DATA516/CSED516
Scalable Data Systems and Algorithms

Lecture 7
Advanced Distributed Query Processing

Announcements

- Today lecture:
  - Part 1: guest lecturer Mingxi Wu, Tigergraph
  - Part 2: finish discussing distributed queries
- Reading assignment postponed for next week; you can update if you submitted
- HW4 = 3 mini homeworks + 1 theory to be posted tomorrow
- Next Tuesday: last regular lecture
- Dec. 1st: 1-on-1 discussion of your projects
- Dec. 8th : project presentations

Review: Distributed Join

Two algorithms for distributed join

- Hash-partition join
- Broadcast join

This lecture: how to compute general queries *without* one join at a time

The Load

- We know the sizes of the input tables: |R|, |S|, |T|, …
  - Sometimes they are all equal, then we denote this with N
- We run an algorithm on p servers

The *load* of the algorithm, L, is the largest number of tuples received by any server

Example: Hash Join

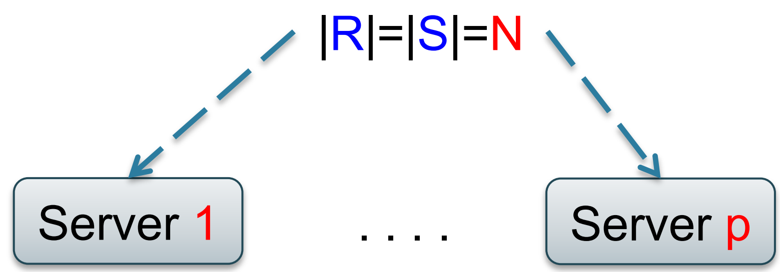
Join(x,y,z) = R(x,y) ∧ S(y,z)

R

x	y
a	e
a	f
b	f
c	f

S

y	z
e	m
e	n
f	m
f	k



Round 1: each server

- Hash partition R(x,y) and S(y,z) by y

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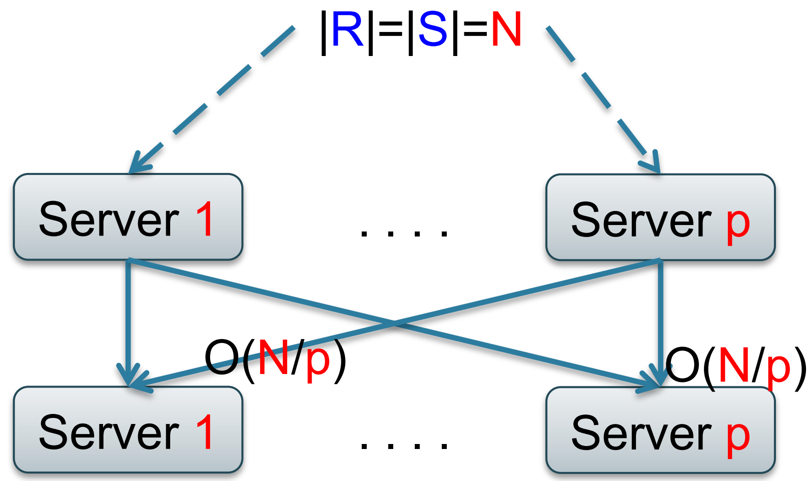

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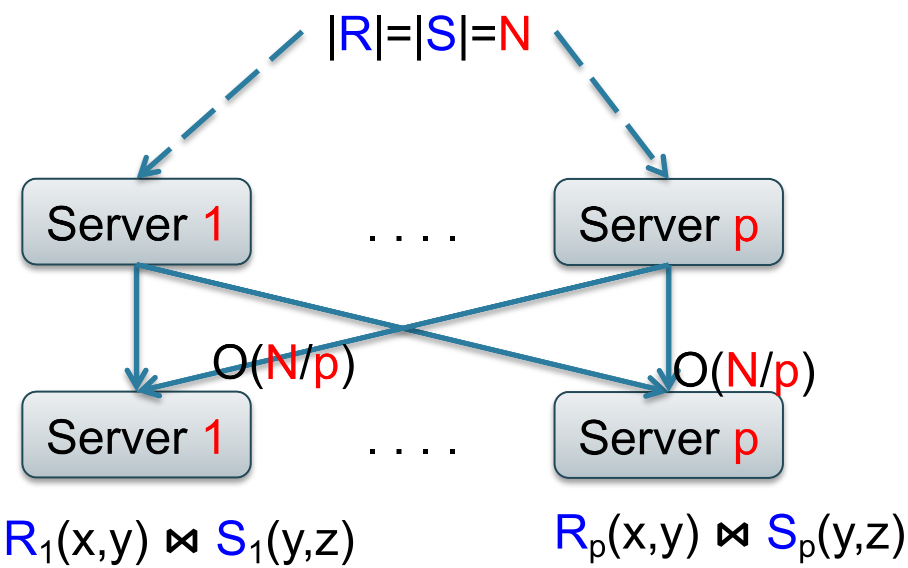

Round 1: each server

- Hash partition R(x,y) and S(y,z) by y

Output: each server u:

- local join $R_u(x,y) \bowtie S_u(y,z)$

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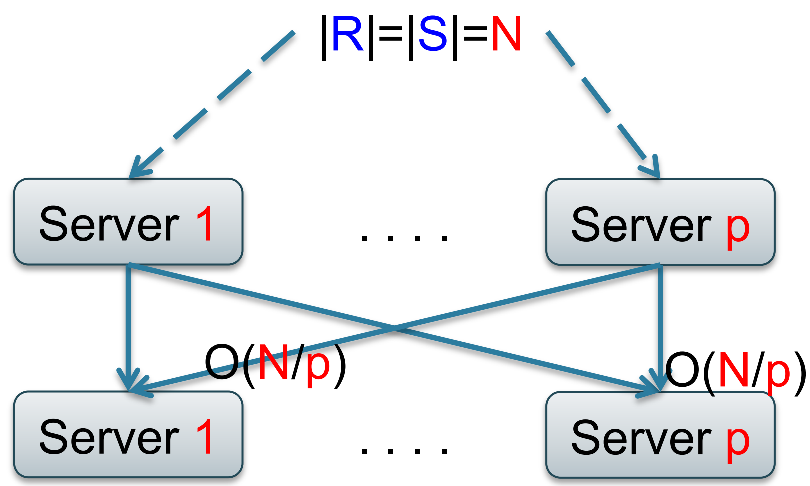

$R_1(x,y) \bowtie S_1(y,z)$ $R_p(x,y) \bowtie S_p(y,z)$

Round 1: each server

- Hash partition $R(x,y)$ and $S(y,z)$ by y

$\boxed{L = O(N/p)}$

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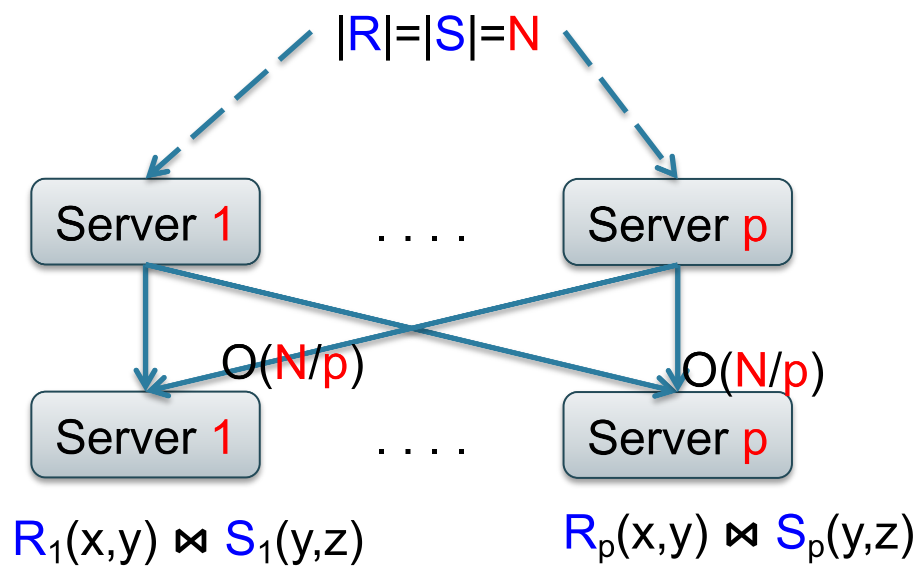

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- Hash partition $R(x,y)$ and $S(y,z)$ by y

$L = O(N/p)$ w.h.p.

Assuming no skew

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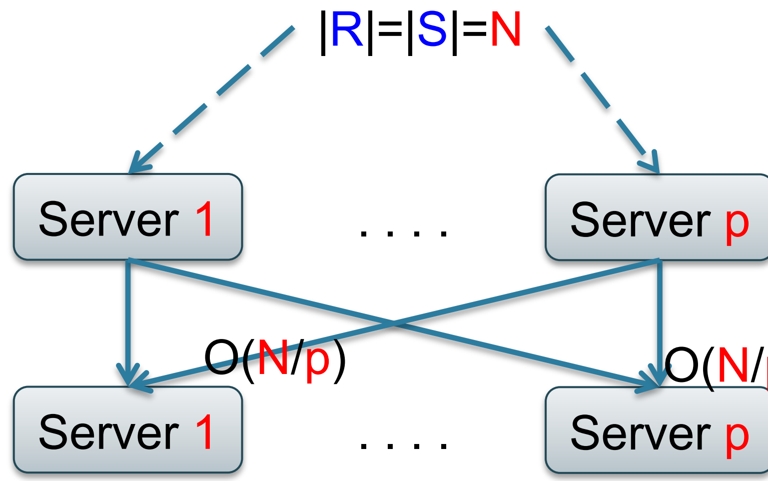

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Round 1: each server

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$L = O(N/p)$ w.h.p.

Assuming no skew

Skew threshold: N/p or lower

Output: each server u:

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Broadcast Join

$Join(x,y,z) = R(x,y) \wedge S(y,z)$

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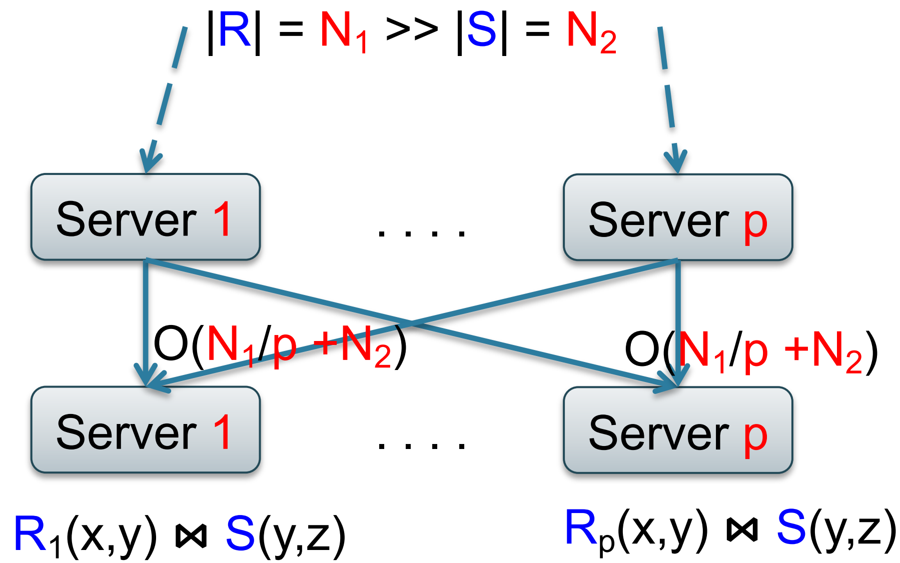
a	f
b	f
c	f

S

y	z
e	m
f	k



Round 1: each server

- Broadcast $S(y,z)$ to all servers

$$\boxed{L = O(N_1/p + N_2)}$$

Skew no problem

Output: each server

- local join $R_u(x,y) \bowtie S(y,z)$

$|R| = |S| = |T| = N$ tuples

The Triangles Query

$Q(x,y,z) = R(x,y) \wedge S(y,z) \wedge T(z,x)$

Round 1: $Temp(x,y,z) = R(x,y) \wedge S(y,z)$

Round 2: $Q(x,y,z) = Temp(x,y,z) \wedge T(z,x)$

Problem: $|Temp| \gg N$

$|R| = |S| = |T| = N$ tuples

The Triangles Query

$Q(x,y,z) = R(x,y) \wedge S(y,z) \wedge T(z,x)$

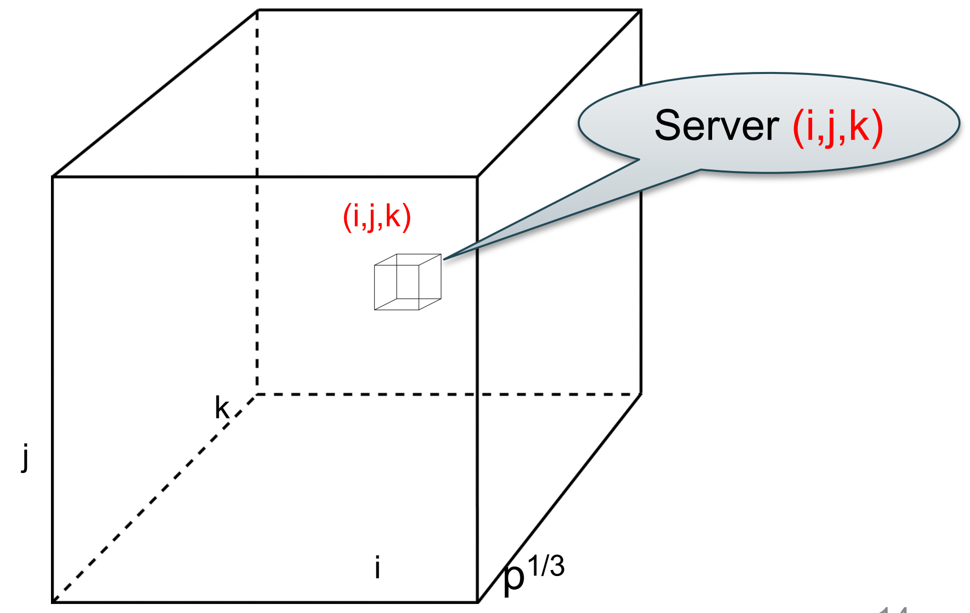
Algorithm in one round!

- [Afrati'10] Shares Algo (MapReduce)
- [Beame'13,'14] HyperCube Algo (MPC)

Triangles in One Round

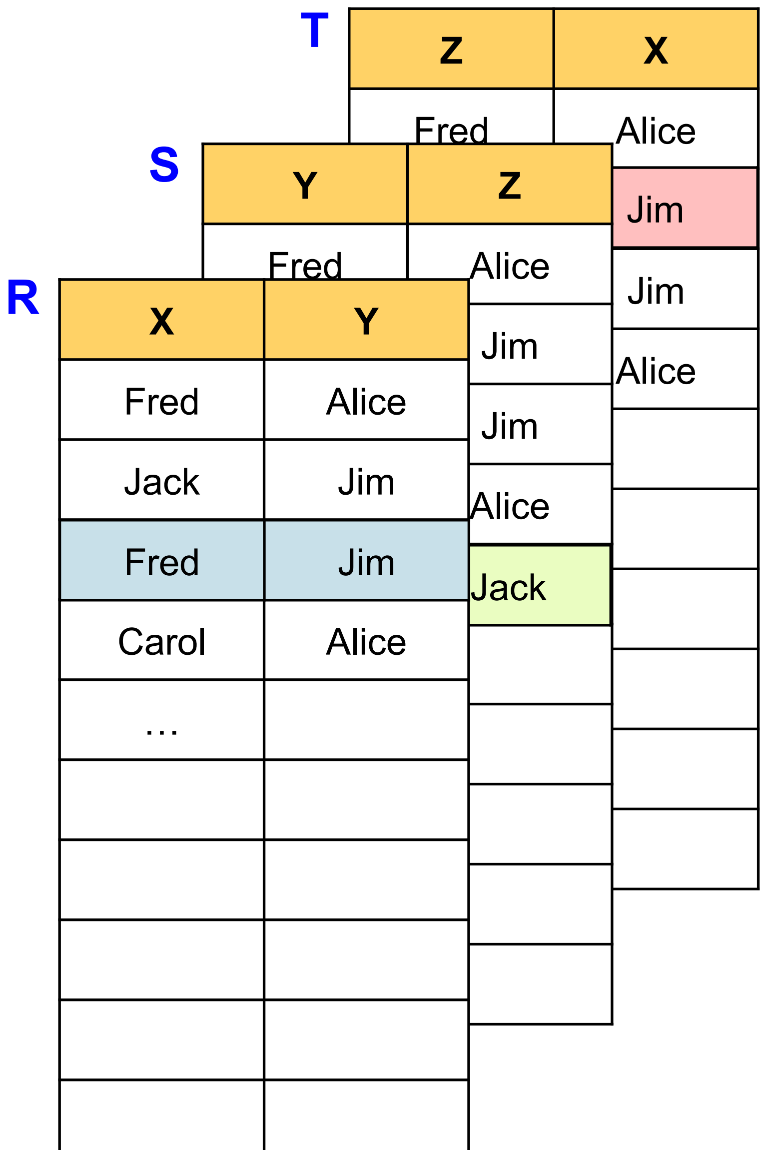
- Place servers in a cube $p = p^{1/3} \times p^{1/3} \times p^{1/3}$
- Each server identified by (i,j,k)

- Choose 3 random, independent hash functions:
  h_1 : Dom → $[p^{1/3}]$
  h_2 : Dom → $[p^{1/3}]$
  h_3 : Dom → $[p^{1/3}]$



$Q(x,y,z) = R(x,y) \wedge S(y,z) \wedge T(z,x)$ $|R| = |S| = |T| = N$ tuples

Triangles in One Round



Round 1:

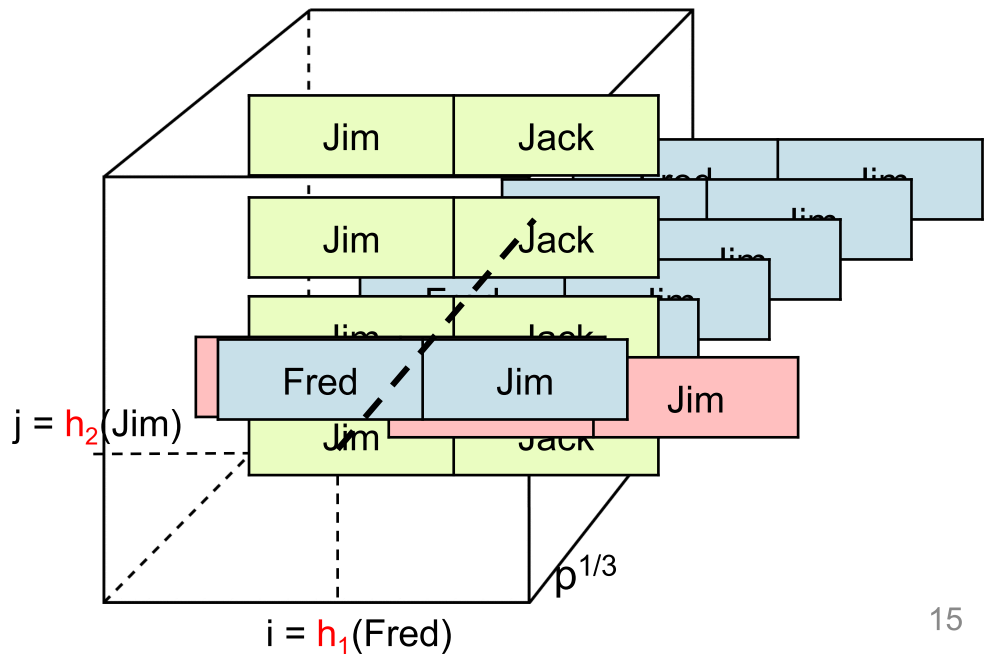
Send $R(x,y)$ to all servers $(h_1(x), h_2(y), *)$

Send $S(y,z)$ to all servers $(*, h_2(y), h_3(z))$

Send $T(z,x)$ to all servers $(h_1(x), *, h_3(z))$

Output:

compute locally $R(x,y) \wedge S(y,z) \wedge T(z,x)$



Communication Cost

Theorem HyperCube has load $L = O(N/p^{2/3})$ w.h.p., on any input database without skew.

Skew threshold: $N/p^{1/3}$ or lower

This load is optimal, even for data without skew

HyperCube Algorithm

- In general, we have a multi-join query.
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Main challenge: compute the *shares* $p_1, p_2, \dots, p_k$ to minimize the load L

Example: Join

$$Q(x, y, z) = R(x, y) \wedge S(y, z)$$

- Hash join: $p_1 = 1, p_2 = p, p_3 = 1$
- Broadcast join: $p_1 = 1, p_2 = 1, p_3 = p$

Which relation is broadcast?

Computing the Shares

- The secret to computing the shares lies in understanding a very simple query: the cartesian product of two, or more relations

Cartesian Product

An important special case: $Q = R \times S$

- In our notation: $Q(x, y) = R(x) \wedge S(y)$
- Assume: $|R| = N_1$, $|S| = N_2$

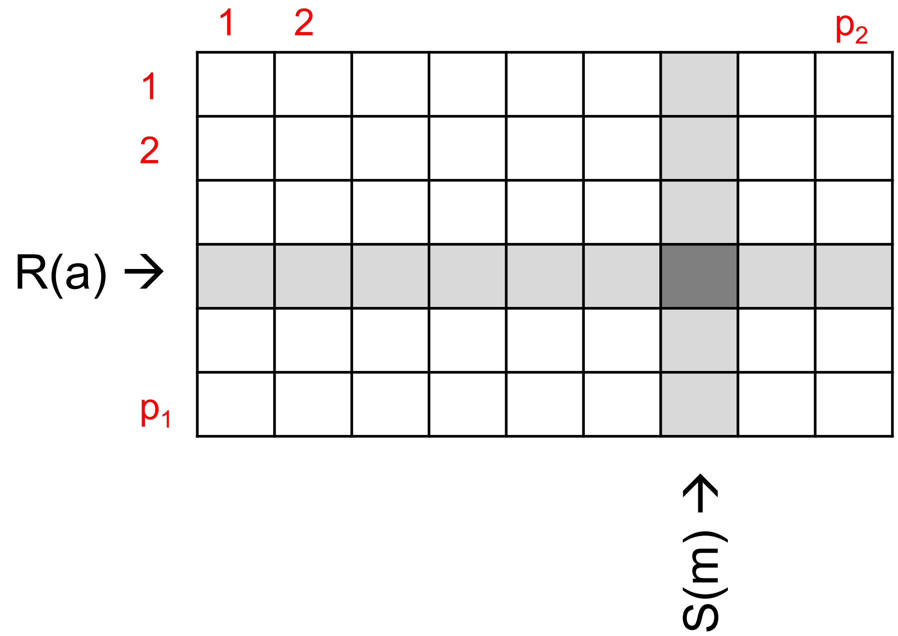
- Algorithm:
  - Choose shares such that $p = p_1 \cdot p_2$
  - Distribute $R(x)$ to row $h_1(x)$
  - Distribute $S(y)$ to column $h_2(y)$

Cartesian Product

$$|R| = N_1,\ |S| = N_2$$

R
x
a
b
c
d
e
f
g

S
Y
m
n
p
q



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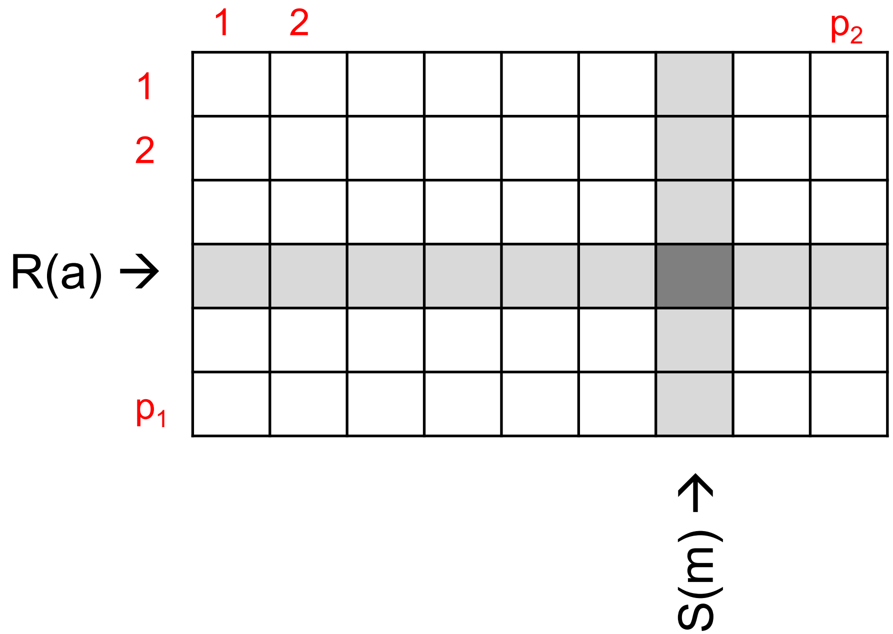

Problem: minimize $\mathrm{L} = \frac{N_1}{p_1} + \frac{N_2}{p_2}$ such that $p = p_1 \cdot p_2$

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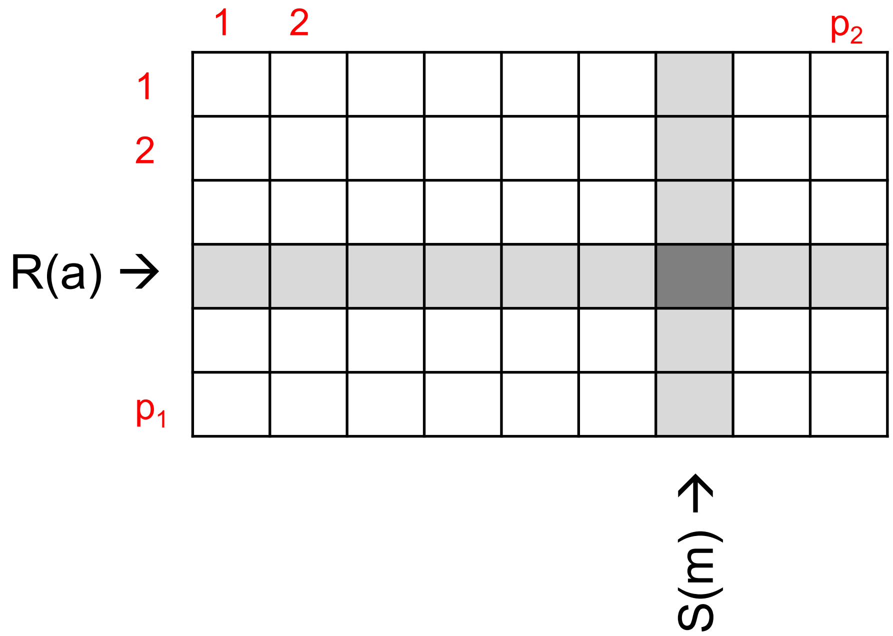

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Cartesian Product

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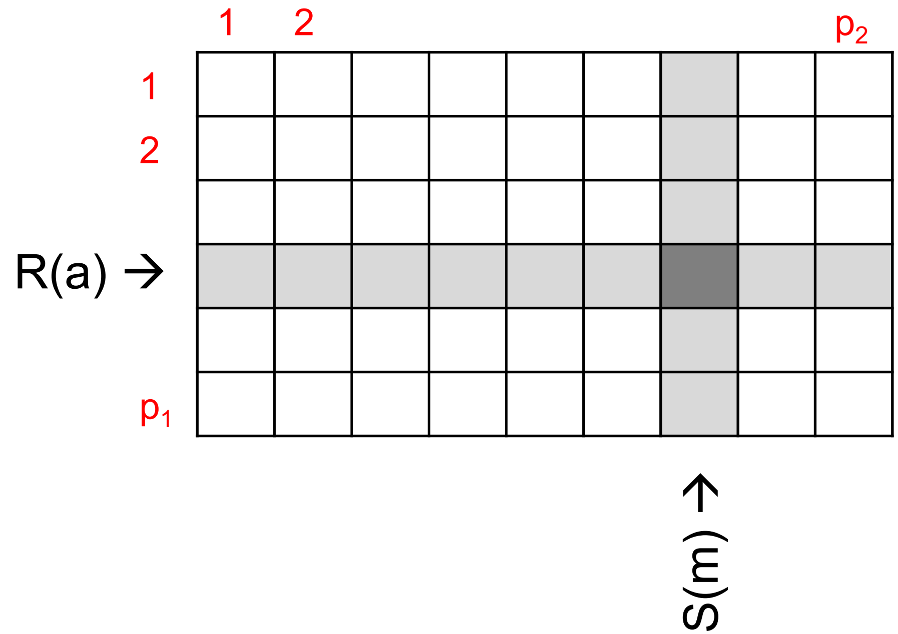
R	S
x	**Y**
a	m
b	n
c	p
d	q
e	
f	
g	



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THIS is the optimal load L_{opt}

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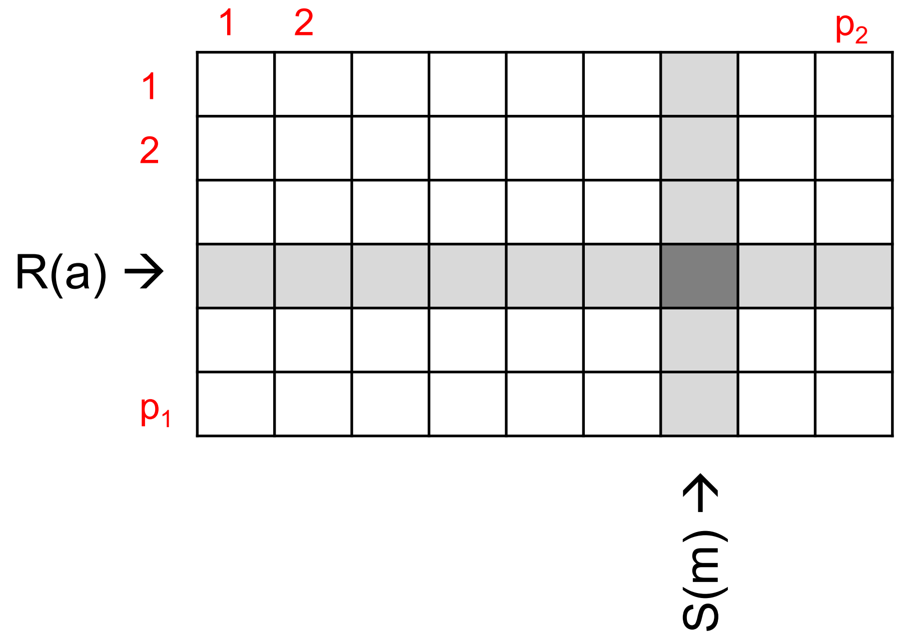

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THIS is the optimal load L_{opt}

From here we can compute the shares: $\frac{N_1}{p_1} = \sqrt{\frac{N_1 N_2}{p}}$ so $p_1 = \cdots$

Discussion

- Special case: when $N_1 = N_2 = N$ then:

$$L_{opt} = \frac{N}{\sqrt{p}} \quad \text{and} \quad p_1 = p_2 = \sqrt{p}$$

- ”Virtual servers” don’t work:
  - Let p=100, hence L_{opt}=N/10
  - Suppose we use $p_{virtual}$=40000: $L_{opt,virt}$=N/200
  - Each real server must simulate 400 virtual
  - Real load is L_{real}=N/200*400=2N ☹

- Reason: $\frac{N}{\sqrt{p}}$ means“sub-linear speedup”

General Cartesian Product

$Q = R_1 \times R_2 \times \cdots \times R_c$

- Assume: $|R_1| = N_1, \ldots, |R_c| = N_c$

Optimal load L_{opt}

Solution: $L = \frac{N_1}{p_1} + \cdots + \frac{N_c}{p_c} \geq c\left(\frac{N_1 \cdots N_c}{p_1 \cdots p_c}\right)^{\frac{1}{c}} = c\left(\frac{N_1 \cdots N_c}{p}\right)^{\frac{1}{c}}$

Edge Packing

$$Q(x_1, \dots, x_k) = R_1(vars_1) \wedge \cdots \wedge R_m(vars_m)$$

An *edge packing* is a subset of relations $R_{i_1}, R_{i_2}, \dots, R_{i_c}$ that do not share variables

Fact. For any edge packing of size c, the load of any 1-round algorithm is:

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Proof (in class)

By example, for $Q(x, y, z, u) = R(x, y) \wedge S(y, z) \wedge T(z, u) \wedge K(u, x)$

- Consider packing $R(x, y)$, $T(z, u)$. Claim: the algorithm *must* compute $R \times T$

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- Assume not; then two tuples $R(a, b), T(c, d)$ do not meet at any server.

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- The tuples $R(a, b), T(c, d)$ still do not meet (why?), hence algorithm is incorrect

Fractional Edge Packing

$$Q(x_1, \dots, x_k) = R_1(vars_1) \wedge \cdots \wedge R_m(vars_m)$$

A *fractional edge packing* is a set of weights $w_1, \dots, w_m$ such that, for every variable, the sum of weights that contain it is ≤ 1.

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Theorem. For any fractional edge packing, the load of any 1-round algorithm is:

$$L \geq \left(\frac{N_1^{w_1} \cdots N_m^{w_m}}{p}\right)^{\frac{1}{w_1 + \cdots + w_m}}$$

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Moreover, there exists shares for which the HyperCube algorithm has a load:

$$L_{opt} = O\left(\max_{w_1,\ldots,w_m} \left(\frac{N_1^{w_1} \cdots N_m^{w_m}}{p}\right)^{\frac{1}{w_1+\cdots+w_m}}\right)$$

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The formula gives us L_{opt} up to some small constant factor (which we ignore). Once you know L_{opt} you can usually compute the optimal shares for HyperCube.

Discussion

$$L_{opt} = O\left(\max_{w_1,\ldots,w_m}\left(\frac{N_1^{w_1}\cdots N_m^{w_m}}{p}\right)^{\frac{1}{w_1+\cdots+w_m}}\right)$$

- We want the *minimal load*, yet the formula above asks us to compute a *max*;
- The reason is that the formula is only a lower bound; it happens that the max has a matching algorithm (the proof is non-trivial)

Example: Join

$$Q(x, y, z) = R(x, y) \wedge S(y, z)$$

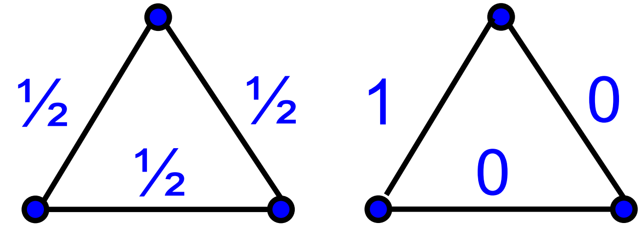
$$L = \left(\frac{N_1^{w_1} \cdot N_2^{w_2}}{p}\right)^{\frac{1}{w_1+w_2}}$$

- Fractional edge packing: 1,0: $L = \frac{N_1}{p}$
- Fractional edge packing: 0,1: $L = \frac{N_2}{p}$
- Assume $N_1 \geq N_2$. We obtain the shares:

$\frac{N_1}{p_1 p_2} = \frac{N_1}{p}$ and $\frac{N_2}{p_2 p_3} = \frac{N_1}{p}$ ➔ $p_1 = \frac{N_1}{N_2}$, $\mathrm{p}_2 = \mathrm{p}\frac{N_2}{N_1}$, $p_3 = 1$

Discuss connection to hash-, broadcast-join

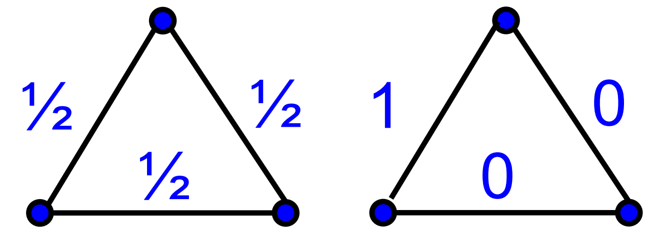
Example



$$Q(x,y,z) = R(x,y) \wedge S(y,z) \wedge T(z,x)$$

When $N_1 = N_2 = N_3 = N$, then the optimal load is $L_{opt} = O(N/p^{2/3})$
What if their sizes are different?

Example



$$Q(x,y,z) = R(x,y) \wedge S(y,z) \wedge T(z,x)$$

When $N_1 = N_2 = N_3 = N$, then the optimal load is $L_{opt} = O(N/p^{2/3})$
What if their sizes are different?

Fractional edge packing w_1, w_2, w_3	$\left(\frac{N_1^{w_1} \cdot N_2^{w_2} \cdot N_3^{w_3}}{p}\right)^{\frac{1}{w_1+w_2+w_3}}$
½, ½, ½	
1,0,0	
0,1,0	
0,0,1	
0,0,0	

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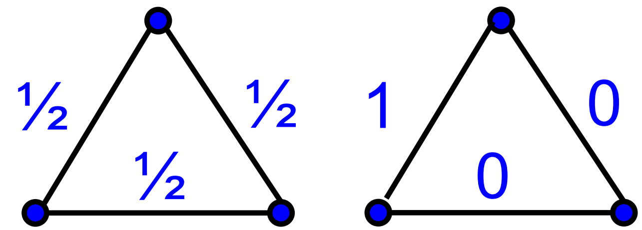

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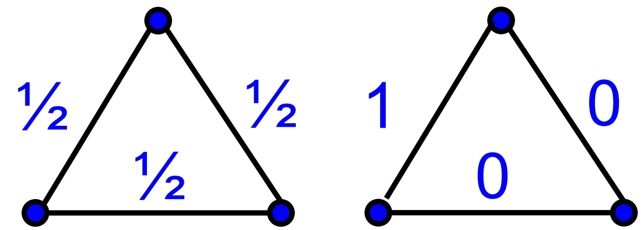

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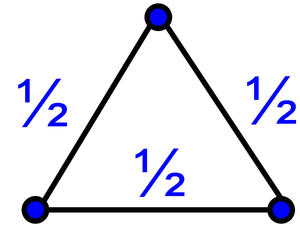

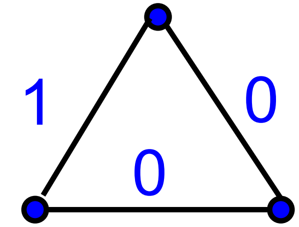

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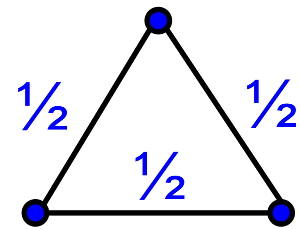

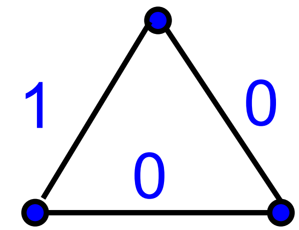

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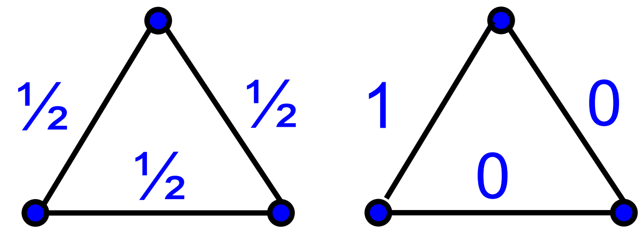

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Optimal load L_{opt} is the *maximum* of this column

Example (cont'd)

$Q(x, y, z) = R(x, y) \wedge S(y, z) \wedge T(z, x)$

Need max of $\frac{(N_1 \cdot N_2 \cdot N_3)^{1/3}}{p^{2/3}}, \frac{N_1}{p}, \frac{N_2}{p}, \frac{N_3}{p}$

Suppose w.l.o.g. $N_1 \geq N_2 \geq N_3$

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- Case 1: $\frac{(N_1 \cdot N_2 \cdot N_3)^{1/3}}{p^{2/3}} \leq \frac{N_1}{p} = L_{opt}$

  The share of z is $p_3 = 1$, hence
  "cartesian product $S \times T$, distribute R"

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Proof: Load due to R: $\frac{N_1}{p_1 p_2} = L_{opt}$, i.e. $\frac{N_1 p_3}{p} = \frac{N_1}{p}$

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When $p \leq \frac{N_1^2}{N_2 N_3}$ then Case 1, linear speedup; otherwise case 2, sublinear

Final Special Case

- When all cardinalities are equal, then:

$$\left(\frac{N^{w_1} \dots N^{w_m}}{p}\right)^{\frac{1}{w_1+\dots+w_m}} = \frac{N}{p^{\frac{1}{w_1+\dots+w_m}}}$$

- For a graph G, the quantity

$$\tau^* = \max_{frac\ edge\ packing} (w_1 + \cdots w_m)$$

is called the <u>*fractional edge packing number*</u>

- $L_{opt} = \frac{N}{p^{\frac{1}{\tau^*}}}$

Conclusions

- The HyperCube algorithms combines two strategies: hash-partition, and broadcast
- When p is small, then it can broadcast the smaller relations;
- As p increases, “smaller” relations no longer help, and the load gets closer to the fractional edge covering number τ^*