings derived from γ.
- FOLLOW(X) is the set of terminals that can immediately follow X in some derivation
- All three of these are computed together

Computing FIRST, FOLLOW, and nullable (1)

- Initialization

  set FIRST and FOLLOW to be empty sets

  set nullable to false for all non-terminals

  set FIRST[a] to a for all terminal symbols a

Computing FIRST, FOLLOW, and nullable (2)

```
repeat
    for each production X := Y1 Y2 … Yk
        if Y1 … Yk are all nullable (or if k = 0)
            set nullable[X] = true
        for each i from 1 to k and each j from i+1 to k
            if Y1 … Yi-1 are all nullable (or if i = 1)
                add FIRST[Yi] to FIRST[X]
            if Yi+1 … Yk are all nullable (or if i = k)
                add FOLLOW[X] to FOLLOW[Yi]
            if Yi+1 … Yj-1 are all nullable (or if i+1=j)
                add FIRST[Yj] to FOLLOW[Yi]
Until FIRST, FOLLOW, and nullable do not change
```

Example

- Grammar

  $Z ::= \mathrm{d}$

  $Z ::= X\ Y\ Z$

  $Y ::= \varepsilon$

  $Y ::= \mathrm{c}$

  $X ::= Y$

  $X ::= \mathrm{a}$

	nullable	FIRST	FOLLOW
X			
Y			
Z			

SLR Construction

- This is identical to LR(0) – states, etc., except for the calculation of reduce actions
- Algorithm:

  Initialize *R* to empty

  for each state *I* in *T*

  for each item [A ::= α .] in *I*

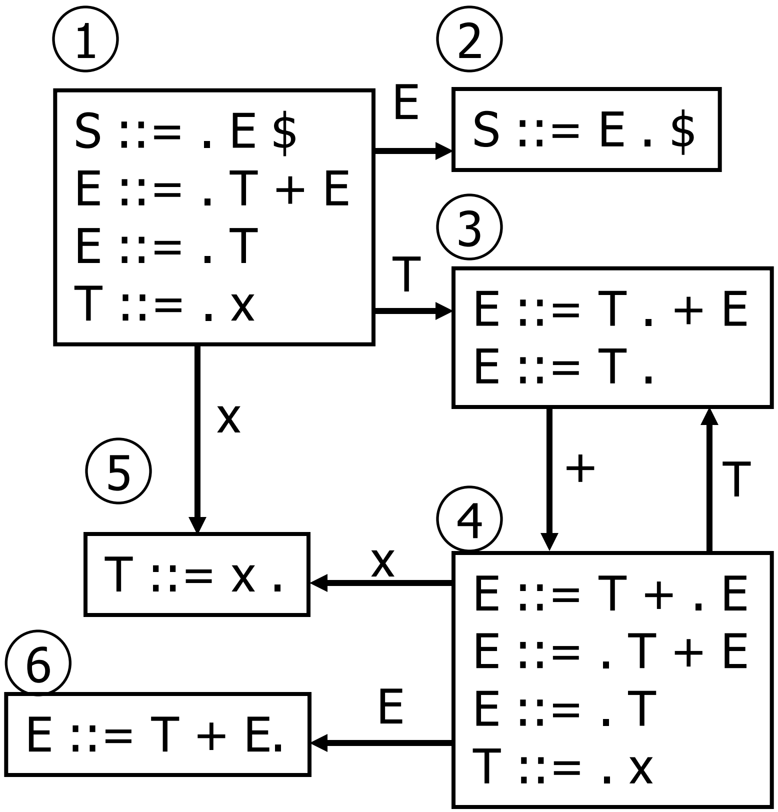
  for each terminal a in FOLLOW(*A*)

  add (*I*, a, A ::= α) to *R*

  - i.e., reduce α to *A* in state *I* only on lookahead a

SLR Parser for

0. S ::= E $
1. E ::= T + E
2. E ::= T
3. T ::= x



	x	+	$	E	T
1	s5			g2	g3
2			acc		
3	r2	s4,r2	r2		
4	s5			g6	g3
5	r3	r3	r3		
6	r1	r1	r1		

On To LR(1)

- Many practical grammars are SLR
- LR(1) is more powerful yet
- Similar construction, but notion of an item is more complex, incorporating lookahead information

LR(1) Items

- An LR(1) item [A ::= α . β, a] is
  - A grammar production (A ::= $\alpha\beta$)
  - A right hand side position (the dot)
  - A lookahead symbol (a)
- Idea: This item indicates that α is the top of the stack and the next input is derivable from βa.
- Full construction: see the book

LR(1) Tradeoffs

- LR(1)
  - Pro: extremely precise; largest set of grammars
  - Con: potentially very large parse tables with many states

LALR(1)

- Variation of LR(1), but merge any two states that differ only in lookahead
  - Example: these two would be merged

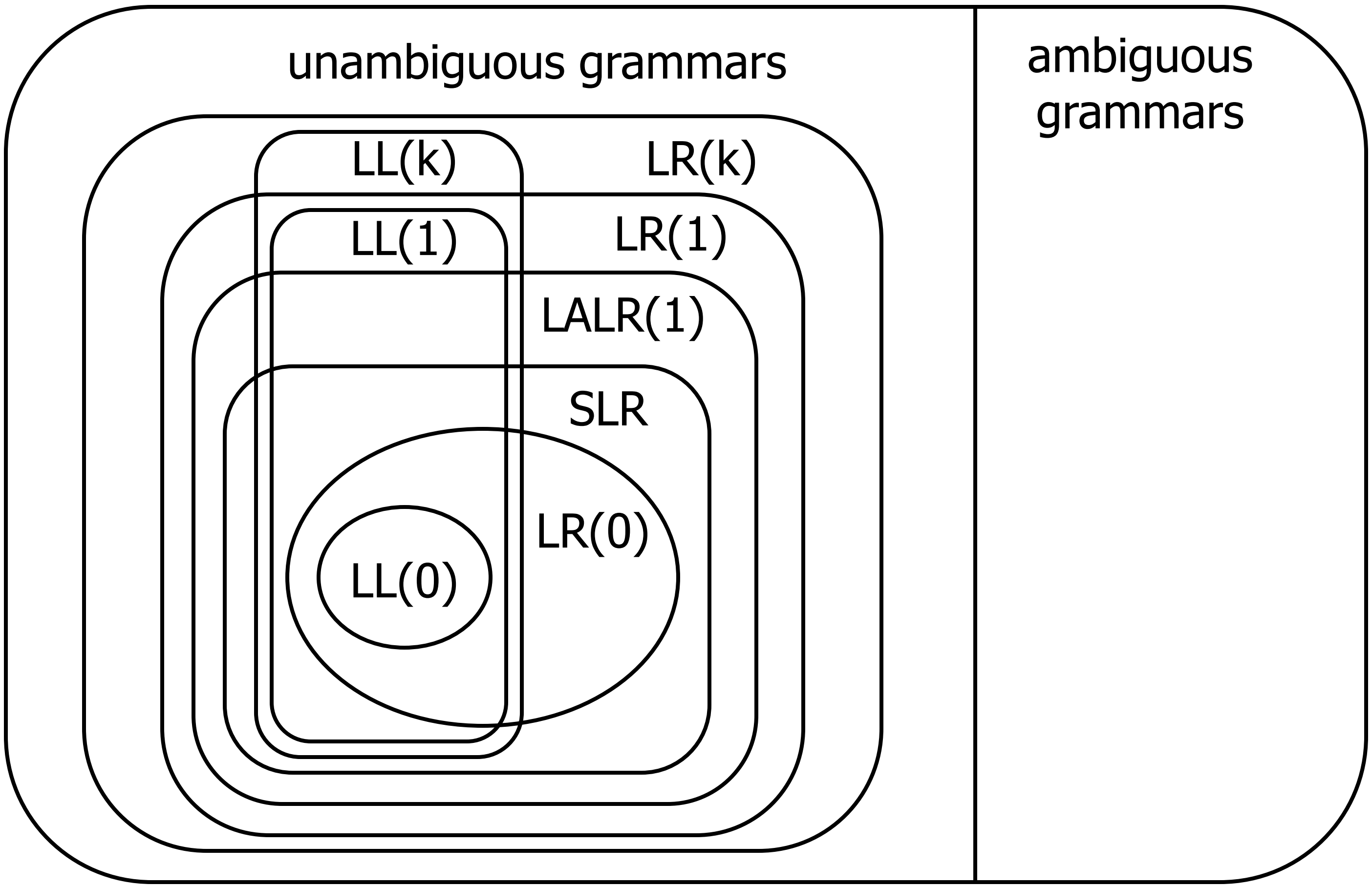
    [A ::= x . , a]

    [A ::= x . , b]

LALR(1) vs LR(1)

- LALR(1) tables can have many fewer states than LR(1)
- LALR(1) may have reduce conflicts where LR(1) would not (but in practice this doesn’t happen often)

Language Heirarchies



Coming Attractions

- LL(k) Parsing – Top-Down
- Recursive Descent Parsers
  - What you can do if you need a parser in a hurry