



CSE P 501 – Compilers

Introduction to Optimization
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Agenda

- Optimization
 - Goals
 - Scope: local, superlocal, regional, global, interprocedural
 - Control flow graphs
 - Value numbering
 - Dominators

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Code Improvement – How?

- Pick a better algorithm(!)
- Use machine resources effectively
 - Instruction selection & scheduling
 - Register allocation

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Code Improvement (2)

- Local optimizations – basic blocks
 - Algebraic simplifications
 - Constant folding
 - Common subexpression elimination (i.e., redundancy elimination)
 - Dead code elimination
 - Specialize computation based on context

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Code Improvement (3)

- Global optimizations
 - Code motion
 - Moving invariant computations out of loops
 - Strength reduction (replace multiplications by repeated additions, for example)
 - Global common subexpression elimination
 - Global register allocation

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“Optimization”

- None of these improvements are truly “optimal”
 - Hard problems
 - Proofs of optimality assume artificial restrictions
- Best we can do is to improve things

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Example: $A[i,j]$

- Without any surrounding context, need to generate code to calculate
$$\text{address}(A) + (i - \text{low}_1(A)) * (\text{high}_2(A) - \text{low}_2(A) + 1) * \text{size}(A) + (j - \text{low}_2(A)) * \text{size}(A)$$
 - low_i and high_i are subscript bounds in dimension i
 - $\text{address}(A)$ is the runtime address of first element of A
- ... And we really should be checking that i, j are in bounds

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Some Optimizations for $A[i,j]$

- With more context, we can do better
- Examples
 - If A is local, with known bounds, much of the computation can be done at compile time
 - If $A[i,j]$ is in a loop where i and j change systematically, we probably can replace multiplications with additions each time around the loop to reference successive rows/columns

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Optimization Phase

- Goal
 - Discover, at compile time, information about the runtime behavior of the program, and use that information to improve the generated code

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Running Example: Redundancy Elimination

- An expression $x+y$ is *redundant* at a program point iff, along every path from the procedure's entry, it has been evaluated and its constituent subexpressions (x & y) have not been redefined
- If the compiler can prove the expression is redundant
 - Can store the result of the earlier evaluation
 - Can replace the redundant computation with a reference to the earlier (stored) result

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Common Problems in Code Improvement

- This strategy is typical of most compiler optimizations
 - First, need to discover opportunities through program analysis
 - Then, need to modify the IR to take advantage of the opportunities
 - Historically, goal usually was to decrease execution time
 - Other possibilities: reduce space, power, ...

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Issues (1)

- Safety – transformation must not change program meaning
 - Must generate correct results
 - Can't generate spurious errors
 - Optimizations must be conservative
 - Large part of analysis goes towards proving safety

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Issues (2)

- Profitability
 - If a transformation is possible, is it profitable?
 - Example: loop unrolling
 - Can increase amount of work done on each iteration, i.e., reduce loop overhead
 - Can eliminate duplicate operations done on separate iterations

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Issues (3)

- Downside risks
 - Even if a transformation is generally worthwhile, need to factor in potential problems
 - Sample issues
 - Transformation might need more temporaries, putting additional pressure on registers
 - Increased code size could cause cache misses, or in bad cases, increase page working set

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Value Numbering

- Technique for eliminating redundant expressions: assign an identifying number $VN(n)$ to each expression
 - $VN(x+y) = VN(j)$ if $x+y$ and j have the same value
 - Use hashing over value numbers for efficiency
- Old idea (Balke 1968, Ershov 1954)
 - Invented for low-level, linear IRs
 - Equivalent methods exist for tree IRs, e.g., build a DAG

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Uses of Value Numbers

- Improve the code
 - Replace redundant expressions
 - Simplify algebraic identities
 - Discover, fold, and propagate constant valued expressions

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Local Value Numbering

- Algorithm
 - For each operation $o = \langle op, o1, o2 \rangle$ in the block
 1. Get value numbers for operands from hash lookup
 2. Hash $\langle op, VN(o1), VN(o2) \rangle$ to get a value number for o (If op is commutative, sort $VN(o1), VN(o2)$ first)
 3. If o already has a value number, replace o with a reference to the value
 4. If $o1$ and $o2$ are constant, evaluate o at compile time and replace with an immediate load
- If hashing behaves well, this runs in linear time

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Example

Code

```
a = x + y
b = x + y
a = 17
c = x + y
```

Rewritten

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Bug in Simple Example

- If we use the original names, we get in trouble when a name is reused
- Solutions
 - Be clever about which copy of the value to use (e.g., use $c=b$ in last statement)
 - Create an extra temporary
 - Rename around it (best!)

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Renaming

- Idea: give each value a unique name
 - a_i^j means i^{th} definition of a with $VN = j$
- Somewhat complex notation, but meaning is clear
- This is the idea behind SSA (Static Single Assignment) IR
 - Popular modern IR – exposes many opportunities for optimizations

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Example Revisited

Code	Rewritten
$a = x + y$	
$b = x + y$	
$a = 17$	
$c = x + y$	

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Simple Extensions to Value Numbering

- Constant folding
 - Add a bit that records when a value is constant
 - Evaluate constant values at compile time
 - Replace op with load immediate
- Algebraic identities: $x+0$, $x*1$, $x-x$, ...
 - Many special cases
 - Switch on op to narrow down checks needed
 - Replace result with input VN

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Larger Scopes

- This algorithm works on straight-line blocks of code (basic blocks)
 - Best possible results for single basic blocks
 - Loses all information when control flows to another block
- To go further we need to represent multiple blocks of code and the control flow between them

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Basic Blocks

- Definition: A *basic block* is a maximal length sequence of straight-line code
- Properties
 - Statements are executed sequentially
 - If any statement executes, they all do (barring exceptions)
- In a linear IR, the first statement of a basic block is often called the *leader*

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Control Flow Graph (CFG)

- Nodes: basic blocks
 - Possible representations: linear 3-address code, expression-level AST, DAG
- Edges: include a directed edge from n1 to n2 if there is *any* possible way for control to transfer from block n1 to n2 during execution

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Constructing Control Flow Graphs from Linear IRs

- Algorithm
 - Pass 1: Identify basic block leaders with a linear scan of the IR
 - Pass 2: Identify operations that end a block and add appropriate edges to the CFG to all possible successors
 - See your favorite compiler book for details
- For convenience, ensure that every block ends with conditional or unconditional jump
 - Code generator can pick the most convenient "fall-through" case later and eliminate unneeded jumps

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Scope of Optimizations

- Optimization algorithms can work on units as small as a basic block or as large as a whole program
- Local information is generally more precise and can lead to locally optimal results
- Global information is less precise (lose information at join points in the graph), but exposes opportunities for improvements across basic blocks

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Optimization Categories (1)

- *Local methods*
 - Usually confined to basic blocks
 - Simplest to analyze and understand
 - Most precise information

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Optimization Categories (2)

- *Superlocal methods*
 - Operate over *Extended Basic Blocks* (EBBs)
 - An EBB is a set of blocks b_1, b_2, \dots, b_n where b_1 has multiple predecessors and each of the remaining blocks b_i ($2 \leq i \leq n$) have only b_1 as its unique predecessor
 - The EBB is entered only at b_1 , but may have multiple exits
 - A single block b_1 can be the head of multiple EBBs (these EBBs form a tree rooted at b_1)
 - Use information discovered in earlier blocks to improve code in successors

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Optimization Categories (3)

- *Regional methods*
 - Operate over scopes larger than an EBB but smaller than an entire procedure/function/method
 - Typical example: loop body
 - Difference from superlocal methods is that there may be merge points in the graph (i.e., a block with two or more predecessors)

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Optimization Categories (4)

- Global methods
 - Operate over entire procedures
 - Sometimes called *intraprocedural* methods
 - Motivation is that local optimizations sometimes have bad consequences in larger context
 - Procedure/method/function is a natural unit for analysis, separate compilation, etc.
 - Almost always need global *data-flow* analysis information for these

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Optimization Categories (5)

- Whole-program methods
 - Operate over more than one procedure
 - Sometimes called *interprocedural* methods
 - Challenges: name scoping and parameter binding issues at procedure boundaries
 - Classic examples: inline method substitution, interprocedural constant propagation
 - Fairly common in aggressive JIT compilers and optimizing compilers for object-oriented languages

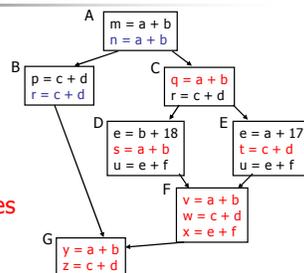
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Value Numbering Revisited

- Local Value Numbering
 - 1 block at a time
 - Strong local results
 - No cross-block effects
- Missed opportunities



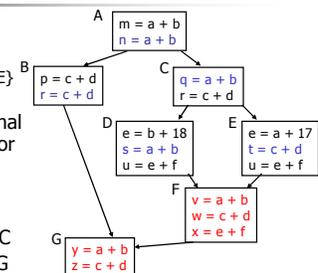
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Superlocal Value Numbering

- Idea: apply local method to EBBs
 - {A,B}, {A,C,D}, {A,C,E}
- Final info from A is initial info for B, C; final info from C is initial for D, E
- Gets reuse from ancestors
- Avoid reanalyzing A, C
- Doesn't help with F, G



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SSA Name Space (from before)

- | Code | Rewritten |
|-------------------------|-------------------------|
| $a_0^3 = x_0^1 + y_0^2$ | $a_0^3 = x_0^1 + y_0^2$ |
| $b_0^3 = x_0^1 + y_0^2$ | $b_0^3 = a_0^3$ |
| $a_1^4 = 17$ | $a_1^4 = 17$ |
| $c_0^3 = x_0^1 + y_0^2$ | $c_0^3 = a_0^3$ |
- Unique name for each definition
 - Name \Leftrightarrow VN
 - a_0^3 is available to assign to c_0^3

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SSA Name Space

- Two Principles
 - Each name is defined by exactly one operation
 - Each operand refers to exactly one definition
- Need to deal with merge points
 - Add Φ functions at merge points to reconcile names
 - Use subscripts on variable names for uniqueness

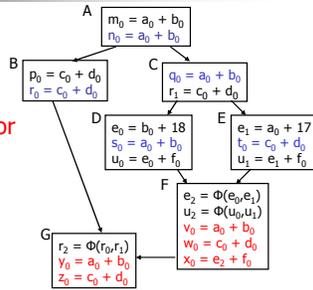
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Superlocal Value Numbering with All Bells & Whistles

- Finds more redundancies
- Little extra cost
- Still does nothing for F and G



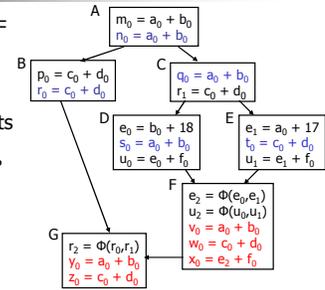
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Larger Scopes

- Still have not helped F and G
- Problem: multiple predecessors
- Must decide what facts hold in F and in G
 - For G, combine B & F?
 - Merging states is expensive
 - Fall back on what we know



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Dominators

- Definition
 - x *dominates* y iff every path from the entry of the control-flow graph to y includes x
- By definition, x dominates x
- Associate a Dom set with each node
 - $|\text{Dom}(x)| \geq 1$
- Many uses in analysis and transformation
 - Finding loops, building SSA form, code motion

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Immediate Dominators

- For any node x , there is a y in $\text{Dom}(x)$ closest to x
- This is the *immediate dominator* of x
 - Notation: $\text{IDom}(x)$

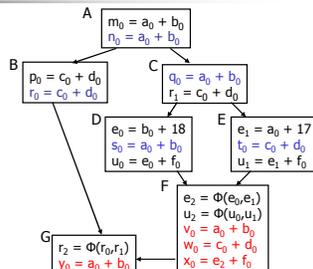
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Dominator Sets

Block Dom IDom



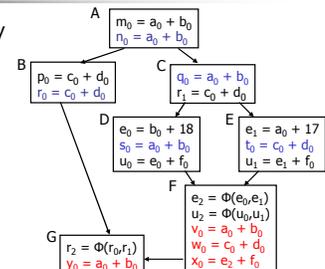
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Dominator Value Numbering

- Still looking for a way to handle F and G
- Idea: Use info from $\text{IDom}(x)$ to start analysis of x
 - Use C for F and A for G
- Dominator VN Technique (DVNT)



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DVNT algorithm

- Use superlocal algorithm on extended basic blocks
 - Use scoped hash tables & SSA name space as before
- Start each node with table from its IDOM
- No values flow along back edges (i.e., loops)
- Constant folding, algebraic identities as before

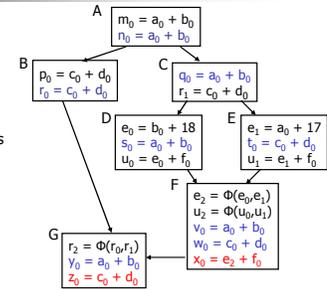
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Dominator Value Numbering

- Advantages
 - Finds more redundancy
 - Little extra cost
- Shortcomings
 - Misses some opportunities (common calculations in ancestors that are not IDOMs)
 - Doesn't handle loops or other back edges



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The Story So Far...

- Local algorithm
- Superlocal extension
 - Some local methods extend cleanly to superlocal scopes
- Dominator VN Technique (DVNT)
- All of these propagate along forward edges
- None are global

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Coming Attractions

- Data-flow analysis
 - Provides global solution to redundant expression analysis
 - Catches some things missed by DVNT, but misses some others
 - Generalizes to many other analysis problems, both forward and backward
- Transformations
 - A catalog of some of the things a compiler can do with the analysis information

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