



# CSE P 501 – Compilers

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SSA

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# Agenda

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- Overview of SSA IR
  - Constructing SSA graphs
  - SSA-based optimizations
  - Converting back from SSA form
  
- Source: Appel ch. 19, also an extended discussion in Cooper-Torczon sec. 9.3



# Def-Use (DU) Chains

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- Common dataflow analysis problem: Find all sites where a variable is used, or find the definition site of a variable used in an expression
- Traditional solution: def-use chains – additional data structure on the dataflow graph
  - Link each statement defining a variable to all statements that use it
  - Link each use of a variable to its definition



# DU-Chain Drawbacks

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- Expensive: if a typical variable has  $N$  uses and  $M$  definitions, the total cost is  $O(N * M)$ 
  - Would be nice if cost were proportional to the size of the program
- Unrelated uses of the same variable are mixed together
  - Complicates analysis



# SSA: Static Single Assignment

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- IR where each variable has only one definition in the program text
  - This is a single *static* definition, but it may be in a loop that is executed dynamically many times



# SSA in Basic Blocks

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We've seen this before when looking at value numbering

- Original

$a := x + y$

$b := a - 1$

$a := y + b$

$b := x * 4$

$a := a + b$

- SSA

$a_1 := x + y$

$b_1 := a_1 - 1$

$a_2 := y + b_1$

$b_2 := x * 4$

$a_3 := a_2 + b_2$



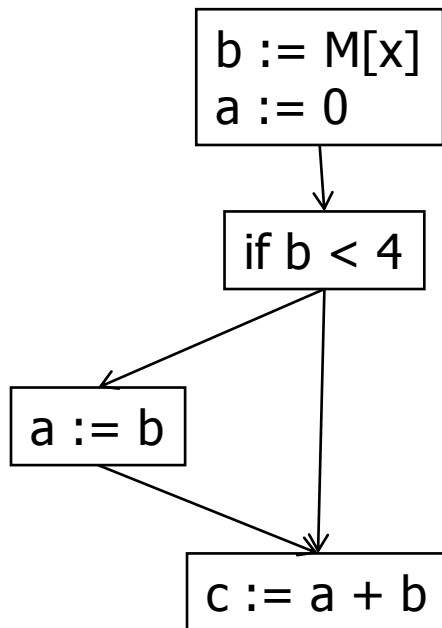
# Merge Points

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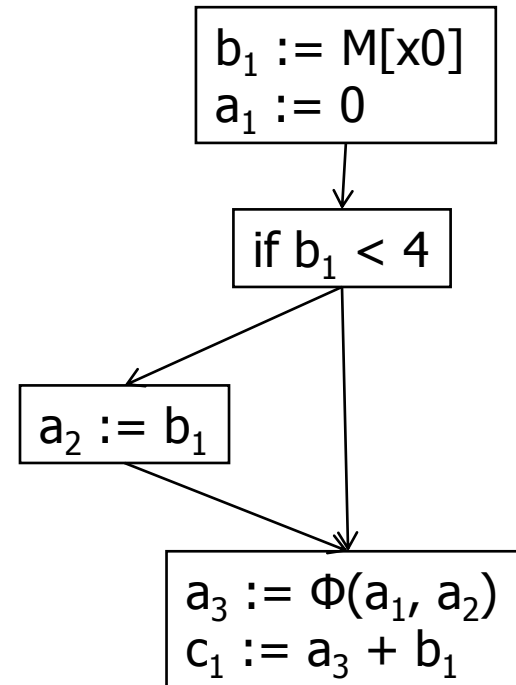
- The issue is how to handle merge points
- Solution: introduce a  $\Phi$ -function
$$a_3 := \Phi(a_1, a_2)$$
- Meaning:  $a_3$  is assigned either  $a_1$  or  $a_2$  depending on which control path is used to reach the  $\Phi$ -function

# Example

## Original



## SSA





# How Does $\Phi$ “Know” What to Pick?

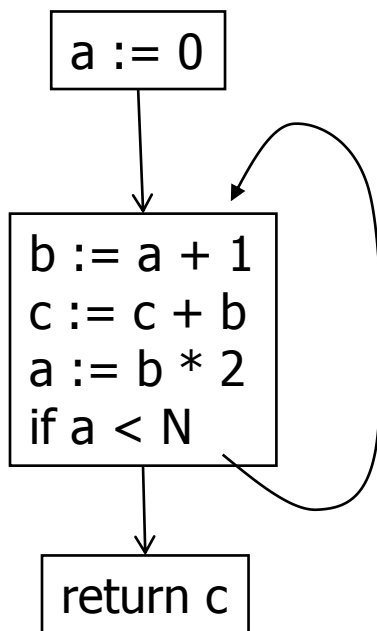


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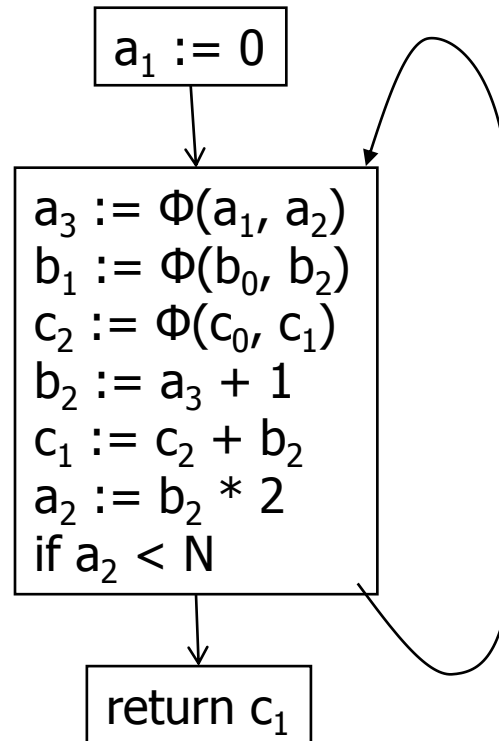
- It doesn't
  - When we translate the program to executable form, we can add code to copy either value to a common location on each incoming edge
  - For analysis, all we may need to know is the connection of uses to definitions – no need to “execute” anything

# Example With Loop

## Original



## SSA



## Notes:

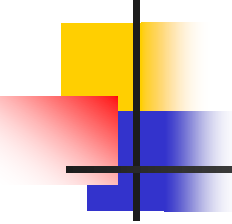
- $a_0, b_0, c_0$  are initial values of  $a, b, c$  on block entry
- $b_1$  is dead – can delete later
- $c$  is live on entry – either input parameter or uninitialized



# Converting To SSA Form

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- Basic idea
  - First, add  $\Phi$ -functions
  - Then, rename all definitions and uses of variables by adding subscripts



# Inserting $\Phi$ -Functions

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- Could simply add  $\Phi$ -functions for every variable at every join point(!)
- But
  - Wastes way too much space and time
  - Not needed



# When to Insert a $\Phi$ -Function

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- Insert a  $\Phi$ -function for variable  $a$  at point  $z$  when
  - There are blocks  $x$  and  $y$ , both containing definitions of  $a$ , and  $x \neq y$
  - There are nonempty paths from  $x$  to  $z$  and from  $y$  to  $z$
  - These paths have no common nodes other than  $z$
  - $z$  is not in both paths prior to the end (it may appear in one of them)



# Details

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- The start node of the flow graph is considered to define every variable (even if to “undefined”)
- Each  $\Phi$ -function itself defines a variable, so we need to keep adding  $\Phi$ -functions until things converge



# Dominators and SSA

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- One property of SSA is that definitions dominate uses; more specifically:
  - If  $x := \Phi(\dots, x_i, \dots)$  in block  $n$ , then the definition of  $x$  dominates the  $i$ th predecessor of  $n$
  - If  $x$  is used in a non- $\Phi$  statement in block  $n$ , then the definition of  $x$  dominates block  $n$



# Dominance Frontier (1)

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- To get a practical algorithm for placing  $\Phi$ -functions, we need to avoid looking at all combinations of nodes leading from  $x$  to  $y$
- Instead, use the dominator tree in the flow graph





# Dominance Frontier (2)

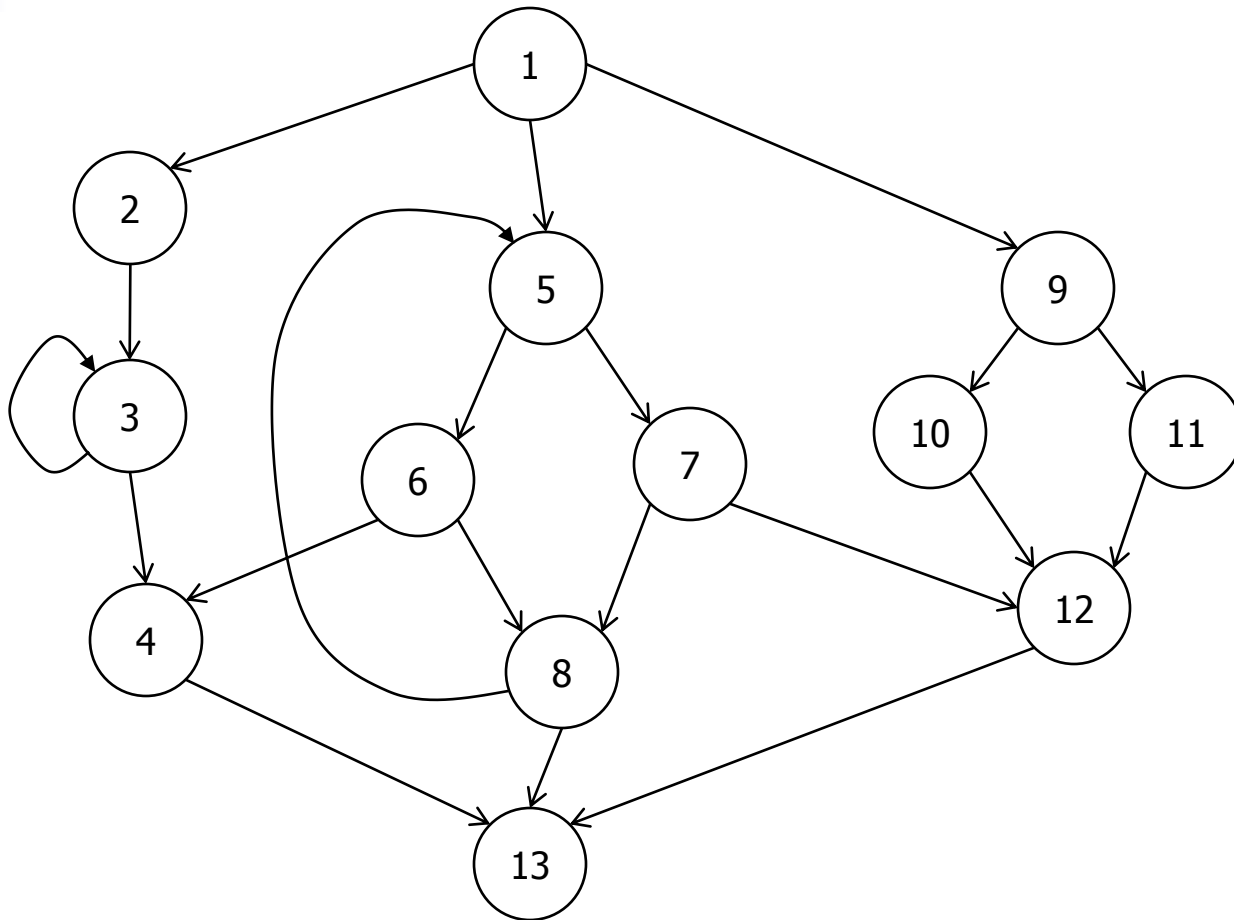
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- Definitions

- $x$  *strictly dominates*  $y$  if  $x$  dominates  $y$  and  $x \neq y$
- The *dominance frontier* of a node  $x$  is the set of all nodes  $w$  such that  $x$  dominates a predecessor of  $w$ , but  $x$  does not strictly dominate  $w$

- Essentially, the dominance frontier is the border between dominated and undominated nodes

# Example





# Placing $\Phi$ -Functions

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- If a node  $x$  contains the definition of variable  $a$ , then every node in the dominance frontier of  $x$  needs a  $\Phi$ -function for  $a$ 
  - Since the  $\Phi$ -function itself is a definition, this needs to be iterated until it reaches a fixed-point
- Theorem: this algorithm places exactly the same set of  $\Phi$ -functions as the path criterion given previously



# Placing $\Phi$ -Functions: Details

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- We won't give the full constructions here. The basic steps are:
  1. Compute the dominance frontiers for each node in the flowgraph
  2. Insert just enough  $\Phi$ -functions to satisfy the criterion. Use a worklist algorithm to avoid reexamining nodes unnecessarily
  3. Walk the dominator tree and rename the different definitions of variable  $a$  to be  $a_1, a_2, a_3, \dots$



# SSA Optimizations

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- A sampler of optimizations that exploit SSA form
- First, what do we know? (i.e., what information is kept in the SSA graph?)



# SSA Data Structures

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- Statement: links to containing block, next and previous statements, variables defined, variables used. Statement kinds are: ordinary,  $\Phi$ -function, fetch, store, branch
- Variable: link to definition (statement) and use sites
- Block: List of contained statements, ordered list of predecessors, successor(s)



# Dead-Code Elimination

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- A variable is live iff its list of uses is not empty(!)
- Algorithm to delete dead code:
  - while there is some variable  $v$  with no uses
    - if the statement that defines  $v$  has no other side effects, then delete it
  - Need to remove this statement from the list of uses for its operand variables – which may cause those variables to become dead



# Simple Constant Propagation

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- If  $c$  is a constant in  $v := c$ , any use of  $v$  can be replaced by  $c$ 
  - Then update every use of  $v$  to use constant  $c$
- If the  $c_i$ 's in  $v := \Phi(c_1, c_2, \dots, c_n)$  are all the same constant  $c$ , we can replace this with  $v := c$
- Can also incorporate copy propagation, constant folding, and others in the same worklist algorithm





# Simple Constant Propagation

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$W :=$  list of all statements in SSA program

while  $W$  is not empty

    remove some statement  $S$  from  $W$

    if  $S$  is  $v := \Phi(c, c, \dots, c)$ , replace  $S$  with  $v := c$

    if  $S$  is  $v := c$

        delete  $S$  from the program

        for each statement  $T$  that uses  $v$

            substitute  $c$  for  $v$  in  $T$

            add  $T$  to  $W$



# Converting Back from SSA

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- Unfortunately, real machines do not include a  $\Phi$  instruction
- So after analysis, optimization, and transformation, need to convert back to a “ $\Phi$ -less” form for execution



# Translating $\Phi$ -functions

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- The meaning of  $x := \Phi(x_1, x_2, \dots, x_n)$  is “set  $x := x_1$  if arriving on edge 1, set  $x := x_2$  if arriving on edge 2, etc.”
- So, for each  $i$ , insert  $x := x_i$  at the end of predecessor block  $i$
- Rely on copy propagation and coalescing in register allocation to eliminate redundant moves



# SSA

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- There are (obviously) many details needed to fully implement SSA, but these are the main ideas
- SSA is used in most modern optimizing compilers & has been retrofitted into many older ones (gcc is a well-known example)